PB84-122936

Evaluation of Low Gravity Dense Media Cyclone Performance in Cleaning Fine Coal

Bituminous Coal Research, Inc., Monroeville, PA

Prepared for

Industrial Environmental Research Lab.
Research Triangle Park, NC

Nov 83

U.S. Department of Commerce
National Technical Information Service
NTIS

PB84-122936

EPA-600/7-83-057
November 1983

[redacted]

EVALUATION OF LOW GRAVITY DENSE MEDIA CYCLONE PERFORMANCE IN CLEANING FINE COAL

[redacted]

by

John L. Zalar
[redacted]

Bituminous Coal Research, Inc.
350 Hochberg Road, P.O. Box 278
Monroeville, PA 15146

Contract No. 68-02-3124

EPA Project Officer:

James D. Kilgroe
Industrial Environmental Research Laboratory
Research Triangle Park, NC 27711

INDUSTRIAL ENVIRONMENTAL RESEARCH LABORATORY
OFFICE OF RESEARCH AND DEVELOPMENT
U.S. ENVIRONMENTAL PROTECTION AGENCY
RESEARCH TRIANGLE PARK, NC 27711

REPRODUCED BY
NATIONAL TECHNICAL
INFORMATION SERVICE
U.S. DEPARTMENT OF COMMERCE
SPRINGFIELD, VA. 22161

TECHNICAL REPORT DATA

(Please read Instructions on the reverse before completing)

1. REPORT NO. EPA-600/7-83-057	2.	3. RECIPIENT'S ACCESSION NO. PB84 122936
4. TITLE AND SUBTITLE Evaluation of Low-gravity Dense-medium Cyclone Performance in Cleaning Fine Coal		5. REPORT DATE November 1983
		6. PERFORMING ORGANIZATION CODE
7. AUTHOR(S) John L. Zalar		8. PERFORMING ORGANIZATION REPORT NO.
9. PERFORMING ORGANIZATION NAME AND ADDRESS Bituminous Coal Research, Inc. 350 Hochberg Road (P.O. Box 278) Monroeville, Pennsylvania 15146		10. PROGRAM ELEMENT NO.
		11. CONTRACT/GRANT NO. 68-02-3124
12. SPONSORING AGENCY NAME AND ADDRESS EPA, Office of Research and Development Industrial Environmental Research Laboratory Research Triangle Park, NC 27711		13. TYPE OF REPORT AND PERIOD COVERED Final; 3/82 - 7/83
		14. SPONSORING AGENCY CODE EPA/600/13

15. SUPPLEMENTARY NOTES IERL-RTP project officer is James D. Kilgroe, Mail Drop 4, 919/541-2854.

16. ABSTRACT The report gives results of 36 pilot plant tests, conducted to evaluate the performance of dense-medium cyclones in cleaning fine coal (9x100 mesh) at low specific gravity (1.3). Test variables included two orifice sizes, three medium-to-coal ratios, and six flow rates. The clean coal and refuse from these tests were separated into three size fractions and float/sink tested at eight specific gravities. Performance criteria used in these evaluations included: four dependent criteria (recovery efficiency, misplaced material, yield error, and ash error), three independent criteria (probable error, error area, and imperfection), and three sulfur-based criteria (percent sulfur in coal, percent sulfur reduction, and percent reduction in pounds per million Btu). Final results of the statistical analysis were regression equations relating each performance criterion to particle size, orifice size, flow rate, and medium-to-coal ratio. Particle size was statistically related to cyclone performance for all criteria examined, except weight recovery, efficiency, and percent reduction in pounds per million Btu. Generally, the best performance occurred at the 9x14 mesh size range and deteriorated as particle size decreased. Cyclone performance improved with increasing flow rate up to 120 gpm, then decreased beyond this value.

17. KEY WORDS AND DOCUMENT ANALYSIS

a. DESCRIPTORS	b. IDENTIFIERS/OPEN ENDED TERMS	c. COSATI Field/Group
Pollution	Pollution Control	13B
Coal Preparation	Stationary Sources	08I
Coal		08G
Cyclone Separators		07A, 13I
Tests		14B
Particle Size		14G

18. DISTRIBUTION STATEMENT Release to Public	19. SECURITY CLASS *(This Report)* Unclassified	21. NO. OF PAGES 254
	20. SECURITY CLASS *(This page)* Unclassified	22. PRICE

EPA Form 2220-1 (9-73)

NOTICE

This document has been reviewed in accordance with U.S. Environmental Protection Agency policy and approved for publication. Mention of trade names or commercial products does not constitute endorsement or recommendation for use.

ABSTRACT

The coal industry commonly uses a dense medium composed of finely ground magnetite and water to separate coal and mineral particles in cyclones called dense medium cyclones. Thirty-six pilot plant tests were conducted to evaluate the performance of dense medium cyclones in cleaning fine coal (9x100 mesh)* at a low specific gravity of separation (1.3). Test variables included two orifice sizes, three medium-to-coal weight ratios, and six flow rates. The clean coal (overflow) and refuse (underflow) from these tests were separated into three size fractions (9x14, 14x28, and 28x100 mesh) and float/sink tested at eight specific gravities. This report gives results of statistical analyses which were performed to evaluate these tests. Performance criteria used in these evaluations included: four dependent criteria (recovery efficiency, misplaced material, yield error, and ash error); three independent criteria (probable error, error area, and imperfection); and three sulfur-based criteria (percent sulfur in coal, percent sulfur reduction, and percent reduction in pounds per million Btu). Final results of the statistical analysis were regression equations relating each performance criterion to particle size, orifice size, flow rate, and medium-to-coal ratio. Particle size was statistically related to cyclone performance for all criteria examined, except weight recovery, efficiency, and percent reduction in pounds per million Btu. Generally, the best performance occurred at the 9x14 mesh size range and deteriorated as particle size decreased. Cyclone performance improved with increasing flow rate up to 120 gpm, then decreased beyond this value. Cyclone performance, as measured by the dependent criteria, improved slightly when a 1.5 in. orifice was used. Percent coal in slurry was a statistically significant predictor of all measures of performance criteria, except weight recovery efficiency. In general, best performance occurred at a nominal medium-to-coal ratio of 7:1 and deteriorated as the percentage of coal in the slurry increased. The cyclone operating parameters were, in general, more highly correlated with the independent measures of cyclone performance than with dependent or sulfur-based criteria.

---

*Readers more familiar with metric units may use the conversion figures at the back of this Summary.

TABLE OF CONTENTS

Page

I. INTRODUCTION.. 1

II. CONCLUSIONS, LIMITATIONS, AND RECOMMENDATIONS........................ 3

A. Introduction... 3

B. Conclusions.. 3

C. Limitations.. 6

D. Recommendations for Future Work.......................... 7

III. TEST CONDITIONS, EQUIPMENT AND PROCEDURES........................ 8

A. Test Conditions.. 8

B. Test Equipment and Procedures............................ 8

IV. PERFORMANCE CRITERIA... 14

A. Dependent Criteria....................................... 14

1. Recovery Efficiency.................................. 14

2. Misplaced Material................................... 14

3. Ash Error.. 15

4. Yield Error.. 15

B. Independent Criteria..................................... 15

1. Distribution Curve................................... 15

2. Specific Gravity of Separation....................... 15

3. Probable Error....................................... 16

4. Error Area.. 16

5. Imperfection.. 18

C. Sulfur-Based Criteria... 18

1. Percent Sulfur in Clean Coal.............................. 18

2. Percent Sulfur Reduction.................................. 18

3. Normalized Percent Sulfur Reduction....................... 18

V. ANALYSIS METHODS.. 19

A. Introduction.. 19

B. Data Base... 19

C. Distribution Curve Generation................................ 19

D. Statistical Evaluations...................................... 20

VI. RESULTS.. 22

A. Introduction.. 22

B. Statistical Evaluation of Dependent Performance Criteria....... 25

C. Statistical Evaluation of Independent Performance Criteria..... 25

D. Statistical Evaluation of Sulfur-Based Performance Criteria.... 34

VII. REFERENCES AND BIBLIOGRAPHY..................................... 48

A. References.. 48

B. Bibliography.. 49

CONVERSION FACTORS... 51

APPENDIX A - DATA SUMMARIES AND DISTRIBUTION CURVES.................. A-1

APPENDIX B - ESTIMATED BTU AND NORMALIZED SULFUR CONTENT VALUES.......... B-1

APPENDIX C - CUBIC SPLINE METHODS.................................... C-1

APPENDIX D - MULTIPLE REGRESSION ANALYSIS............................ D-1

LIST OF TABLES

Table Page

1 Dense Medium Cyclone Operating Conditions, By Test Number...... 10

2 Means, Standard Deviations, and Correlations for Cyclone Performance Criteria.......... 23

3A Results of Regression Analysis for Weight Recovery Efficiency.. 26

3B Predicted and Observed Values of Weight Recovery Efficiency by Particle Size Range.......... 27

4A Results of Regression Analysis for Misplaced Material.......... 28

4B Predicted and Observed Values of Misplaced Material by Particle Size Range.......... 29

5A Results of Regression Analysis for Ash Error.......... 30

5B Predicted and Observed Values of Ash Error by Particle Size Range.......... 31

6A Results of Regression Analysis for Yield Error.......... 32

6B Predicted and Observed Values of Yield Error by Particle Size Range.......... 33

7A Results of Regression Analysis for Error Area.......... 35

7B Predicted and Observed Values of Error Area by Particle Size Range.......... 36

8A Results of Regression Analysis for Probable Error.......... 37

8B Predicted and Observed Values of Probable Error by Particle Size Range.......... 38

9A Results of Regression Analysis for Imperfection.......... 39

9B Predicted and Observed Values of Imperfection by Particle Size Range.......... 40

10A Results of Regression Analysis for Percent Sulfur in Clean Coal.......... 41

10B Predicted and Observed Values of Percent Sulfur in Clean Coal by Particle Size Range.......... 42

11A Results of Regression Analysis for Percent Sulfur Reduction.... 43

LIST OF TABLES (Continued)

Table | Page

11B Predicted and Observed Values of Percent Sulfur Reduction by Particle Size Range.. 44

12A Results of Regression Analysis for Normalized Percent Sulfur Reduction.. 45

12B Predicted and Observed Values of Normalized Percent Sulfur Reduction by Particle Size Range........................ 46

LIST OF FIGURES

Figure | Page

1 Test Matrix of Cyclone Operating Conditions.................... 9

2 Dense Medium Cyclone.. 11

3 Cyclone Flow Pattern.. 11

4 DOE Dense Medium Cyclone Pilot Plant........................ 12

5 Distribution Curve Showing Specific Gravity of Separation...... 16

6 Probable Error.. 17

7 Error Area.. 17

I. INTRODUCTION

The Multi-Stream Coal Cleaning Strategy (MCCS) is being tested by the Pennsylvania Electric Company (PENELEC) and the New York State Electric & Gas Corporation (NYSEG) to control emissions of sulfur oxides from coal-fired boilers at the Homer City, Pennsylvania, Generating Station. The MCCS is based on physical coal cleaning using dense medium cyclones in which mineral impurities including pyritic sulfur are removed from raw coal. Dense medium cyclones are used because they are very efficient in separating organic coal particles from the more dense mineral particles. The apparent density of the medium is controlled by mixing finely ground magnetite with water.

In a dense medium cyclone, centrifugal force is used to separate particles of different specific gravities. The specific gravity of organic coal particles may range from 1.3 to 1.9 while the specific gravity of coal mineral particles may range from 2.2 to 5.0.

To remove most of the mineral contaminants from coal it is necessary to crush the raw coal to fine particle sizes and separate these particles at a low specific gravity. Prior to this project there is little data on the performance of dense medium cyclones operating on fine size coal particles at low specific gravity.

To provide additional data on dense medium cyclone performance a cooperative pilot plant test program was conducted by the Department of Energy, the U.S. Environmental

Protection Agency and the Pennsylvania Electric Company. Thirty six tests were conducted in the Department of Energy's dense medium pilot plant at the Pittsburgh Energy Technology Center. These tests are documented in DOE/PECTC/TR-83/1 (DE83009250), Dense-Medium Cycloning of Fine Coal at Low Specific Gravities.(1)

All of the dense medium tests were run with an 8-in diameter dense medium cyclone operated at a nominal 1.3 s.g. with 9x100 mesh coal. The operating variables included two orifice sizes (1-1/2 and 1-3/4 in); three medium-to-coal ratios (7:1, 5:1 and 3:1) and six flow rates (92, 100, 110, 120, 140 and 160 gpm). The overflow (clean coal) and underflow (refuse) from each test was divided into three size fractions (9x14 mesh, 14x28 mesh and 28x100 mesh) and float sink tested at 8 specific gravities. Each size and specific gravity increment was analyzed for moisture, ash and total sulfur.

The Bituminous Coal Research Inc. (BCR) was awarded a contract to perform statistical evaluations of the thirty-six dense medium pilot plant tests. Performance criteria used in these evaluations included: 4 dependent criteria (recovery efficiency, misplaced material, yield error, and ash error); 3 independent criteria (probable error, error area and imperfection); and three sulfur based criteria (percent sulfur in coal, percent sulfur reduction, normalized percent sulfur reduction*).

This report documents the results of these statistical analyses.

---

*Percent reduction in lbs/10^6Btu.

II. CONCLUSIONS, LIMITATIONS, AND RECOMMENDATIONS

A. Introduction

The mathematical and statistical evaluations of dense medium cyclones conducted during this project have yielded several important conclusions regarding the relationships between cyclone operating parameters and the various performance criteria examined. These conclusions are presented and discussed in this section. In order that they may serve as operating guidelines, all conclusions are stated in terms of the operating parameters as they relate to measured cyclone performance. The limitations of the stated conclusions are discussed and recommendations are made for further work in evaluating cyclone performance.

B. Conclusions

1. Dense medium cyclone performance improves as particle size of the feed coal increases.

Particle size is found to be statistically related to cyclone performance for all criteria examined, except weight recovery efficiency and normalized percent sulfur reduction. Generally, best performance occurs for the 9 x 14 size range and deteriorates as particle size decreases. Only in the case of percent sulfur reduction is this tendency reversed; i.e., slightly higher percent reductions occur as particle size decreases. When percent sulfur reduction is normalized for Btu value, this particle size effect is not observed.

2. Cyclone performance, as measured by the dependent criteria and percent sulfur reduction, is statistically related to flow rate level.

Flow rate is a statistically significant predictor of cyclone performance as measured by the four dependent criteria and percent sulfur reduction. In all cases, this relationship is best characterized using a second-order polynomial expression for flow rate. This is due to the fact that cyclone performance improves with increasing flow rate up to a value of 120 gpm, then deteriorates as the flow rate increases beyond this value. Based upon the flow-rate levels examined in the test matrix and the statistical models derived in this investigation, 120 gpm appears to produce overall best results for the criteria mentioned above. It is possible, however, that the true optimal flow rate is slightly higher or lower than 120 gpm.

3. Cyclone performance, as measured by the dependent criteria, is related to the size of the inlet orifice.

Inlet-orifice size is found to be statistically related to the dependent measures of cyclone performance. In all cases, performance is slightly improved when a 1.5-inch orifice is used.

4. Dense medium cyclone performance improves as the percent coal in slurry decreases.

Percent coal in slurry is a statistically significant predictor of all measures of cyclone performance examined, except weight recovery efficiency. In general, best performance occurs at a nominal medium-to-coal ratio of 7:1 and deteriorates as the percentage of coal in slurry increases. This tendency is most pronounced for independent measures of cyclone performance.

5. The cyclone operating parameters investigated are, in general, more highly correlated with independent measures of cyclone performance than with dependent or sulfur-based criteria.

A comparison of the results obtained for the various performance criterthat a major portion of the variability (over 80 percent) in the independent measures can be accounted for through a knowledge of the operating parameters being considered. In contrast, among the dependent and sulfur-based measures, variability explained by the operating parameters generally ranges from about 25 to 70 percent. (In the case of sulfur in clean coal, less than 10 percent of the variability is explained; however, this is a poor measure of sulfur performance, as explained in Section VI of this report.)

These results are not surprising when one considers that error area, probable error, and imperfection are essentially independent of the washability characteristics of the raw coal. As such, these criteria are more characteristic of the performance of the coal-cleaning unit itself. However, some important and consistent relationships between operating parameters and dependent criteria have emerged from this evaluation. The observed correlations between cyclone operating parameters and the performance measures do indicate particular operating conditions under which cyclone performance is likely to be improved.

6. For the test matrix under investigation, cyclone operating conditions corresponding to Test No. 84 appear to produce best overall cyclone performance.

The results of all statistical analyses performed in this investigation suggest that there is one area of the test matrix that is optimal or very near optimal for all cyclone performance criteria examined. The operating conditions of Test No. 84; i.e., 1.5-inch orifice size, 120 gpm flow rate, and 7:1 medium-to-coal ratio; seem to produce improved cyclone performance as compared to other cyclone operating configurations. While true optimal performance may not occur exactly at the conditions of Test No. 84, generally this is the best set of operating conditions in the test matrix under consideration.

C. Limitations

1. The results and conclusions of this evaluation are strictly applicable to 8-inch cyclones. Generalizations of these results to larger or smaller cyclones should be made with caution. Values of specific operating parameters used in this investigation, such as inlet-orifice sizes and flow rates, may not be appropriate for larger or smaller cyclones.

2. Extrapolations and interpolations of the relationships and prediction equations derived in this evaluation should be made with caution. The results of this analysis are not necessarily applicable beyond the ranges of values for the operating variables that were used in this investigation. Further, interpolation of these results is limited by the number of levels of each of the operating parameters actually incorporated in the test matrix, particularly inlet-orifice size, which had only two possible values.

3. The results and conclusions of this evaluation, pertaining to dependent and sulfur-based criteria, are not necessarily generalizable to other coal types having different washability characteristics. The independent performance criteria, however, should be applicable to other coal types.

4. The actual performance curves and performance criteria derived from these curves may be somewhat different from those derived using other curve fitting procedures, e.g., hand drawn or other curve fitting routines.

D. Recommendations for Future Work

Verification of the relationships and prediction equations derived from this analysis should be carried out through additional cyclone testing. These tests should include other levels of the operating parameters, such as additional flow rates and inlet-orifice sizes. Also, additional cyclone operating parameters should be examined through matrix testing. Emphasis should be placed on improving correlations for the dependent performance criteria and, in particular, the sulfur-based measurers.

The results of the present investigation should be extended to larger cyclones. Finally, the results should be verified for all coal types to be processed.

III. TEST CONDITIONS, EQUIPMENT AND PROCEDURES

A. Test Conditions

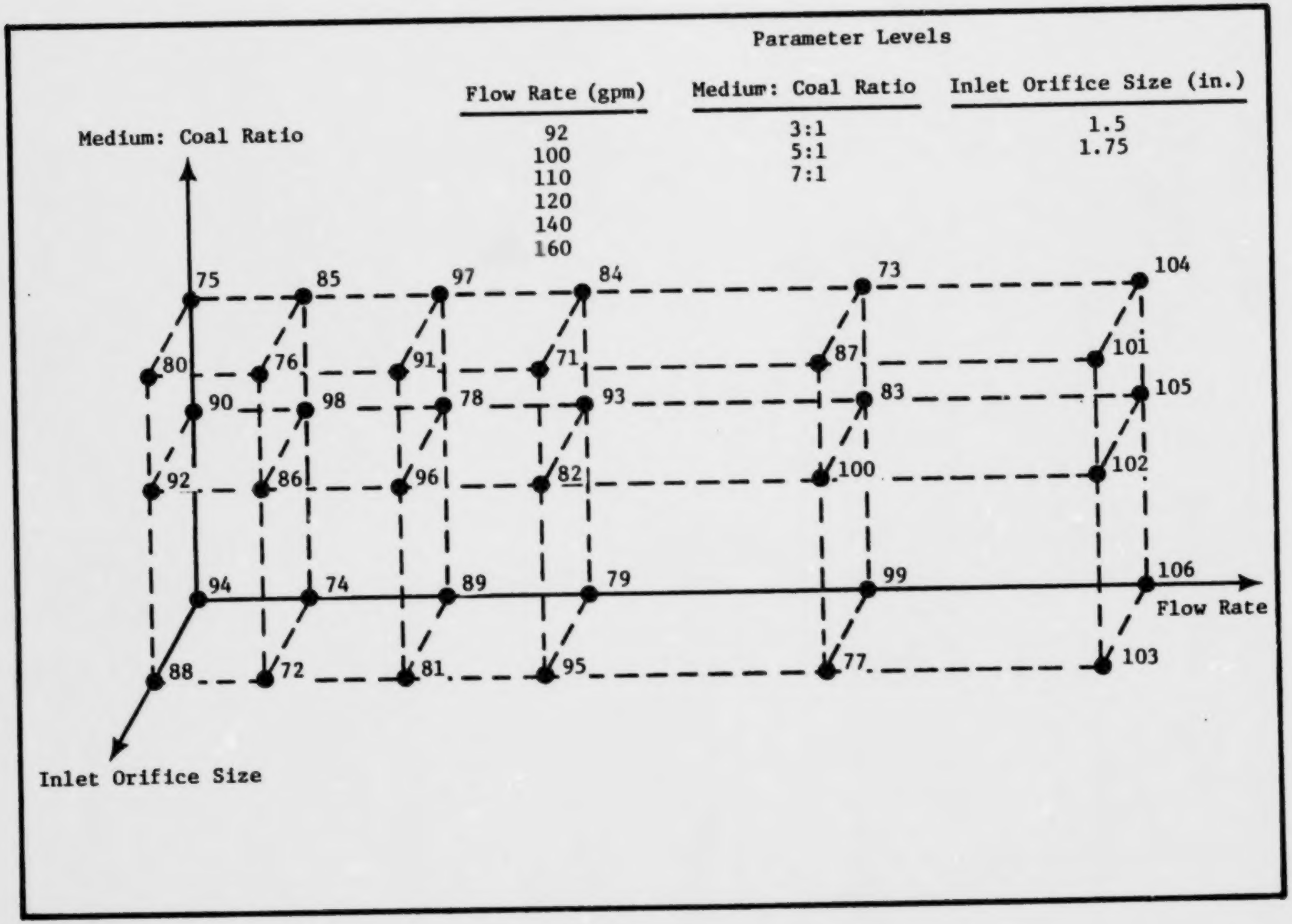
Thirty-six tests, using an 8-inch dense medium cyclone, were conducted at the pilot plant of the U.S. Department of Energy, in Bruceton, Pennsylvania. The testing provided performance data for the cyclone operating under various conditions on coal particle mesh sizes of 9x14, 14x28, 28x100, and 9x100. The operating variables investigated included two inlet orifice sizes (1-1/2 and 1-3/4 in.); three medium-to-coal ratios (7:1, 5:1, and 3:1); and six flow rates (92, 100, 110, 120, 140, and 160 gpm). The test matrix of cyclone operating conditions is depicted in Figure 1. Operating conditions for each of the dense medium cyclone tests are given in Table 1.

B. Test Equipment and Procedures

The typical dense medium cyclone is illustrated in Figure 2. The idealized flow of medium through the cyclone is shown in Figure 3. For further information on the theory of operation of the dense medium cyclone, see "Coal Preparation," J.W. Leonard and D.R. Mitchell, Ed.(2)

The pilot plant circuit contained an 8-in. diameter Heyl & Patterson cyclone with a 14° included angle, a full-stream slurry sampler, a 3-in. x 2-in. centrifugal pump with a variable speed drive, a nuclear density gauge, a magnetic flowmeter, and a full-stream pressure sensor and gauge.(1) A schematic of the dense medium pilot plant is shown in Figure 4.

The circuit was a closed-loop, batch system that recirculated the slurry. Each test began with 60 gallons of water mixed with enough magnetite to produce the desired medium density, as indicated by the density gauge. The flow rate was set using a flowmeter and variable speed pump. The inlet pressure was read from



Parameter Levels

Flow Rate (gpm)	Medium: Coal Ratio	Inlet Orifice Size (in.)
92	3:1	1.5
100	5:1	1.75
110	7:1	
120		
140		
160		

Figure 1. Test Matrix of Cyclone Operating Conditions

TABLE 1. DENSE MEDIUM CYCLONE OPERATING CONDITIONS, BY TEST NUMBER

Test No.	Inlet Orifice in.	Inlet Pressure psi	Flow Rate gpm	Feed Coal tph	Medium-to-coal Ratio	Coal in Slurry, wt %
71	1.75	7.0	120	4.2	7:1	11.2
72	1.75	5.0	100	7.5	3:1	23.3
73	1.50	10.0	140	4.7	7:1	10.9
74	1.50	5.5	100	7.3	3:1	22.7
75	1.50	4.5	92	3.6	7:1	12.3
76	1.75	5.0	100	3.8	7:1	12.0
77	1.75	9.0	140	10.8	3:1	24.1
78	1.50	6.5	110	5.0	5:1	14.6
79	1.50	7.5	120	8.9	3:1	23.3
80	1.75	4.0	92	3.7	7:1	12.8
81	1.75	5.5	110	8.4	3:1	24.0
82	1.75	6.5	120	6.1	5:1	16.3
83	1.50	10.0	140	7.5	5:1	17.3
84	1.50	8.0	120	4.5	7:1	12.1
85	1.50	6.0	100	3.9	7:1	12.5
86	1.75	5.0	100	5.2	5:1	16.5
87	1.75	8.5	140	5.2	7:1	12.2
88	1.75	5.0	92	6.6	3:1	22.7
89	1.50	6.5	110	8.4	3:1	24.4
90	1.50	5.0	92	4.9	5:1	17.2
91	1.75	6.0	110	4.1	7:1	12.1
92	1.75	4.5	92	4.2	5:1	14.6
93	1.50	7.0	120	4.6	5:1	15.2
94	1.50	5.0	92	7.0	3:1	24.0
95	1.75	6.5	120	9.6	3:1	25.5
96	1.75	6.5	110	4.8	5:1	14.2
97	1.50	6.5	110	4.2	7:1	12.5
98	1.50	6.0	100	4.8	5:1	15.5
99	1.50	10.0	140	11.1	3:1	25.2
100	1.75	8.5	140	6.6	5:1	15.3
101	1.75	11.0	160	6.5	7:1	13.3
102	1.75	11.0	160	7.3	5:1	15.0
103	1.75	11.0	160	12.5	3:1	24.8
104	1.50	12.0	160	6.1	7:1	12.3
105	1.50	12.0	160	8.1	5:1	16.5
106	1.50	12.0	160	12.9	3:1	25.6

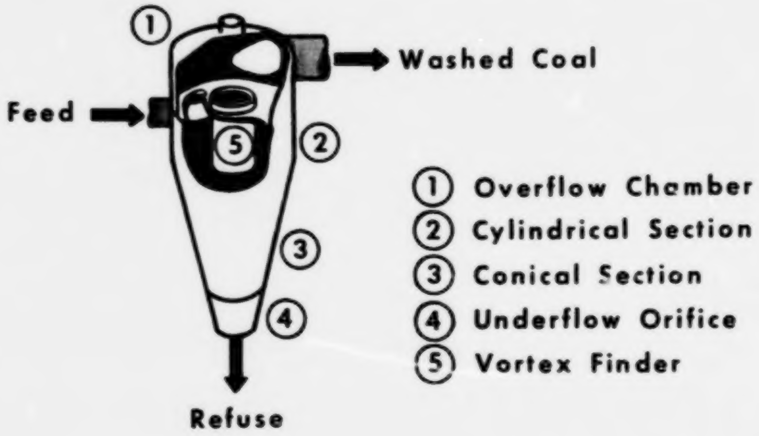


Figure 2. Dense Medium Cyclone

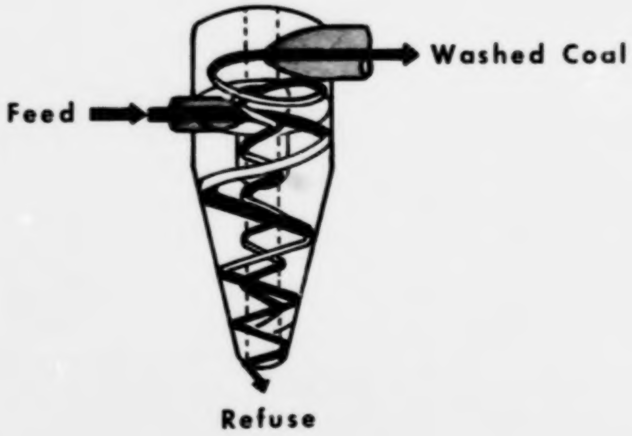


Figure 3. Cyclone Flow Pattern

BCR 6015G1

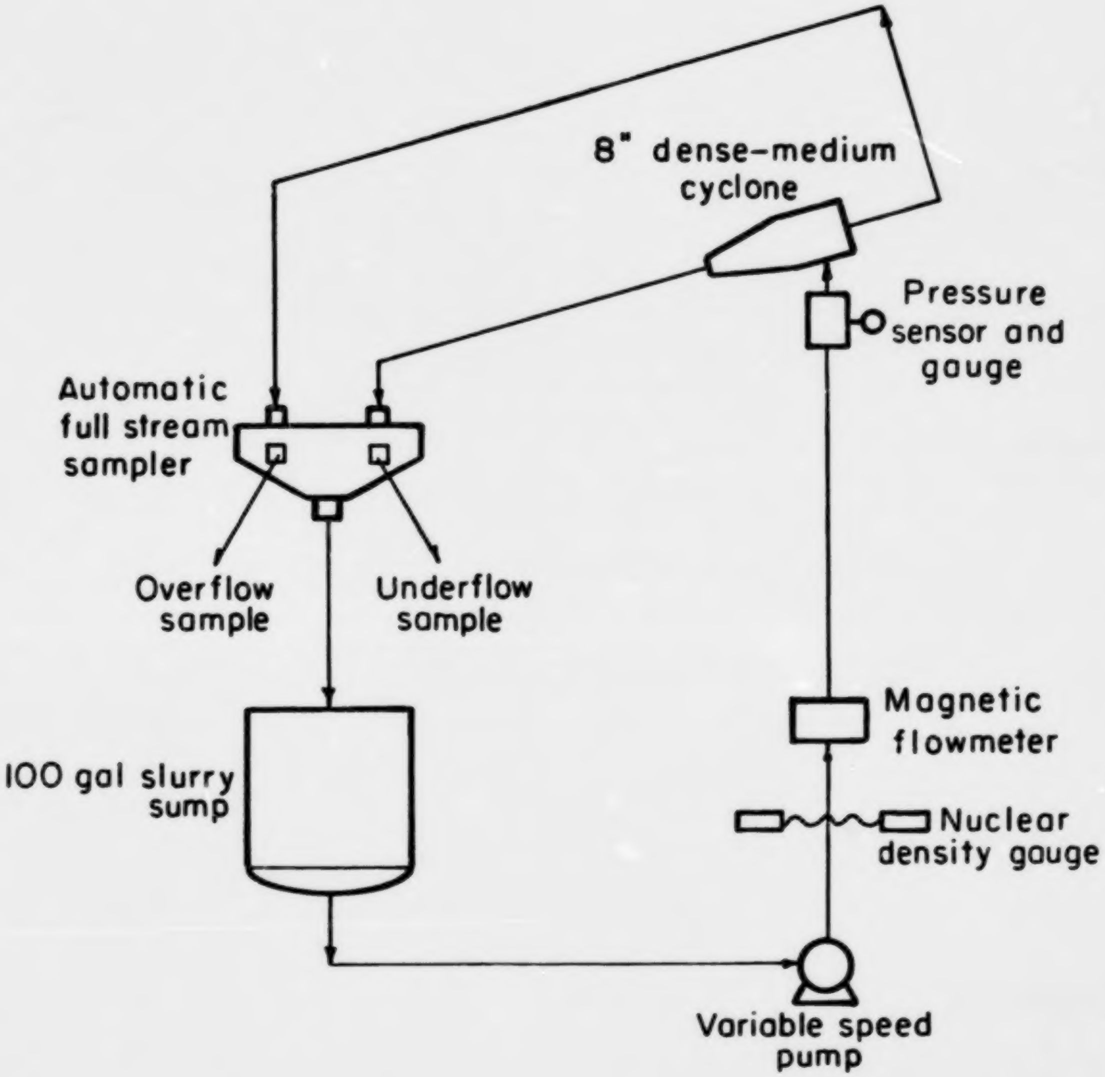


Figure 4. -DOE dense-medium cyclone pilot plant.

the pressure sensor near the inlet. Once steady state was obtained, full-stream samples of the recirculating medium were taken to measure the medium distribution between the overflow and the underflow.

At this point, a measured weight of 9x100 mesh coal (100 to 200 lb) based on the recirculating medium density was added to obtain the desired medium-to-coal ratio. The coal slurry was recirculated for only about two minutes, in order to minimize degradation while still allowing time for good mixing. Between 8 and 16 consecutive full-stream cuts through the product streams were then taken, depending on the flow rate and concentration. The sampler was designed to take identical, simultaneous cuts of the clean coal (overflow) and refuse (underflow) streams so that the yield could be based on the weights of the samples. About ten pounds of each product (clean coal and refuse solids) were necessary for subsequent washability analysis.

The weight and volume of slurry samples were measured and the samples were wet-screened to remove magnetite. The products were dried and then checked with a hand magnet to remove any magnetite before the final weighing.

Each whole sample was screened into 9x14 mesh, 14x28 mesh, 28x100 mesh, and 100 mesh x 0 size fractions (Tyler screens). The three coarsest fractions were float-sink tested in organic heavy liquids at specific gravities of 1.27, 1.29, 1.32, 1.35, 1.40, 1.50, 1.60, and 1.80. Each float-sink increment was analyzed for moisture, ash, and total sulfur.

IV. PERFORMANCE CRITERIA

Several criteria are traditionally used to evaluate performance of coal-cleaning equipment. This section provides definitions of the performance criteria used in this report. A more detailed discussion of performance criteria can be found in the publications by Geer and Yancey.(3,4,5) The various performance criteria are usually classified according to the degree to which they are dependent on the characteristics of the particular coal being cleaned. Some are dependent, either directly or indirectly, and some are, under certain circumstances, essentially independent.

A. Dependent Criteria

Performance criteria that depend on the characteristics of the coal being cleaned are called dependent criteria. The dependent criteria that have been defined for this study include recovery efficiency, misplaced material, ash error, and yield error.

1. Recovery Efficiency

Recovery efficiency is defined as the ratio, expressed in percentage, of the yield of cleaned coal to the yield of float coal of the same ash content shown, by the specific gravity analysis, to be present in the feed.

2. Misplaced Material

Total misplaced material is that percentage of the feed which reports to the wrong product. It is the sum of the sink material in the cleaned coal and the float material in the refuse, expressed as a percentage of the raw coal.

3. Ash Error

Ash error is the numerical difference between the actual and theoretical ash content of cleaned coal at the yield of cleaned coal obtained.

4. Yield Error

Yield error is the difference between the yield of coal actually obtained and the theoretical yield at the ash content of the cleaned coal.

B. Independent Criteria

The methods of evaluating performance that are characteristic of the coal-cleaning unit and are substantially independent of the specific-gravity composition of the raw coal are referred to as the sharpness-of-separation criteria and include probable error, error area, and imperfection.

1. Distribution Curve

The distribution curve, illustrated in Figure 5, is a plot of the percentage of each specific-gravity fraction of the feed recovered in the clean-coal product as the ordinate, or y-value, against the median of the specific-gravity fraction as the abscissa, or x-value.(6) The distribution curve is a measure for evaluating the sharpness with which a coal-cleaning unit separates coal from its impurities. It provides information that is related to the inherent characteristics of the coal-cleaning unit.

2. Specific Gravity of Separation

The value of the specific gravity corresponding to the weight-percent 50, in Figure 5, is defined as the specific gravity of separation, that is, the specific gravity of the material in the raw feed that is divided equally between clean coal and refuse.

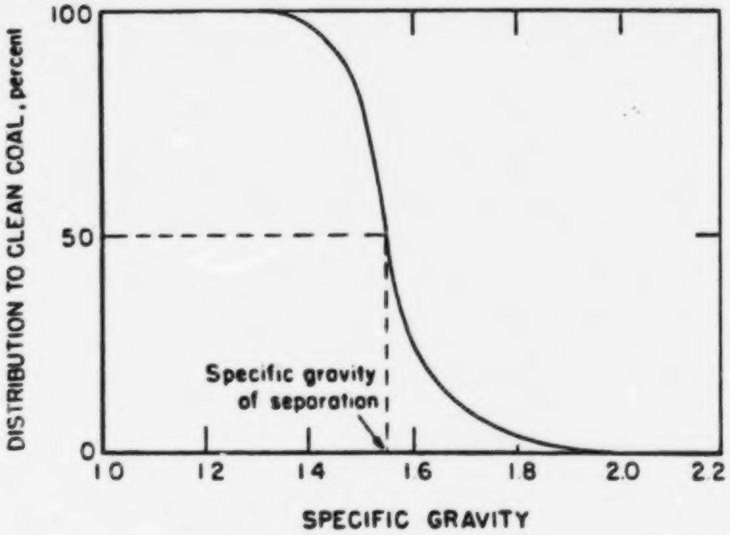


Figure 5. Distribution Curve Showing Specific Gravity of Separation(6)

3. Probable Error

Probable error, shown in Figure 6, is measured directly from the distribution curve. It equals one-half the specific gravity difference between the 25- and 75-percent ordinate values on the curve. Thus, probable error evaluates only the portion of the curve in the range of intermediate density.

4. Error Area

Error area, shown in Figure 7, is defined as the area between the actual distribution curve and the theoretically perfect distribution curve. Unlike probable error, error area takes into account the entire distribution curve. For a theoretically perfect separation, the error area is zero.

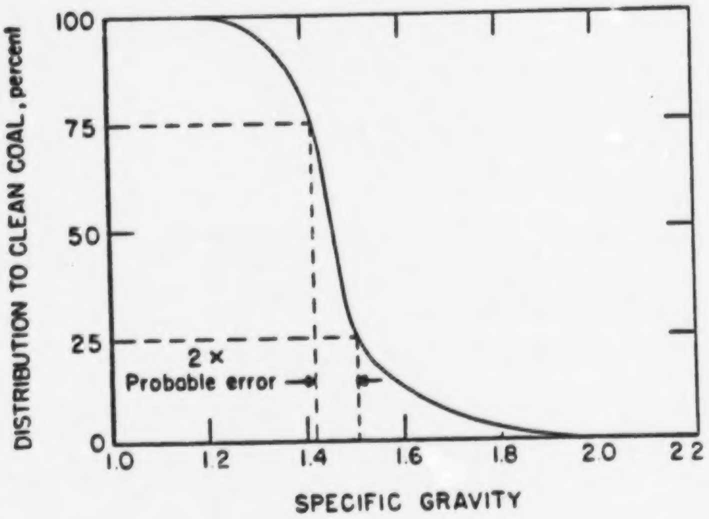


Figure 6. Probable Error(6)

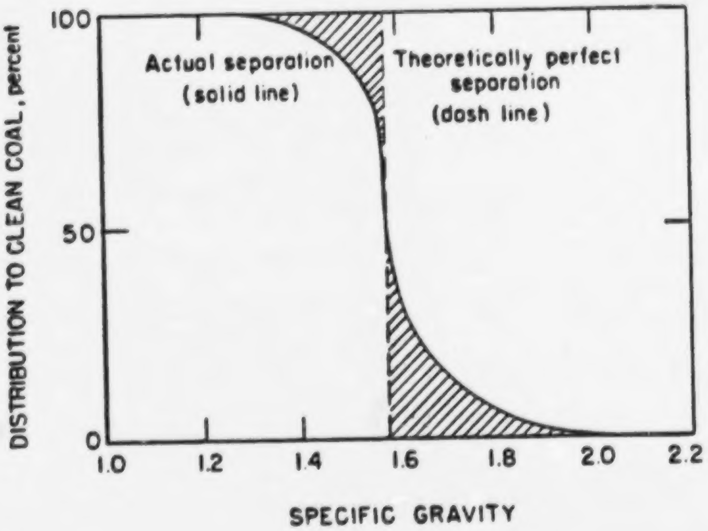


Figure 7. Error Area(6)

5. Imperfection

Imperfection was developed as a criterion for sharpness of separation that would be independent of the specific gravity of separation. Imperfection is equal to the probable error divided by the specific gravity of separation minus one.

C. Sulfur-based Criteria

1. Percent Sulfur in Clean Coal

The percent sulfur in clean coal was determined from laboratory analyses. These values were obtained directly from the PENELEC/DOE data summaries.

2. Percent Sulfur Reduction

The percent sulfur reduction was computed by taking the difference between percent sulfur in feed and percent sulfur in clean coal and forming the ratio of this difference to percent sulfur in feed. The resulting value was expressed as a percentage.

3. Normalized Percent Sulfur Reduction

This measure of percent sulfur reduction was computed (as in C-2 above) using sulfur values for clean and feed coal expressed as pounds per million Btu. Since no Btu values were available in the original data set, they were estimated by PENELEC from the ash content values.

V. ANALYSIS METHODS

A. Introduction

This section describes the methods BCR used in performing its statistical analysis of the dense medium cyclone performance data. Descriptions of database construction, verification, and management; mathematical procedures for generating distribution curves; and statistical techniques for evaluating cyclone performance are provided.

B. Data Base

Dense medium cyclone performance data for 36 tests conducted at the pilot plant of DOE in Bruceton, Pennsylvania, were supplied to BCR by PENELEC. These data were collected according to the test plan described in Section III and shown in Figure 1 and Table 1 of this report. BCR computer programmers developed computer software for data entry and verification. Data files were created on a test-by-test basis and included all summary information provided by PENELEC and DOE (i.e., operating parameters and cyclone performance measures). Additional software was developed to generate data summaries for each test, as well as summary statistics (means and standard deviations) for any desired combination of tests. Data summaries for each cyclone test are displayed in Appendix A. Data used in the calculation of normalized percent sulfur reduction (i.e., estimated Btu values and normalized sulfur contents) are shown in Appendix B.

C. Distribution Curve Generation

In order to compute data values for the three independent performance criteria (probable error, error area, and imperfection), distribution curves were generated by BCR from the distribution data values. Four distribution curves were generated for each cyclone test--one for each of the three particle size ranges of coal and one for the composite size range. The technique used by BCR to fit curves to the distribution data is a mathematical procedure known as the cubic spline method.(7) This method is described in Appendix C.

The fitting of cubic splines to the dense media cyclone distribution data and evaluations of the resulting spline functions were accomplished using two computer programs developed by BCR. The first program fits the cubic spline to the data and performs the smoothing process to produce spline functions for each set of distribution values. These spline functions are then evaluated by a second computer program. In particular, this program reads specific gravity values from the cubic spline curve corresponding to distribution values of 25, 50, and 75 percent to clean coal. These numbers are then used to calculate probable error and imperfection. This program also algebraically integrates the spline functions in order to compute the area between the observed distribution curve and the curve representing perfect separation. This value is equal to the independent performance criterion known as error area. The resulting distribution curves and independent criteria values are discussed in Section VI of this report and displayed in Appendix A on a test-by-test basis.

D. Statistical Evaluations

In order to determine the effects of operating parameters on cyclone performance, BCR performed a multiple regression analysis of the test data. The theory and techniques of regression analysis are given in reference 8 thru 11. The techniques used in this study are described in Appendix D. Specifically, the objective of these analyses was to quantify the relationships between the operating variables under investigation (particle size range, inlet orifice size, flow rate, and media-to-coal ratio) and the various dependent, independent, and sulfur-based performance criteria defined in Section IV of this report. The nature of these relationships would then suggest the most efficient set of operating conditions for each criterion.

Various functional forms of the independent variables were examined by BCR in the course of the regression analysis. Since orifice size had only two possible values in the test matrix (1.50 in. and 1.75 in.), only these specific values were used in constructing regression models. However, for flow rate, media-to-coal ratio, and particle size, polynominal forms of higher order as well as other transformations of these variables (logarithmic, exponential, inverse, etc.) were evaluated as possible terms in the regression models. In addition, product or interaction terms involving two or more of the independent variables were examined. In all cases, models were constructed which used the independent variables in their simplest functional forms without sacrificing precision. For a detailed list of the steps followed by BCR in developing each regression model, see Appendix D, page D-6. The results of BCR's regression analyses are presented and discussed in Section VI of this report.

VI. RESULTS

A. Introduction

This section details the results of BCR's statistical evaluation of dense medium cyclone performance, utilizing the methods described in Section V, Appendix C, and Appendix D of this report.

The mean values, standard deviations and relative standard deviations for each performance criterion are given in Table 2. These statistical parameters are computed from test values at each of 3 size ranges for 36 tests (N=108). Data are also presented for the simple correlations between each criterion and each cyclone operating parameter.

The relative standard deviation (standard deviation ÷ MEAN) provides a relative measure of the variation in each of the performance criteria during the 36 tests. The weight recovery efficiency, percent sulfur in clean coal, percent sulfur reduction and normalized percent sulfur reduction are not very sensitive to changes in the operating variables. The misplaced material and yield error show moderate sensitivity. The ash error, error area, probable error and imperfection are the most sensitive.

The simple correlations provide an estimate of the strength and direction of the relationship between a performance criterion and single operating variable. The only operating variable which shows a strong correlation with the performance criteria is particle size. The large negative correlations for misplaced material, ash error, yield error, error area, probable error and imperfection show that the value of these performance criteria increase significantly with decreasing particle size.

TABLE 2. MEANS, STANDARD DEVIATIONS, AND CORRELATIONS FOR CYCLONE PERFORMANCE CRITERIA (N = 108)

				Correlations			
Criteria	Mean	Standard Deviation	Relative Standard Deviation	Orifice Size	Flow Rate	Percent Coal in Slurry	Particle Size
Weight Recovery Efficiency	77.96	8.22	0.11	-0.342	-0.108	-0.132	-0.008
Misplaced Material	14.62	3.75	0.26	0.201	-0.173	0.244	-0.615
Ash Error	1.14	0.73	0.64	0.222	0.103	0.240	-0.649
Yield Error	11.44	5.18	0.45	0.301	0.102	0.161	-0.496
Error Area	32.71	22.81	0.70	0.091	-0.036	0.176	-0.826
Probable Error	0.042	0.03	0.71	0.053	-0.043	0.233	-0.799
Imperfection	0.129	0.08	0.62	0.055	-0.049	0.203	-0.803
Percent Sulfur in Clean Coal	1.00	0.15	0.15	-0.002	0.019	0.196	-0.200
Percent Sulfur Reduction	68.50	9.30	0.14	0.059	-0.504	-0.215	-0.249
Normalized Percent Sulfur Reduction	75.65	7.90	0.10	0.065	-0.515	-0.223	-0.120

Results of statistical analyses on dependent performance criteria (weight-recovery efficiency, misplaced material, ash error, and yield error) are presented in Part B. Pact C details results of analyses performed on the independent criteria (error area, probable error, and imperfection), including a discussion of distribution curves generated for the test data. Finally, Part D presents the results of statistical analyses on sulfur-based performance criteria (percent sulfur in clean coal, percent sulfur reduction, and normalized percent sulfur reduction).

All statistical results are presented on a criterion-by-criterion basis. Stepwise regression results are displayed for predicting each criterion from the relevant cyclone operating conditions. Relevant operating conditions are statistically related (at the 95-percent confidence level) to the given performance criterion and, thus, explain (account for) a significant portion of the observed variability in the criterion. The amount of explained variability is measured by the coefficient of multiple determination (MCC^2). Specifically, the value MCC^2 (the square of the correlation coefficient) represents the fraction of total variability in the given performance criterion that is accounted for by the regression model. The final prediction equation (regression model) is presented for each criterion, as well as predicted and observed values for each model (Note: see limitation No. 2, Section II, C.). These values are used to indicate the overall fit of the regression model to the data as well as to select the set of cyclone operating conditions that will most likely produce the best results of the given performance criterion.

B. Statistical Evaluation of Dependent Performance Criteria

A preliminary examination of the correlations between the cyclone operating parameters and the dependent performance criteria indicated that all four operating parameters should be tested for inclusion in the final prediction models for the dependent criteria. First-order expressions for orifice size (O), percent coal in slurry (R), and particle size (P) were selected as potential predictor variables. A second-order polynomial expression was used in the case of flow rate (F,F^2).

Tables 3A through 6B present the results of stepwise multiple regression analyses performed on the dependent performance criteria. Final prediction equations are shown, along with predicted and observed values, on a test-by-test basis. Final models include those independent variables that are statistically significant at the 95-percent confidence level using the procedure outlined in Appendix C. The final models for three of the dependent performance criteria (misplaced material, ash error, and yield error) include all four independent variables (O, R, P, and F,F^2). Only O and F,F^2 terms are significant for weight recovery efficiency. The coefficients of multiple determination (MCC^2) for the final models range from 0.274 for weight recovery efficiency to 0.658 for ash error. Thus, these models are able to account for about 27.4 to 65.8 percent of the variability in the various dependent performance criteria.

C. Statistical Evaluation of Independent Performance Criteria

Distribution curves, generated using the cubic spline method described in Appendix C of this report, are displayed in Appendix A on a test-by-test basis. Four curves are shown for each test, one for each particle-size range and one for the composite. Values for the three (text continued on Page 34.)

TABLE 3A. RESULTS OF REGRESSION ANALYSIS FOR WEIGHT RECOVERY EFFICIENCY

Step	Independent Variable	MCC	MCC^2	F-ratio
1	F, F^2	0.397	0.158	19.63*
2	O	0.524	0.274	16.72*
3	R	0.538	0.290	2.25
4	P	0.538	0.290	0.01

Final Prediction Equation

$$\hat{Y}_i = 1.71\ F_i - 0.007\ F_i^2 - 22.37\ O_i + 12.74$$

$$MCC = 0.524$$

$$MCC^2 = 0.274$$

$$SEE = 7.01$$

O - Orifice size
F - Flow rate
R - Percent coal in slurry
P - Particle size
MCC - Multiple correlation coefficient
MCC^2 - Coefficient of multiple determination
F-ratio - Statistic for test of significance of regression model
* - Significant at 95-percent confidence level
$\hat{Y}$ - Predicted value for criterion variable
SEE - Standard error of estimate

TABLE 3B. PREDICTED AND OBSERVED VALUES OF WEIGHT RECOVERY EFFICIENCY BY PARTICLE SIZE RANGE

Test No.	(9 x 14)		(14 x 28)		(28 x 100)	
	Predicted	Observed	Predicted	Observed	Predicted	Observed
71	78.9	92.6	78.9	93.3	78.9	89.5
72	75.2	83.5	75.2	83.1	75.2	73.4
73	82.7	86.0	82.7	90.0	82.7	78.9
74	80.8	74.8	80.8	78.2	80.8	79.7
75	77.8	97.3	77.8	76.8	77.8	78.3
76	75.2	74.1	75.2	75.5	75.2	73.3
77	77.1	78.9	77.1	85.4	77.1	80.4
78	83.4	88.7	83.4	84.4	83.4	77.7
79	84.5	75.5	84.5	82.6	84.5	79.1
80	72.2	74.2	72.2	60.4	72.2	64.4
81	77.8	82.9	77.8	76.2	77.8	73.7
82	78.9	73.5	78.9	75.3	78.9	79.1
83	82.7	88.4	82.7	84.3	82.7	85.0
84	84.5	100.0	84.5	96.8	84.5	87.3
85	80.8	77.0	80.8	79.3	80.8	75.5
86	75.2	87.1	75.2	76.6	75.2	74.4
87	77.1	83.3	77.1	78.9	77.1	83.2
88	72.2	68.4	72.2	68.1	72.2	71.5
89	83.4	64.7	83.4	73.0	83.4	77.0
90	77.8	79.0	77.8	84.2	77.8	73.8
91	77.8	59.8	77.8	67.7	77.8	74.0
92	72.2	87.3	72.2	76.0	72.2	71.1
93	84.5	83.0	84.5	81.1	84.5	79.7
94	77.8	85.0	77.8	83.3	77.8	73.9
95	78.9	78.5	78.9	80.9	78.9	78.5
96	77.8	73.3	77.8	80.8	77.8	70.8
97	83.4	80.4	83.4	80.7	83.4	100.0
98	80.8	84.0	80.8	78.5	80.8	74.0
99	82.7	78.6	82.7	78.6	82.7	82.2
100	77.1	65.9	77.1	73.9	77.1	77.2
101	69.7	50.7	69.7	64.6	69.7	72.2
102	69.7	59.9	69.7	67.9	69.7	75.9
103	69.7	67.6	69.7	72.8	69.7	77.3
104	75.3	77.3	75.3	77.8	75.3	79.9
105	75.3	72.9	75.3	78.3	75.3	82.4
106	75.3	64.8	75.3	72.0	75.3	79.1

TABLE 4A. RESULTS OF REGRESSION ANALYSIS FOR MISPLACED MATERIAL

Step	Independent Variable	MCC	MCC^2	F-ratio
1	P	0.615	0.378	64.36*
2	F, F^2	0.685	0.469	17.91*
3	R	0.728	0.530	13.35*
4	O	0.757	0.573	10.22*

Final Prediction Equation

$$\hat{Y}_i = -0.53 F_i + 0.002 F_i^2 - 4.61 P_i + 0.18 R_i + 6.18 O_i + 39.97$$

MCC = 0.757

MCC^2 = 0.573

SEE = 2.45

O - Orifice size

F - Flow rate

R - Percent coal in slurry

P - Particle size

MCC - Multiple correlation coefficient

MCC^2 - Coefficient of multiple determination

F-ratio - Statistic for test of significance of regression model

* - Significant at 95-percent confidence level

$\hat{Y}$ - Predicted value for criterion variable

SEE - Standard error of estimate

TABLE 4B. PREDICTED AND OBSERVED VALUES OF MISPLACED MATERIAL BY PARTICLE SIZE RANGE

Test	(9 x 14)		(14 x 28)		(28 x 100)	
No.	Predicted	Observed	Predicted	Observed	Predicted	Observed
71	10.2	6.9	13.5	9.0	15.8	7.2
72	14.4	12.2	17.6	15.6	20.0	16.2
73	8.4	5.9	11.6	9.4	14.0	12.6
74	12.7	11.7	15.9	14.1	18.3	15.2
75	12.0	5.9	15.2	16.1	17.6	18.1
76	12.3	11.2	15.5	13.8	17.9	17.4
77	12.3	7.6	15.6	13.0	17.9	14.4
78	10.1	10.2	13.3	13.6	15.7	16.0
79	10.9	8.3	14.2	13.5	16.5	13.8
80	13.6	9.5	16.8	15.0	19.2	22.9
81	13.3	16.2	16.6	20.2	18.9	18.1
82	11.2	13.5	14.4	16.1	16.8	17.6
83	9.5	6.0	12.8	9.2	15.1	13.0
84	8.9	10.5	12.1	13.2	14.5	14.2
85	10.8	13.6	14.0	15.1	16.4	18.5
86	13.1	12.3	16.3	14.7	18.7	18.0
87	10.1	11.4	13.4	14.1	15.7	15.2
88	15.4	17.4	18.7	19.3	21.0	20.0
89	11.9	13.4	15.1	18.1	17.5	18.2
90	12.9	11.5	16.1	15.1	18.5	20.2
91	11.1	14.4	14.4	17.1	16.7	21.6
92	13.9	11.9	17.2	15.2	19.5	20.1
93	9.4	10.4	12.7	15.3	15.0	17.4
94	14.1	15.7	17.4	20.0	19.7	19.7
95	12.9	11.8	16.1	17.8	18.5	17.7
96	11.5	14.2	14.8	18.0	17.1	19.7
97	9.7	6.3	12.9	11.3	15.3	15.1
98	11.4	14.7	14.6	18.5	17.0	18.2
99	11.0	14.3	14.2	17.5	16.6	16.0
100	10.7	16.1	13.9	15.1	16.3	17.9
101	11.7	14.1	14.9	18.5	17.3	19.2
102	12.0	13.1	15.2	14.9	17.6	18.1
103	13.8	12.6	17.0	17.6	19.4	17.2
104	9.9	7.5	13.2	11.9	15.5	15.9
105	10.7	9.8	13.9	14.2	16.3	15.6
106	12.4	12.4	15.6	18.9	18.0	18.0

TABLE 5A. RESULTS OF REGRESSION ANALYSIS FOR ASH ERROR

Step	Independent Variable	MCC	MCC^2	F-ratio
1	P	0.649	0.421	77.01*
2	F, F^2	0.745	0.555	31.45*
3	R	0.779	0.607	13.40*
4	O	0.811	0.658	15.36*

Final Prediction Equation

$$\hat{Y}_i = -0.14 F_i + 0.0006 F_i^2 - 0.95 P_i + 0.03 R_i + 1.32 O_i + 7.52$$

$MCC = 0.811$

$MCC^2 = 0.658$

$SEE = 0.43$

O - Orifice size
F - Flow rate
R - Percent coal in slurry
P - Particle size
MCC - Multiple correlation coefficient
MCC^2 - Coefficient of multiple determination
F-ratio - Statistic for test of significance of regression model
* - Significant at 95-percent confidence level
$\hat{Y}$ - Predicted value for criterion variable
SEE - Standard error of estimate

TABLE 5B. PREDICTED AND OBSERVED VALUES OF ASH ERROR BY PARTICLE SIZE RANGE

Test	(9 x 14)		(14 x 28)		(28 x 100)	
No.	Predicted	Observed	Predicted	Observed	Predicted	Observed
71	0.2	0.3	0.9	0.3	1.3	0.4
72	0.9	0.7	1.6	0.9	2.1	2.8
73	0.0	0.4	0.7	0.4	1.2	1.5
74	0.6	0.5	1.2	1.1	1.7	1.7
75	0.5	0.1	1.1	1.1	1.6	1.3
76	0.5	0.7	1.2	0.8	1.7	2.0
77	0.8	0.7	1.4	0.6	1.9	2.0
78	0.1	0.2	0.7	0.6	1.2	1.7
79	0.3	0.6	0.9	0.7	1.4	1.8
80	0.8	0.8	1.5	3.3	1.9	2.1
81	0.7	0.5	1.4	1.1	1.9	2.7
82	0.4	0.6	1.0	0.8	1.5	1.5
83	0.2	0.3	0.9	0.6	1.4	1.0
84	-0.1	0.0	0.6	0.1	1.0	0.7
85	0.2	0.4	0.9	0.6	1.4	1.6
86	0.7	0.3	1.3	0.8	1.8	2.0
87	0.4	0.3	1.0	0.6	1.5	1.1
88	1.1	1.0	1.8	1.5	2.3	2.6
89	0.4	1.1	1.1	1.3	1.6	2.1
90	0.6	0.6	1.3	0.9	1.8	2.0
91	0.3	1.2	1.0	0.9	1.5	1.8
92	0.9	0.3	1.5	1.0	2.0	2.3
93	0.0	0.5	0.7	0.8	1.2	1.4
94	0.8	0.4	1.5	0.6	2.0	2.1
95	0.7	0.8	1.3	0.8	1.8	2.3
96	0.4	0.5	1.1	0.5	1.5	2.6
97	0.0	0.4	0.7	0.4	1.2	0.0
98	0.3	0.5	1.0	0.8	1.5	1.9
99	0.5	0.6	1.1	1.0	1.6	1.6
100	0.5	0.6	1.1	0.9	1.6	1.8
101	1.0	1.5	1.7	1.5	2.2	2.4
102	1.1	1.4	1.7	1.6	2.2	2.3
103	1.4	1.4	2.0	1.6	2.5	2.7
104	0.6	0.8	1.3	1.0	1.8	2.0
105	0.8	1.0	1.4	1.1	1.9	1.9
106	1.1	1.3	1.7	1.6	2.2	2.4

TABLE 6A. RESULTS OF REGRESSION ANALYSIS FOR YIELD ERROR

Step	Independent Variable	MCC	MCC^2	F-ratio
1	P	0.496	0.246	34.51*
2	F, F^2	0.639	0.408	28.49*
3	O	0.706	0.498	18.58*
4	R	0.722	0.522	5.01*

Final Prediction Equation

$$\hat{Y}_i = -1.09\ F_i + 0.004\ F_i^2 - 5.13\ P_i + 12.56\ O_i + 0.16\ R_i + 58.27$$

$MCC = 0.722$

$MCC^2 = 0.522$

$SEE = 3.58$

O - Orifice size

F - Flow rate

R - Percent coal in slurry

P - Particle size

MCC - Multiple correlation coefficient

MCC^2 - Coefficient of multiple determination

F-ratio - Statistic for test of significance of regression model

* - Significant at 95-percent confidence level

$\hat{Y}$ - Predicted value for criterion variable

SEE - Standard error of estimate

TABLE 6B. PREDICTED AND OBSERVED VALUES OF YIELD ERROR BY PARTICLE SIZE RANGE

Test	(9 x 14)		(14 x 28)		(28 x 100)	
No.	Predicted	Observed	Predicted	Observed	Predicted	Observed
71	6.4	2.5	10.0	3.1	12.6	5.7
72	10.7	6.7	14.3	8.8	16.9	14.6
73	4.3	4.2	7.9	4.3	10.5	13.0
74	7.5	10.3	11.0	11.4	13.7	11.1
75	7.8	0.7	11.4	11.1	14.0	12.2
76	8.9	8.9	12.5	11.9	15.2	16.1
77	9.5	7.6	13.1	7.1	15.7	12.0
78	4.5	3.7	8.1	7.5	10.8	13.7
79	5.1	6.8	8.7	8.6	11.4	12.9
80	11.0	8.3	14.6	23.5	17.2	24.7
81	9.1	6.3	12.7	12.6	15.4	16.6
82	7.2	9.4	10.8	12.1	13.4	12.3
83	5.3	3.9	8.9	7.8	11.5	8.9
84	3.4	0.0	7.0	1.4	9.6	7.4
85	5.9	7.7	9.5	9.5	12.1	15.0
86	9.6	4.5	13.2	12.7	15.9	16.2
87	7.6	5.2	11.2	10.3	13.9	10.2
88	12.5	13.2	16.1	17.3	18.8	19.1
89	6.1	13.4	9.7	14.2	12.3	16.3
90	8.6	7.8	12.1	7.5	14.8	18.3
91	7.3	16.4	10.9	16.3	13.5	17.7
92	11.3	4.5	14.9	13.3	17.5	19.8
93	3.9	6.1	7.5	9.4	10.1	13.8
94	9.6	5.5	13.2	8.7	15.8	18.7
95	8.6	8.6	12.2	10.5	14.8	16.2
96	7.6	8.3	11.2	8.9	13.9	20.8
97	4.2	6.4	7.8	8.7	10.5	0.0
98	6.3	6.2	9.9	11.7	12.6	17.3
99	6.5	8.3	10.1	13.0	12.8	13.5
100	8.1	14.1	11.7	14.1	14.4	16.3
101	12.4	19.9	16.0	18.3	18.7	19.8
102	12.7	16.7	16.3	18.4	18.9	16.8
103	14.2	13.7	17.8	16.3	20.5	16.6
104	9.2	8.5	12.7	11.2	15.4	14.4
105	9.8	10.1	13.4	11.8	16.0	12.8
106	11.2	15.3	14.8	16.7	17.5	15.5

independent criteria, along with the specific gravity of separation, are reported for each curve. As indicated in Section V, these values were read or computed from the distribution curve shown.

A preliminary examination of the correlations between the cyclone operating parameters and the independent performance criteria indicated two statistically significant parameters to be tested for inclusion in the final prediction models for the independent criteria. A second-order polynomial expression for particle size (P,P^2) and a first-order expression for percent coal in slurry (R) were selected as potential predictor variables.

Tables 7A through 9B present the results of stepwise multiple regression analyses performed on the independent performance criteria. Final prediction equations are shown, along with predicted and observed values, on a test-by-test basis. Final models for all three independent performance criteria (error area, probable error, and imperfection) include both P,P^2 and R terms. The coefficients of multiple determination (MCC^2) for the final models range from 0.828 for imperfection to 0.881 for error area.

D. Statistical Evaluation of Sulfur-based Performance Criteria

As in the case of the dependent performance criteria, a preliminary examination of the correlations between the cyclone operating parameters and the sulfur-based performance criteria indicated that all four operating parameters should be tested for inclusion in the final models for sulfur-based criteria. First-order expressions for orifice size (O), percent coal in slurry (R), and particle size (P) were, again, selected as potential predictor variables. A second-order polynomial expression was used for flow rate (F,F^2).

Tables 10A through 12B present the results of stepwise multiple regression analyses performed on the three sulfur-based (text continued on Page 47)

TABLE 7A. RESULTS OF REGRESSION ANALYSIS FOR ERROR AREA

Step	Independent Variable	MCC	MCC^2	F-ratio
1	P, P^2	0.922	0.849	592.21*
2	R	0.938	0.881	27.07*

Final Prediction Equation

$$\hat{Y}_i = -0.0001\ P_i + 54.73\ P_i^2 + 0.78\ R_i + 95.70$$

$MCC = 0.938$

$MCC^2 = 0.881$

$SEE = 7.88$

R - Percent coal in slurry
P - Particle size
MCC - Multiple correlation coefficient
MCC^2 - Coefficient of multiple determination
F-ratio - Statistic for test of significance of regression model
* - Significant at 95-percent confidence level
$\hat{Y}$ - Predicted value for criterion variable
SEE - Standard error of estimate

TABLE 7B. PREDICTED AND OBSERVED VALUES OF ERROR AREA BY PARTICLE SIZE RANGE

Test	(9 x 14)		(14 x 28)		(28 x 100)	
No.	Predicted	Observed	Predicted	Observed	Predicted	Observed
71	9.	10.	17.	13.	57.	11.
72	19.	18.	27.	28.	67.	63.
73	9.	8.	17.	14.	57.	44.
74	18.	13.	26.	25.	66.	58.
75	10.	14.	18.	28.	58.	80.
76	10.	13.	18.	18.	58.	60.
77	20.	15.	27.	27.	67.	65.
78	12.	12.	20.	18.	60.	51.
79	19.	11.	27.	18.	67.	60.
80	11.	16.	18.	22.	58.	76.
81	19.	25.	27.	36.	67.	72.
82	13.	11.	21.	25.	61.	65.
83	14.	9.	22.	13.	62.	39.
84	10.	11.	18.	13.	58.	40.
85	10.	11.	18.	16.	58.	58.
86	14.	13.	21.	31.	61.	72.
87	10.	10.	18.	14.	58.	57.
88	18.	18.	26.	34.	66.	74.
89	20.	11.	28.	27.	68.	74.
90	14.	25.	22.	25.	62.	64.
91	10.	13.	18.	24.	58.	64.
92	12.	15.	20.	25.	60.	65.
93	13.	14.	20.	20.	60.	61.
94	19.	8.	27.	20.	67.	64.
95	21.	25.	28.	31.	68.	82.
96	12.	13.	20.	12.	59.	67.
97	10.	7.	18.	6.	58.	50.
98	13.	13.	21.	19.	61.	65.
99	20.	24.	28.	26.	68.	71.
100	13.	15.	20.	26.	60.	68.
101	11.	17.	19.	20.	59.	61.
102	12.	14.	20.	29.	60.	62.
103	20.	17.	28.	27.	68.	74.
104	10.	12.	18.	18.	58.	62.
105	14.	14.	21.	21.	61.	61.
106	21.	16.	28.	26.	68.	68.

TABLE 8A. RESULTS OF REGRESSION ANALYSIS FOR PROBABLE ERROR

Step	Independent Variable	MCC	MCC^2	F-ratio
1	P, P^2	0.891	0.794	405.70*
2	R	0.921	0.849	37.42*

Final Prediction Equation

$$\hat{Y}_i = -0.188\, P_i + 0.070\, P_i^2 + 0.0014\, R_i + 0.117$$

$$MCC = 0.921$$

$$MCC^2 = 0.849$$

$$SEE = 0.012$$

R - Percent coal in slurry
P - Particle size
MCC - Multiple correlation coefficient
MCC^2 - Coefficient of multiple determination
F-ratio - Statistic for test of significance of regression model
* - Significant at 95-percent confidence level
$\hat{Y}$ - Predicted value for criterion variable
SEE - Standard error of estimate

TABLE 8B. PREDICTED AND OBSERVED VALUES OF PROBABLE ERROR BY PARTICLE SIZE RANGE

Test No.	(9 x 14) Predicted	(9 x 14) Observed	(14 x 28) Predicted	(14 x 28) Observed	(28 x 100) Predicted	(28 x 100) Observed
71	0.010	0.017	0.020	0.019	0.071	0.017
72	0.027	0.023	0.037	0.049	0.088	0.077
73	0.010	0.013	0.020	0.017	0.071	0.042
74	0.026	0.020	0.036	0.028	0.087	0.091
75	0.012	0.016	0.022	0.033	0.073	0.103
76	0.011	0.022	0.021	0.030	0.072	0.084
77	0.028	0.018	0.038	0.031	0.089	0.095
78	0.015	0.018	0.025	0.025	0.076	0.063
79	0.027	0.018	0.037	0.026	0.088	0.080
80	0.012	0.017	0.022	0.027	0.073	0.117
81	0.028	0.031	0.038	0.051	0.089	0.097
82	0.017	0.017	0.027	0.023	0.078	0.054
83	0.018	0.012	0.028	0.015	0.080	0.066
84	0.011	0.018	0.021	0.021	0.072	0.071
85	0.012	0.019	0.022	0.019	0.073	0.064
86	0.017	0.018	0.027	0.031	0.079	0.073
87	0.011	0.015	0.021	0.020	0.073	0.057
88	0.026	0.021	0.036	0.032	0.087	0.104
89	0.028	0.018	0.038	0.034	0.089	0.108
90	0.018	0.021	0.028	0.023	0.080	0.099
91	0.011	0.018	0.021	0.023	0.072	0.074
92	0.015	0.020	0.025	0.022	0.076	0.086
93	0.016	0.014	0.026	0.027	0.077	0.070
94	0.028	0.016	0.038	0.038	0.089	0.095
95	0.030	0.021	0.040	0.041	0.091	0.117
96	0.014	0.019	0.024	0.026	0.075	0.072
97	0.012	0.008	0.022	0.010	0.073	0.042
98	0.016	0.022	0.026	0.026	0.077	0.069
99	0.029	0.022	0.039	0.038	0.091	0.100
100	0.016	0.018	0.026	0.020	0.077	0.096
101	0.013	0.019	0.023	0.037	0.074	0.063
102	0.015	0.017	0.025	0.026	0.076	0.078
103	0.029	0.023	0.039	0.041	0.090	0.100
104	0.012	0.015	0.022	0.024	0.073	0.078
105	0.017	0.020	0.027	0.028	0.079	0.078
106	0.030	0.022	0.040	0.040	0.091	0.090

TABLE 9A. RESULTS OF REGRESSION ANALYSIS FOR IMPERFECTION

Step	Independent Variable	MCC	MCC^2	F-ratio
1	P, P^2	0.887	0.787	388.42*
2	R	0.910	0.828	25.04*

Final Prediction Equation

$$\hat{Y}_i = -0.488\ P_i + 0.179\ P_i^2 + 0.0032\ R_i + 0.331$$

$MCC = 0.910$

$MCC^2 = 0.828$

$SEE = 0.034$

R - Percent coal in slurry

P - Particle size

MCC - Multiple correlation coefficient

MCC^2 - Coefficient of multiple determination

F-ratio - Statistic for test of significance of regression model

* - Significant at 95-percent confidence level

$\hat{Y}$ - Predicted value for criterion variable

SEE - Standard error of estimate

TABLE 9B. PREDICTED AND OBSERVED VALUES OF IMPERFECTION BY PARTICLE SIZE RANGE

Test	(9 x 14)		(14 x 28)		(28 x 100)	
No.	Predicted	Observed	Predicted	Observed	Predicted	Observed
71	0.044	0.056	0.075	0.060	0.209	0.055
72	0.083	0.074	0.114	0.150	0.248	0.207
73	0.044	0.043	0.074	0.057	0.208	0.129
74	0.082	0.067	0.112	0.090	0.246	0.250
75	0.048	0.052	0.078	0.111	0.213	0.310
76	0.047	0.077	0.077	0.100	0.212	0.252
77	0.086	0.062	0.116	0.101	0.251	0.253
78	0.055	0.063	0.086	0.085	0.220	0.192
79	0.083	0.062	0.114	0.085	0.248	0.221
80	0.050	0.059	0.080	0.095	0.214	0.350
81	0.086	0.108	0.116	0.161	0.250	0.268
82	0.061	0.059	0.091	0.079	0.226	0.164
83	0.064	0.041	0.094	0.052	0.229	0.196
84	0.047	0.063	0.078	0.068	0.212	0.203
85	0.049	0.067	0.079	0.067	0.213	0.198
86	0.062	0.064	0.092	0.106	0.226	0.213
87	0.048	0.052	0.078	0.070	0.212	0.176
88	0.082	0.074	0.112	0.107	0.246	0.296
89	0.087	0.064	0.117	0.113	0.252	0.300
90	0.064	0.073	0.094	0.079	0.229	0.300
91	0.047	0.063	0.078	0.080	0.212	0.233
92	0.055	0.067	0.086	0.072	0.220	0.253
93	0.057	0.048	0.088	0.089	0.222	0.204
94	0.086	0.058	0.116	0.134	0.250	0.267
95	0.091	0.071	0.121	0.134	0.255	0.303
96	0.054	0.071	0.084	0.088	0.219	0.226
97	0.049	0.030	0.079	0.036	0.213	0.140
98	0.058	0.076	0.089	0.085	0.223	0.220
99	0.090	0.076	0.120	0.123	0.254	0.247
100	0.058	0.062	0.088	0.070	0.222	0.292
101	0.051	0.068	0.081	0.130	0.216	0.191
102	0.057	0.062	0.087	0.089	0.221	0.223
103	0.088	0.078	0.118	0.131	0.253	0.263
104	0.048	0.054	0.078	0.081	0.213	0.221
105	0.062	0.070	0.092	0.093	0.226	0.202
106	0.091	0.077	0.121	0.130	0.256	0.224

TABLE 10A. RESULTS OF REGRESSION ANALYSIS FOR PERCENT SULFUR IN CLEAN COAL

Step	Independent Variable	MCC	MCC^2	F-ratio
1	P	0.200	0.040	4.41*
2	R	0.280	0.078	4.39*
3	F,F^2	0.289	0.083	0.57
4	O	0.289	0.083	0.00

Final Prediction Equation

$$\hat{Y}_i = -0.06\ P_i + 0.006\ R_i + 0.95$$

$$MCC = 0.280$$

$$MCC^2 = 0.078$$

$$SEE = 0.15$$

O - Orifice size

F - Flow rate

R - Percent coal in slurry

P - Particle size

MCC - Multiple correlation coefficient

MCC^2 - Coefficient of multiple determination

F-ratio - Statistic for test of significance of regression model

* - Significant at 95-percent confidence level

$\hat{Y}$ - Predicted value for criterion variable

SEE - Standard error of estimate

TABLE 10B. PREDICTED AND OBSERVED VALUES OF PERCENT SULFUR IN CLEAN COAL BY PARTICLE SIZE RANGE

Test No.	(9 x 14) Predicted	(9 x 14) Observed	(14 x 28) Predicted	(14 x 28) Observed	(28 x 100) Predicted	(28 x 100) Observed
71	0.92	0.87	0.96	0.66	1.00	1.09
72	0.99	1.05	1.04	0.96	1.07	1.23
73	0.92	1.05	0.96	1.09	0.99	0.91
74	0.99	1.09	1.03	1.02	1.06	1.18
75	0.93	0.96	0.97	0.46	1.00	1.00
76	0.93	0.92	0.97	0.89	1.00	1.15
77	1.00	0.94	1.04	0.96	1.07	1.09
78	0.94	1.03	0.98	0.90	1.02	1.10
79	0.99	0.91	1.04	0.89	1.07	1.12
80	0.93	0.88	0.97	0.99	1.01	1.22
81	1.00	1.00	1.04	0.86	1.07	1.32
82	0.95	0.73	0.99	0.70	1.03	1.00
83	0.96	0.90	1.00	1.00	1.03	0.87
84	0.93	1.11	0.97	1.16	1.00	0.90
85	0.93	0.80	0.97	0.80	1.00	0.88
86	0.95	1.19	1.00	0.92	1.03	0.91
87	0.93	0.86	0.97	0.73	1.00	1.13
88	0.99	0.95	1.03	0.99	1.06	1.07
89	1.00	0.87	1.04	0.85	1.07	0.98
90	0.96	1.02	1.00	0.99	1.03	0.94
91	0.93	1.13	0.97	0.93	1.00	0.96
92	0.94	1.01	0.98	0.77	1.02	0.99
93	0.95	1.01	0.99	1.01	1.02	1.01
94	1.00	1.14	1.04	1.17	1.07	1.20
95	1.01	0.78	1.05	0.82	1.08	1.03
96	0.94	1.38	0.98	1.45	1.01	1.30
97	0.93	0.96	0.97	0.94	1.00	0.76
98	0.95	0.97	0.99	0.91	1.02	1.34
99	1.00	1.04	1.05	1.31	1.08	1.21
100	0.95	0.91	0.99	0.79	1.02	1.01
101	0.93	0.96	0.98	0.95	1.01	1.03
102	0.94	0.95	0.99	1.03	1.02	1.07
103	1.00	1.04	1.05	0.97	1.08	1.26
104	0.93	0.97	0.97	0.99	1.00	1.07
105	0.95	0.97	1.00	0.98	1.03	0.97
106	1.01	1.04	1.05	1.02	1.08	1.10

TABLE 11A. RESULTS OF REGRESSION ANALYSIS FOR PERCENT SULFUR REDUCTION

Step	Independent Variable	MCC	MCC^2	F-ratio
1	F, F^2	0.664	0.441	82.94*
2	P	0.709	0.503	12.96*
3	R	0.731	0.535	6.99*
4	O	0.733	0.538	0.66

Final Prediction Equation

$$\hat{Y}_i = 2.02\,F_i - 0.009\,F_i^2 - 4.62\,P_i - 0.32\,R_i - 32.59$$

$MCC = 0.731$

$MCC^2 = 0.535$

$SEE = 6.34$

O - Orifice size
F - Flow rate
R - Percent coal in slurry
P - Particle size
MCC - Multiple correlation coefficient
MCC^2 - Coefficient of multiple determination
F-ratio - Statistic for test of significance of regression model
* - Significant at 95-percent confidence level
$\hat{Y}$ - Predicted value for criterion variable
SEE - Standard error of estimate

TABLE 11B. PREDICTED AND OBSERVED VALUES OF PERCENT SULFUR REDUCTION BY PARTICLE SIZE RANGE

Test	(9 x 14)		(14 x 28)		(28 x 100)	
No.	Predicted	Observed	Predicted	Observed	Predicted	Observed
71	72.4	79.2	75.6	82.2	78.0	80.6
72	66.7	68.6	69.9	71.6	72.3	80.1
73	67.2	70.2	70.4	67.8	72.8	75.5
74	66.9	65.0	70.1	73.4	72.5	78.1
75	67.6	70.6	70.8	86.5	73.2	79.9
76	70.4	73.8	73.6	75.1	76.0	78.9
77	62.9	74.6	66.1	73.1	68.5	78.4
78	71.3	72.2	74.5	70.8	76.9	76.7
79	68.5	75.7	71.7	74.3	74.1	80.4
80	67.4	77.3	70.7	75.2	73.1	76.4
81	68.2	70.6	71.5	75.7	73.8	74.1
82	70.7	77.8	74.0	79.5	76.3	78.4
83	65.1	71.6	68.3	73.0	70.7	81.9
84	72.1	69.3	75.3	68.9	77.7	82.3
85	70.2	73.8	73.4	74.3	75.8	81.4
86	68.9	64.4	72.1	72.6	74.5	79.6
87	66.8	72.9	70.0	72.1	72.4	71.0
88	64.2	65.3	67.5	71.1	69.9	76.6
89	68.1	63.3	71.3	66.3	73.7	68.6
90	66.0	55.7	69.3	62.2	71.6	65.1
91	72.1	57.0	75.3	66.5	77.7	69.9
92	66.9	67.2	70.1	72.0	72.5	73.6
93	71.1	64.3	74.3	68.9	76.7	72.4
94	63.8	57.6	67.1	62.0	69.4	67.4
95	67.7	72.6	71.0	72.8	73.4	69.8
96	71.4	60.6	74.6	55.4	77.0	69.6
97	72.0	64.7	75.2	68.8	77.6	79.2
98	69.2	66.6	72.5	69.2	74.9	63.8
99	62.6	63.6	65.8	54.5	68.2	64.5
100	65.8	65.8	69.0	67.6	71.4	68.6
101	54.1	60.0	57.3	55.2	59.7	57.3
102	53.5	57.2	56.8	53.0	59.1	55.8
103	50.4	25.2	53.6	57.1	56.0	51.7
104	54.4	60.7	57.6	57.9	60.0	55.4
105	53.0	59.1	56.3	55.7	58.7	58.4
106	50.1	55.0	53.3	52.1	55.7	53.0

TABLE 12A. RESULTS OF REGRESSION ANALYSIS FOR NORMALIZED PERCENT SULFUR REDUCTION

Step	Independent Variable	MCC	MCC^2	F-ratio
1	F,F^2	0.676	0.457	88.37*
2	R	0.701	0.491	7.03*
3	P	0.711	0.506	3.00
4	O	0.714	0.510	0.77

Final Prediction Equation

$$\hat{Y}_i = 1.73\ F_i - 0.0075\ F_i^2 - 0.287\ R_i - 14.44$$

MCC = 0.701

MCC^2 = 0.491

SEE = 5.63

O - Orifice size
F - Flow rate
R - Percent coal in slurry
P - Particle size
MCC - Multiple correlation coefficient
MCC^2 - Coefficient of multiple determination
F-ratio - Statistic for test of significance of regression model
* - Significant at 95-percent confidence level
$\hat{Y}$ - Predicted value for criterion variable
SEE - Standard error of estimate

TABLE 12B. PREDICTED AND OBSERVED VALUES OF NORMALIZED PERCENT SULFUR REDUCTION BY PARTICLE SIZE RANGE

Test No.	(9 x 14)		(14 x 28)		(28 x 100)	
	Predicted	Observed	Predicted	Observed	Predicted	Observed
71	81.6	86.7	81.6	87.0	81.6	85.8
72	76.6	77.2	76.6	77.9	76.6	86.3
73	77.1	80.7	77.1	75.9	77.1	81.7
74	76.8	74.1	76.8	79.9	76.8	84.5
75	77.5	80.8	77.5	90.1	77.5	85.3
76	79.9	81.8	79.9	82.2	79.9	84.7
77	73.3	83.3	73.3	80.2	73.3	84.1
78	80.6	80.2	80.6	76.8	80.6	82.2
79	78.1	83.7	78.1	80.5	78.1	85.4
80	77.4	85.0	77.4	81.6	77.4	81.5
81	77.9	78.9	77.9	81.5	77.9	80.5
82	80.1	83.9	80.1	84.3	80.1	83.7
83	75.2	79.4	75.2	79.7	75.2	86.2
84	81.3	77.0	81.3	76.6	81.3	86.4
85	79.7	81.1	79.7	80.0	79.7	85.7
86	78.6	75.0	78.6	79.2	78.6	84.5
87	76.7	80.8	76.7	78.4	76.7	77.3
88	74.5	73.6	74.5	77.3	74.5	81.8
89	77.8	72.0	77.8	73.1	77.8	74.1
90	76.1	66.3	76.1	69.8	76.1	70.7
91	81.3	65.6	81.3	72.7	81.3	75.0
92	76.9	75.4	76.9	77.4	76.9	78.4
93	80.4	72.8	80.4	75.4	80.4	77.3
94	74.2	67.0	74.2	70.0	74.2	73.4
95	77.5	79.7	77.5	78.3	77.5	74.9
96	80.7	71.7	80.7	65.1	80.7	75.3
97	81.2	73.2	81.2	75.0	81.2	83.1
98	78.9	74.2	78.9	75.5	78.9	71.1
99	73.0	72.0	73.0	62.7	73.0	70.0
100	75.8	73.3	75.8	73.7	75.8	73.8
101	65.8	70.1	65.8	63.3	65.8	64.4
102	65.3	66.9	65.3	61.6	65.3	63.3
103	62.5	43.9	62.5	63.1	62.5	60.0
104	66.1	70.5	66.1	66.7	66.1	62.6
105	64.9	69.6	64.9	64.2	64.9	65.0
106	62.2	65.4	62.2	60.8	62.2	60.4

performance criteria. Final prediction equations are shown, along with predicted and observed values, on a test-by-test basis. The final model for sulfur in clean coal includes two significant terms, namely, P and R. For percent sulfur reduction, P, R, and F,F^2 are significant terms in the final model. For normalized percent sulfur reduction, F,F^2, and R terms are significant. The coefficient of multiple determination (MCC^2) is only 0.078 for percent sulfur in clean coal as compared to 0.535 and 0.491 for percent sulfur reduction and normalized percent sulfur reduction, respectively. This large difference is due to variations in sulfur content in the feed coal. Percent sulfur in clean coal includes the variability in feed coal sulfur content, making it a less desirable measure of cyclone performance than either percent sulfur reduction or normalized percent sulfur reduction, both of which adjust for differences in the sulfur content of the feed. Normalizing percent sulfur reduction for Btu value had little effect on the magnitude of the overall correlation; however, it did remove the particle size effect from the final model; i.e., the P term.

VII. REFERENCES AND BIBLIOGRAPHY

A. References

1. Killmeyer, R.P., "Dense-Medium Cycloning of Fine Coal at Low Specific Gravities," U.S. Department of Energy, DOE/PETC/TR-83/1 (DE 83009250), Pittsburgh Energy Technology Center, Pittsburgh, PA, October, 1983.

2. Sokaski, M., Geer, M.R., and Yancey, H.F., "Wet Concentration of Fine Coal: Part 1. Dense Medium Separation," in "Coal Preparation," J.W. Leonard and D.R. Mitchell, Ed., New York: American Institute of Mining, Metallurgical and Petroleum Engineers, Inc., 1968.

3. Geer, M.R. and Yancey, H.F., "Plant Performance and Forecasting Cleaning Results," in "Coal Preparation," J.W. Leonard and D.R. Mitchell, Ed., American Institute of Mining, Metallurgical and Petroleum Engineers, Inc., New York, NY, 1968.

4. Yancey, H.F. and Geer, M.R., "Efficiency and Sharpness-to-Separation in Evaluating Coal-Washery Performance," Trans. AIME 190, 507-517 (1951).

5. Yancey, H.F. and Geer, M.R., "Some Practical Aspects of Appraising Coal-Washing Results," U.S. Bureau of Mines, RI 5153 (1955). 15 pp.

6. Gottfried, B.S. and Jacobsen, P.S., "Generalized distribution Curve for Characterizing the Performance of Coal-Cleaning Equipment," U.S. Bureau of Mines, RI 8238 (1977). 21 pp.

7. Ahlberg, J., Nilson, E., and Walsh, J., "The Theory of Splines and Their Applications," New York: Academic Press, Inc., 1967.

8. Daniel, C. and Wood, F.S., "Fitting Equations to Date," New York: John Wiley & Sons, Inc., 1980.

9. Draper, N.R. and Smith, H., "Applied Regression Analysis," New York: John Wiley & Sons, Inc. 1981.

10. Hoel, P.G., "Introduction to Mathematical Statistics," New York: John Wiley & Sons, Inc., 1971.

11. Mosteller, F. and Tukey, J.W., "Data Analysis and Regression," Reading, Massachusetts: Addison-Wesley Publishing Co., Inc., 1977.

B. Bibliography

Coal Cleaning Equipment

Deurbrouck, A. W. and Hudy, J., Jr., "Performance characteristics of coal-washing equipment: dense-medium cyclones," U.S. Bur. Mines, RI 7673 (1972). 34 pp.

Geer, M. R., Sokaski, M., Jacobsen, P. S., and Yancey, H. F., "Performance of dense-medium cyclone in cleaning fine coal," U.S. Bur. Mines, RI 5732 (1961). 22 pp.

Geer, M. R. and Yancey, H. F., "Plant performance and forecasting cleaning results," in "Coal Preparation," J. W. Leonard and D. R. Mitchell, Ed., New York: The American Institue of Mining, Metallurgical and Petroleum Engineers, Inc., 1968. pp 18-1--18-29.

Gottfried, B. S. and Jacobsen, P. S., "Generalized disribution curve for characterizing the performance of coal-cleaning equipment," U.S. Bur. Mines, RI 8238 (1977). 21 pp.

Lyons, O. R., "Comparative effectiveness of coal cleaning equipment," Trans. AIME 193, 895-902 (1952).

Sokaski, M., Geer, M. R., and Yancey, H. F., "Wet Concentration of Fine Coal: Part 1. Dense Medium Separation," in "Coal Preparation," J. W. Leonard and D. R. Mitchell, Ed., New York: The American Institute of Mining, Metallurgical and Petroleum Engineers, Inc., 1968. pp 10-1--10-31.

Yancey, H. F. and Geer, M. R., "Efficiency and sharpness-to-separation in evaluating coal-washery performance," Trans. AIME 190, 507-517 (1951).

Yancey, H. F. and Geer, M. R., "Some practical aspects of appraising coal-washing results," U.S. Bur. Mines, RI 5153 (1955). 15 pp.

Cubic Spline Method

Ahlberg, J., Nilson, E., and Walsh, J., "The Theory of Splines and Their Applications," New York: Academic Press, Inc., 1967.

Andria, G. D., Byrne, G. D. and Hall, C. A., "Convergence of Cubic Spline Interpolants of Functions Possessing Discontinuities," J. Approximation Theory 8, 150-159 (1973).

Cheney, E. W. and Schurer, F., "Convergence of Cubic Spline Interpolants," J. Approximation Theory 3, 114-116 (1970).

Gerald, C. F., "Applied Numerical Analysis," Reading, Massachusetts: Addison-Wesley Publishing Co., Inc., 1970. pp 290-294.

Schoenberg, I. J., "Approximations with Special Emphasis on Spline Functions," New York: Academic Press, Inc., 1969.

Statistical Procedures

Daniel, C. and Wood, F. S., "Fitting Equations to Data," New York: John Wiley & Sons, Inc., 1980.

Draper, N. R. and Smith, H., "Applied Regression Analysis," New York: John Wiley & Sons, Inc. 1981.

Hoel, P. G., "Introduction to Mathematical Statistics," New York: John Wiley & Sons, Inc., 1971.

Mosteller, F. and Tukey, J. W., "Data Analysis and Regression," Reading, Massachusetts: Addison-Wesley Publishing Co., Inc., 1977.

Metric Equivalents

EPA policy includes use of metric units in all its documents. Although this summary uses nonmetric units for convenience, readers more familiar with the metric system are asked to use the equivalents below:

Nonmetric	Metric
1 in	2.54 cm
1 lb	0.454 kg
1 Btu	1055.06 J
lb/10^6 Btu	429.91 ng/J
1 gpm	3.785 ℓ/min

Tyler Screen Size Mesh Openings

Mesh Size	Sieve Opening	
	in.	mm
9	0.0787	2.00
14	0.0469	1.18
28	0.0234	0.60
100	0.0059	0.15
200	0.0029	0.075

APPENDIX A

DATA SUMMARIES AND DISTRIBUTION CURVES

TABLE A-1. TEST 71 DATA SUMMARY

	9x14	14x28	28x100	9x100
SCREEN ANALYSIS, PERCENT:				
FEED	29.7	28.7	35.2	93.6
CLEAN COAL	22.2	29.9	41.5	93.6
REFUSE	34.9	27.9	30.7	93.6
ASH, PERCENT:				
FEED	33.8	26.7	26.2	28.8
CLEAN COAL	3.4	3.5	3.1	3.3
REFUSE	47.4	44.2	48.2	46.7
TOTAL SULFUR, PERCENT:				
FEED	4.19	3.70	5.62	4.58
CLEAN COAL	0.87	0.66	1.09	0.90
REFUSE	5.67	5.98	9.93	7.16
WEIGHT RECOVERY (YIELD)	30.9	42.9	48.7	41.3
THEORETICAL WEIGHT RECOVERY	33.4	46.0	54.4	45.1
WEIGHT RECOVERY EFFICIENCY	92.6	93.3	89.5	91.5
ASH ERROR	0.3	0.3	0.4	0.3
FLOAT IN REFUSE...% OF PRODUCT	6.7	11.6	13.2	10.7
SINK IN CLEAN COAL	7.4	5.5	0.8	3.9
TOTAL MISPLACED MATERIAL..% OF FEED	6.9	9.0	7.2	7.9
NEAR GRAVITY MATERIAL	49.1	60.7	61.6	57.5
SPECIFIC GRAVITY OF SEPARATION	1.305	1.312	1.312	1.313
PROBABLE ERROR	0.017	0.019	0.017	0.015
IMPERFECTION	0.056	0.060	0.055	0.049
ERROR AREA	10.	13.	11.	11.
DISTRIBUTION, PERCENT TO CLEANED COAL (SPECIFIC GRAVITY FRACTION):				
FLOAT 1.27	95.4	92.5	90.5	92.2
1.27-1.29	81.3	81.2	83.5	82.1
1.29-1.32	52.7	64.7	83.7	69.1
1.32-1.35	3.0	14.8	13.2	9.5
1.35-1.40	0.0	4.3	0.0	1.5
1.40-1.50	0.0	0.0	0.0	0.0
1.50-1.60	0.0	0.0	0.0	0.0
1.60-1.80	0.0	0.0	0.0	0.0
SINK 1.80	0.0	0.0	0.0	0.0

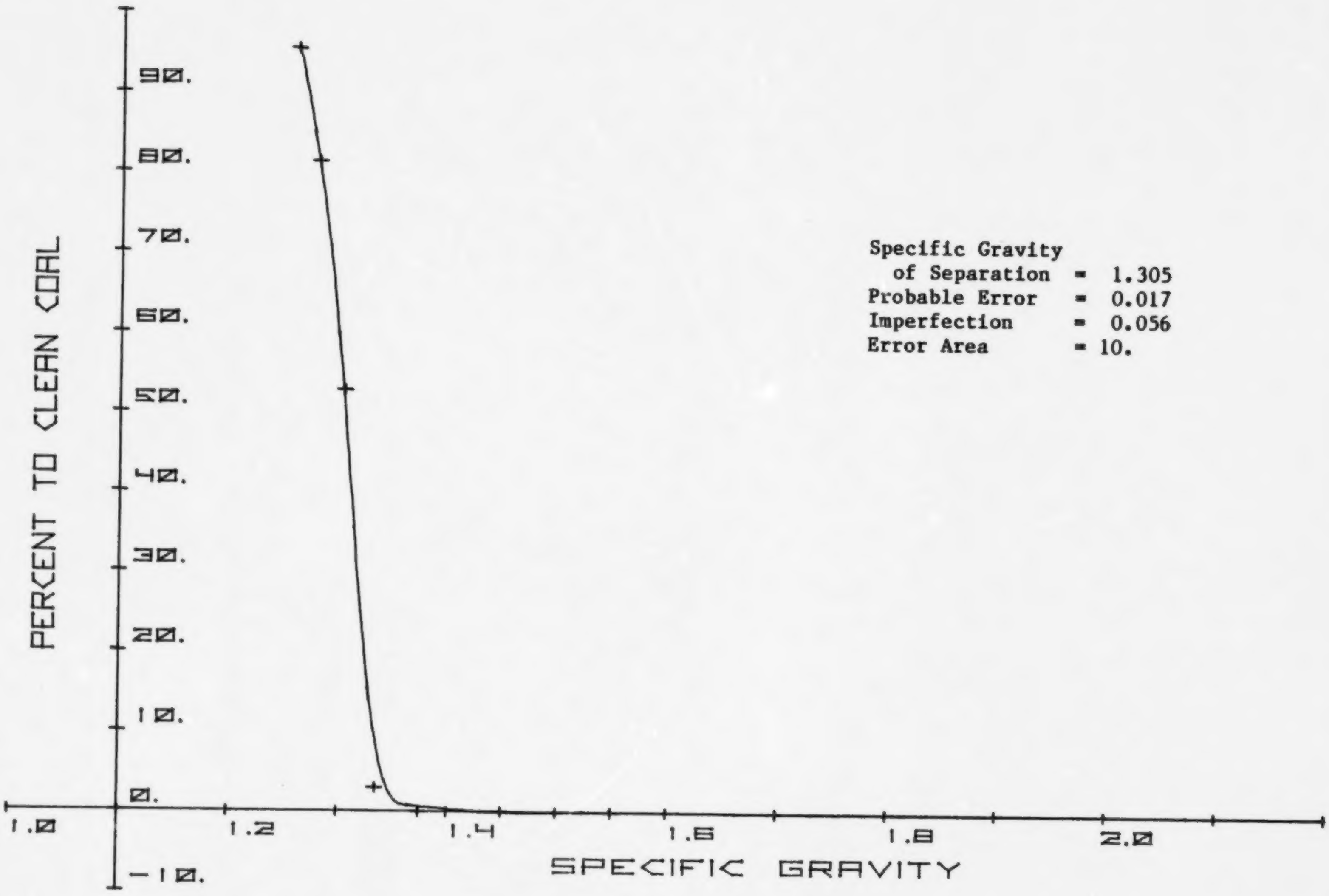
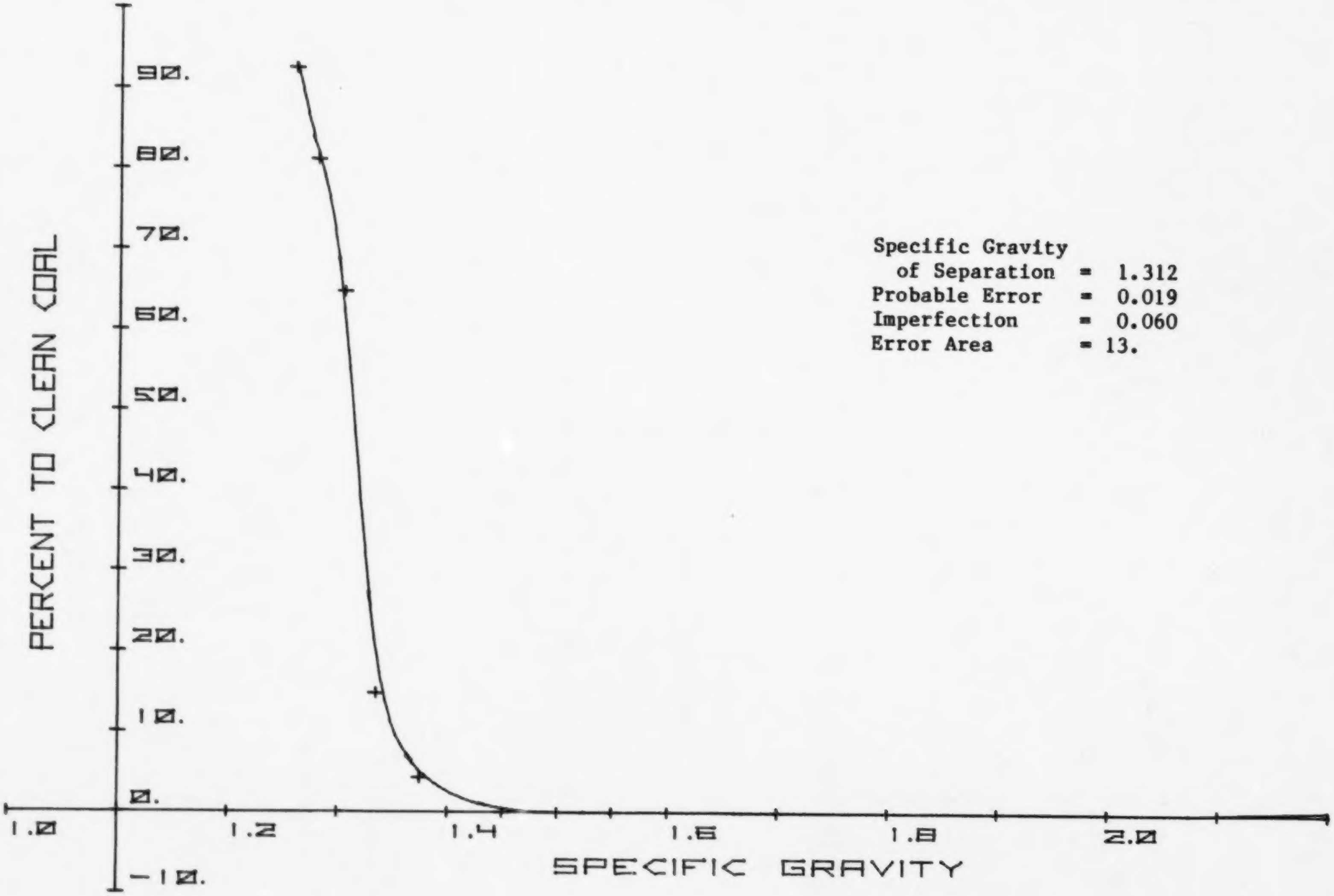


Figure A-1. Distribution Curve, Test No. 71 (9 x 14)



Specific Gravity
  of Separation = 1.312
Probable Error = 0.019
Imperfection = 0.060
Error Area = 13.

Figure A-2. Distribution Curve, Test No. 71 (14 x 28)

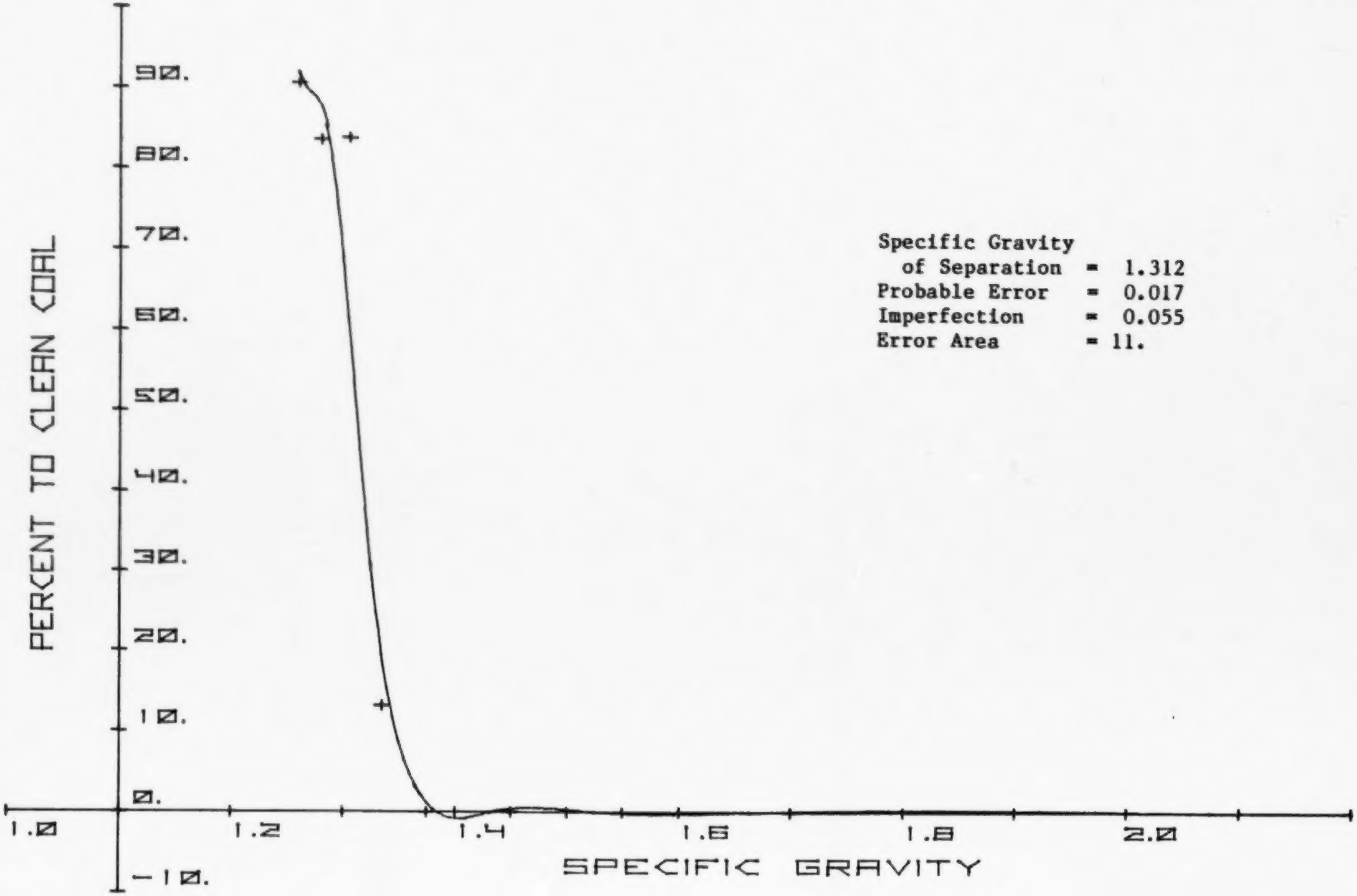
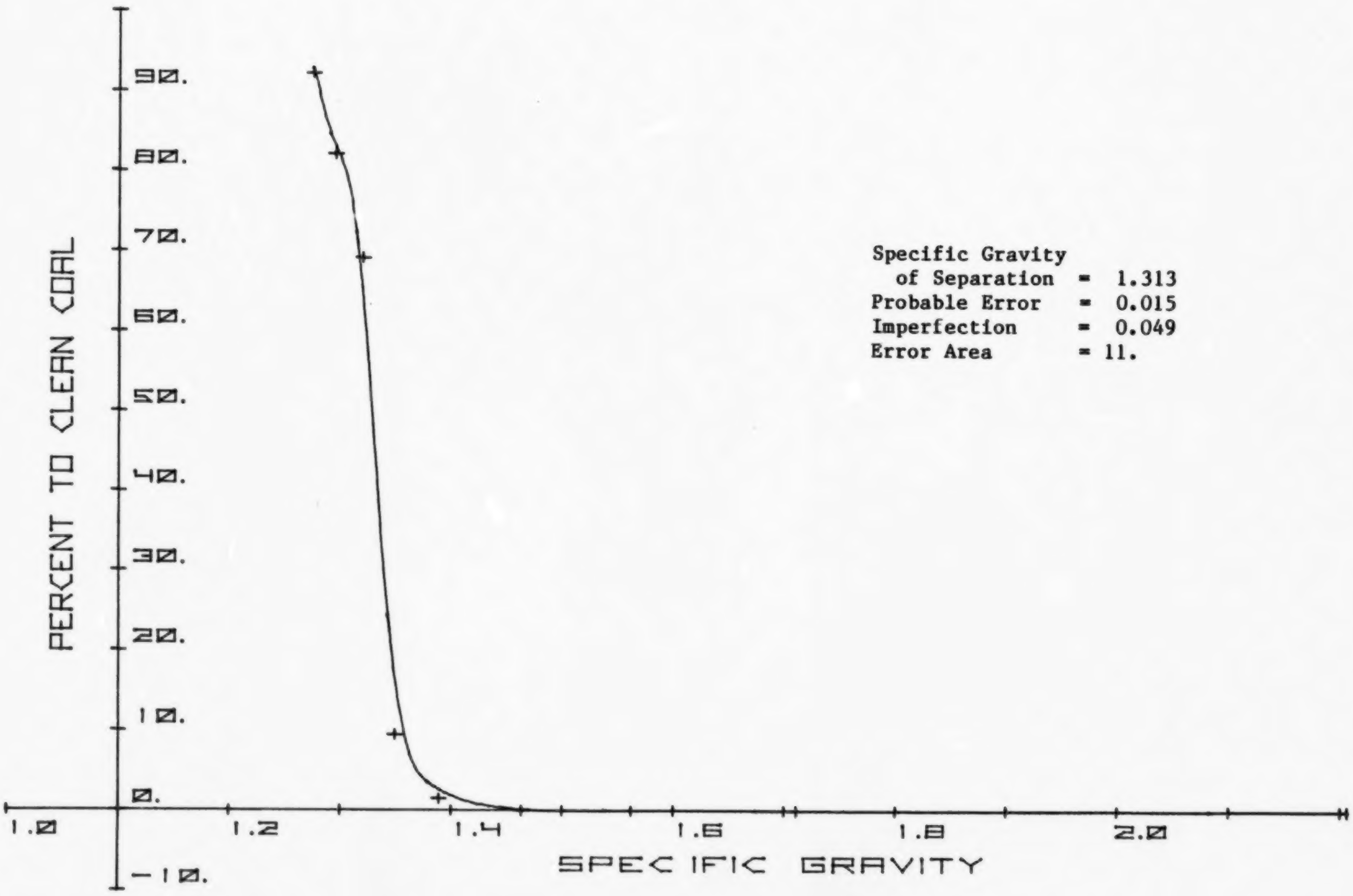


Figure A-3. Distribution Curve, Test No. 71 (28 x 100)

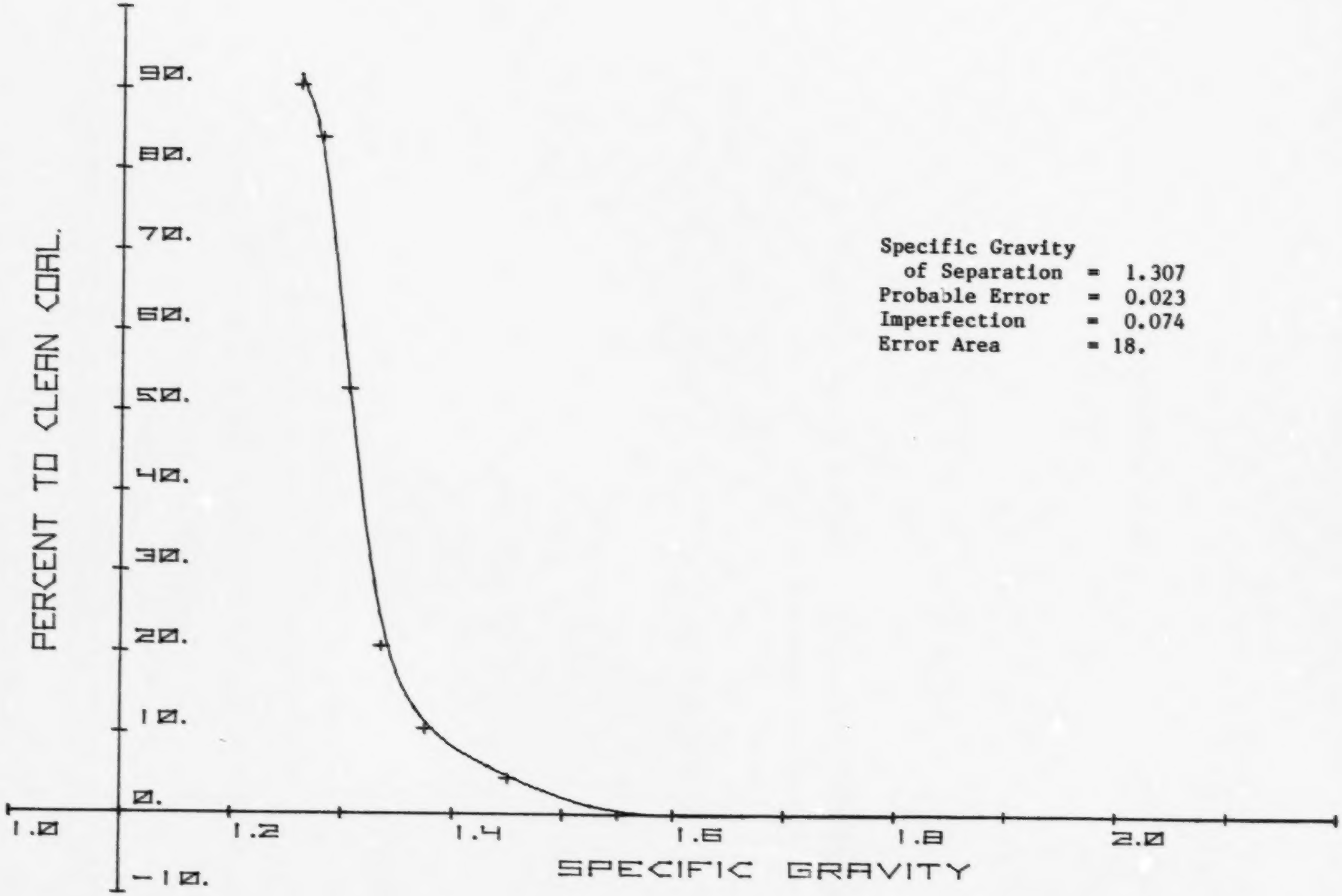


Specific Gravity
  of Separation = 1.313
Probable Error = 0.015
Imperfection = 0.049
Error Area = 11.

Figure A-4. Distribution Curve, Test No. 71 (9 x 100)

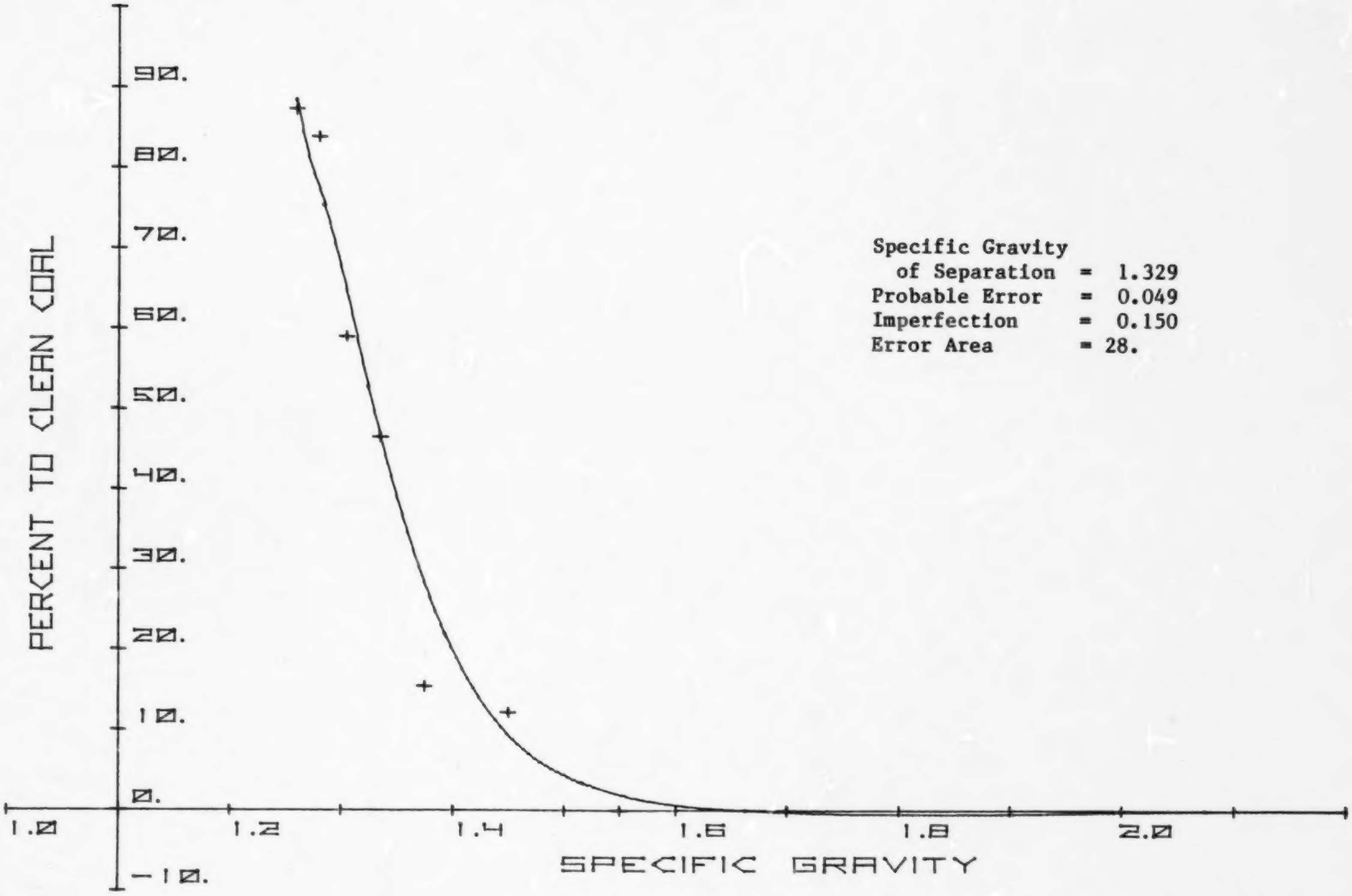
TABLE A-2. TEST 72 DATA SUMMARY

	9x14	14x28	28x100	9x100
SCREEN ANALYSIS, PERCENT:				
FEED............................	34.5	30.8	30.3	95.6
CLEAN COAL......................	30.3	34.0	31.3	95.6
REFUSE..........................	37.2	28.7	29.7	95.7
ASH, PERCENT:				
FEED............................	27.3	22.9	32.3	27.5
CLEAN COAL......................	4.1	4.2	6.6	4.9
REFUSE..........................	39.4	37.1	49.7	41.9
TOTAL SULFUR, PERCENT:				
FEED............................	3.34	3.38	6.18	4.25
CLEAN COAL......................	1.05	0.96	1.23	1.08
REFUSE..........................	4.53	5.21	9.51	6.28
WEIGHT RECOVERY (YIELD)............	34.3	43.1	40.3	39.0
THEORETICAL WEIGHT RECOVERY........	41.0	51.9	54.9	50.8
WEIGHT RECOVERY EFFICIENCY.........	83.5	83.1	73.4	76.8
ASH ERROR..........................	0.7	0.9	2.8	1.5
FLOAT IN REFUSE...% OF PRODUCT.....	9.7	17.9	19.6	14.5
SINK IN CLEAN COAL.................	17.0	12.5	11.2	17.0
TOTAL MISPLACED MATERIAL..% OF FEED	12.2	15.6	16.2	15.5
NEAR GRAVITY MATERIAL..............	57.7	65.4	29.8	58.7
SPECIFIC GRAVITY OF SEPARATION.....	1.307	1.329	1.369	1.321
PROBABLE ERROR.....................	0.023	0.049	0.077	0.043
IMPERFECTION.......................	0.074	0.150	0.207	0.134
ERROR AREA.........................	18.	28.	63.	37.
DISTRIBUTION, PERCENT TO CLEANED COAL (SPECIFIC GRAVITY FRACTION):				
FLOAT 1.27	90.5	87.3	82.7	86.7
1.27-1.29	84.1	83.9	72.9	81.8
1.29-1.32	52.8	59.0	71.5	59.0
1.32-1.35	20.8	46.5	71.7	42.7
1.35-1.40	10.6	15.5	48.1	23.0
1.40-1.50	4.5	12.2	22.7	12.8
1.50-1.60	0.0	0.0	10.6	3.0
1.60-1.80	0.0	0.0	9.0	3.6
SINK 1.80	0.0	0.0	2.7	1.0



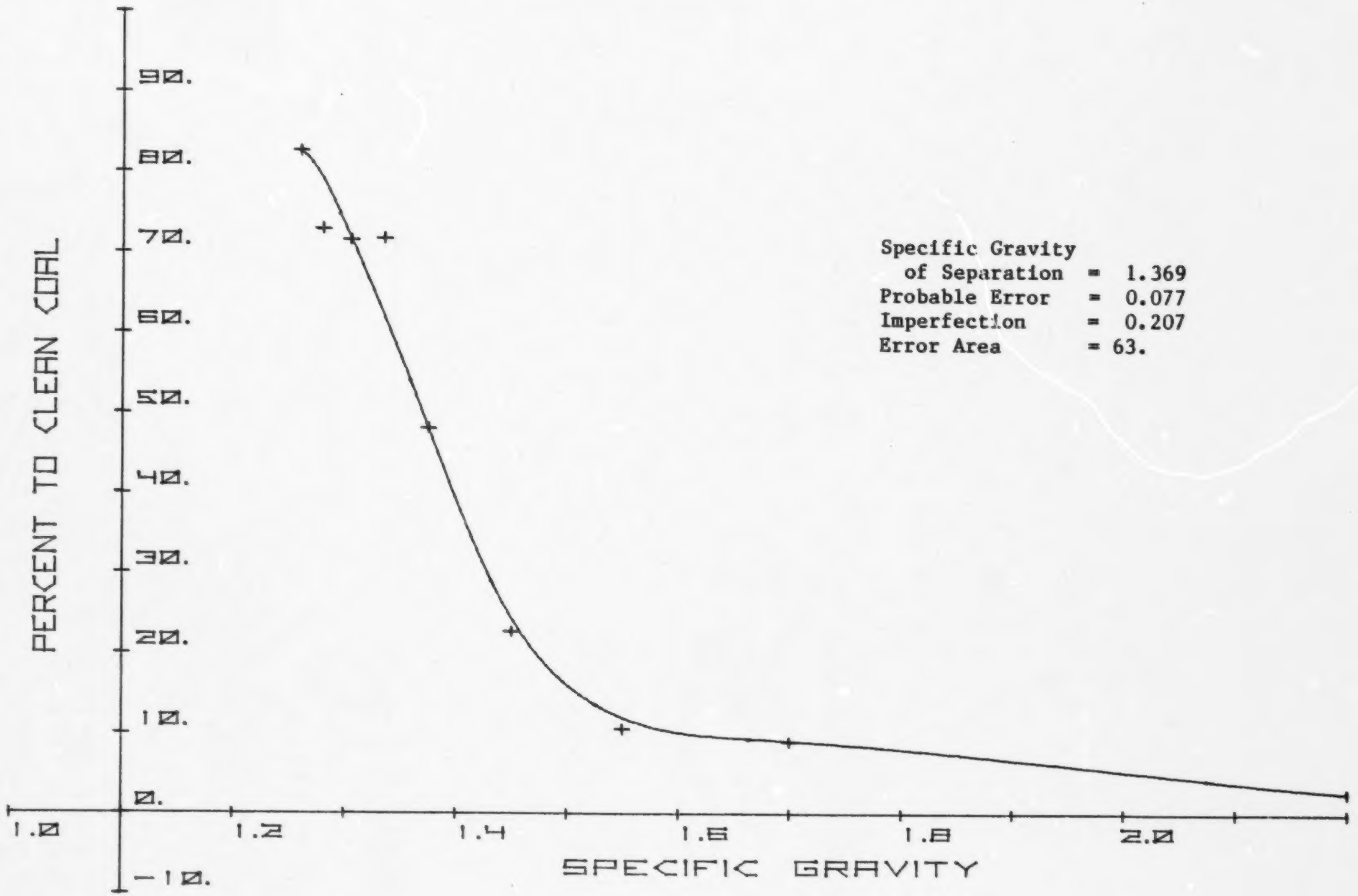
Specific Gravity
of Separation = 1.307
Probable Error = 0.023
Imperfection = 0.074
Error Area = 18.

Figure A-5. Distribution Curve, Test No. 72 (9 x 14)



Specific Gravity
of Separation = 1.329
Probable Error = 0.049
Imperfection = 0.150
Error Area = 28.

Figure A-6. Distribution Curve, Test No. 72 (14 x 28)



Specific Gravity
  of Separation = 1.369
Probable Error = 0.077
Imperfection = 0.207
Error Area = 63.

Figure A-7. Distribution Curve, Test No. 72 (28 x 100)

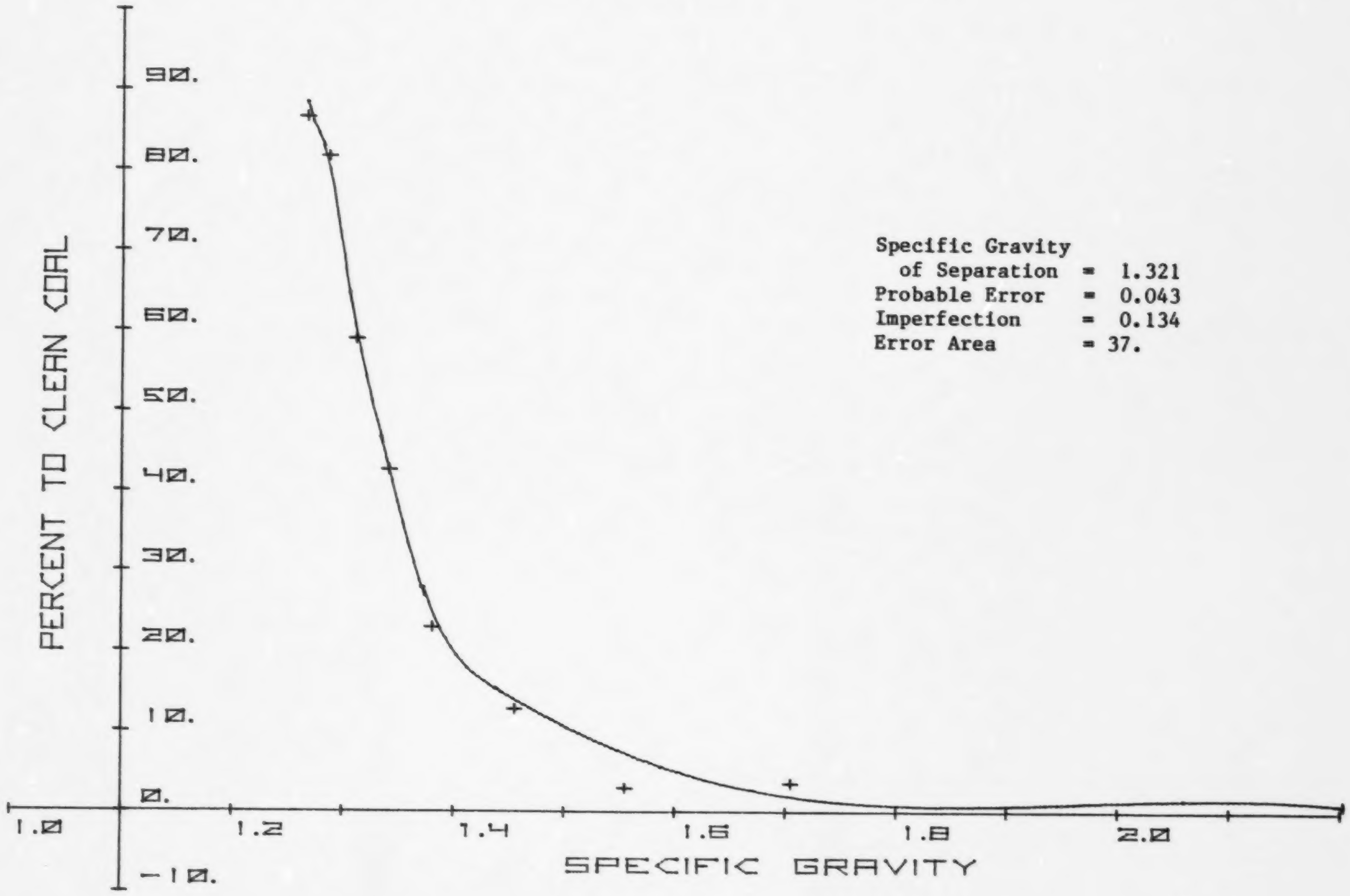
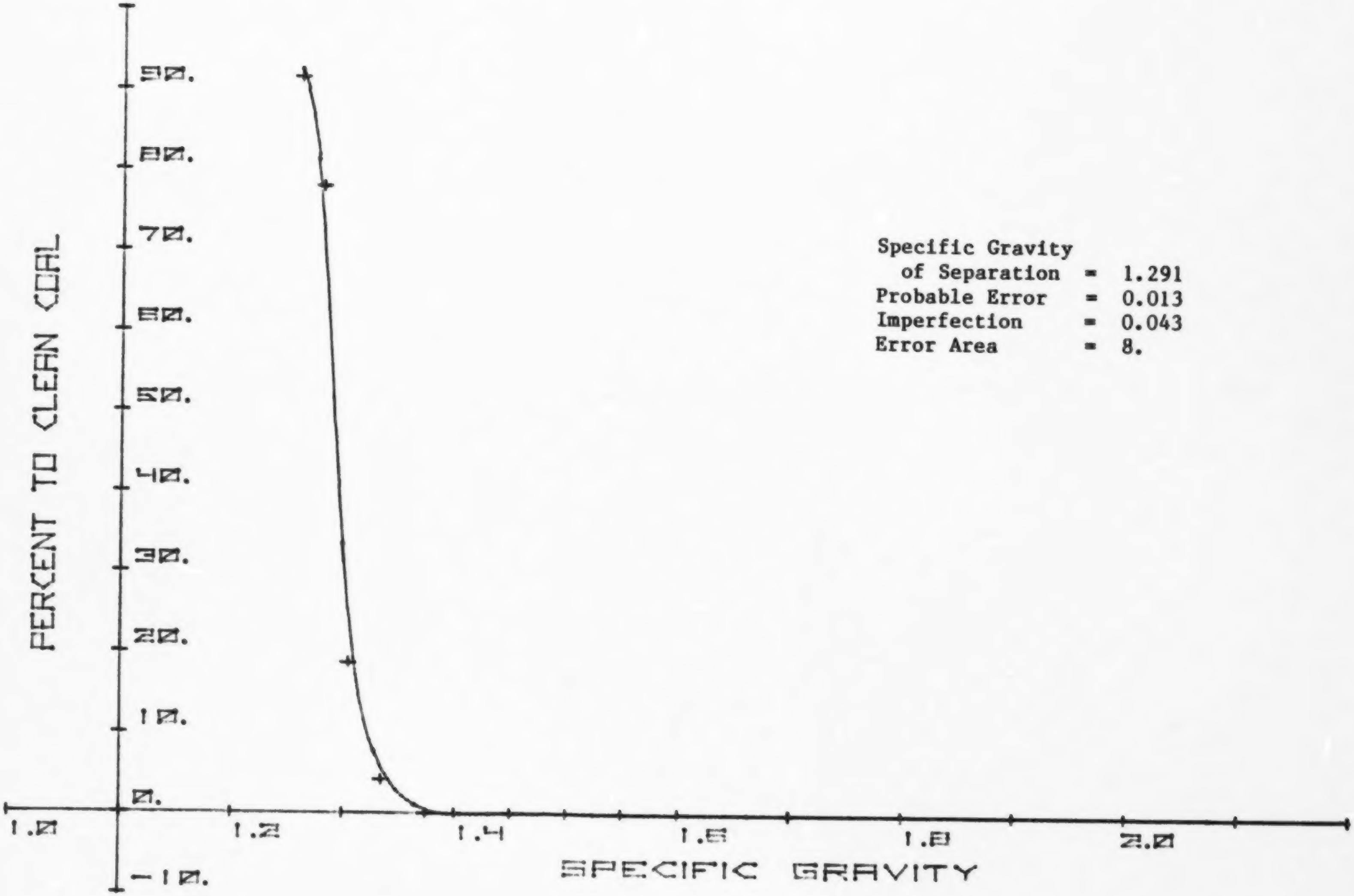


Figure A-8. Distribution Curve, Test No. 72 (9 x 100)

TABLE A-3. TEST 73 DATA SUMMARY

	9x14	14x28	28x100	9x100
SCREEN ANALYSIS, PERCENT:				
FEED	31.3	28.8	34.4	94.5
CLEAN COAL	20.7	28.8	43.3	92.8
REFUSE	38.0	28.8	28.8	95.5
ASH, PERCENT:				
FEED	33.3	24.8	25.9	28.0
CLEAN COAL	3.0	3.1	4.4	3.7
REFUSE	43.7	38.4	46.2	42.9
TOTAL SULFUR, PERCENT:				
FEED	3.52	3.38	3.71	3.55
CLEAN COAL	1.05	1.09	0.91	1.00
REFUSE	4.36	4.83	6.37	5.11
WEIGHT RECOVERY (YIELD)	25.6	38.7	48.7	38.0
THEORETICAL WEIGHT RECOVERY	29.8	43.0	61.7	47.4
WEIGHT RECOVERY EFFICIENCY	86.0	90.0	78.9	80.1
ASH ERROR	0.4	0.4	1.5	0.9
FLOAT IN REFUSE...% OF PRODUCT	6.1	11.2	16.1	10.5
SINK IN CLEAN COAL	5.1	6.4	9.0	10.4
TOTAL MISPLACED MATERIAL..% OF FEED	5.9	9.4	12.6	10.4
NEAR GRAVITY MATERIAL	48.0	60.4	62.9	57.0
SPECIFIC GRAVITY OF SEPARATION	1.291	1.296	1.327	1.300
PROBABLE ERROR	0.013	0.017	0.042	0.022
IMPERFECTION	0.043	0.057	0.129	0.072
ERROR AREA	8.	14.	44.	22.
DISTRIBUTION, PERCENT TO CLEANED COAL (SPECIFIC GRAVITY FRACTION):				
FLOAT 1.27	91.5	89.9	90.2	90.4
1.27-1.29	77.9	80.7	85.3	81.7
1.29-1.32	18.7	33.6	68.3	41.0
1.32-1.35	4.2	10.6	43.8	18.6
1.35-1.40	0.0	6.2	25.2	9.8
1.40-1.50	0.0	3.4	13.8	5.3
1.50-1.60	0.0	0.0	9.4	2.8
1.60-1.80	0.0	0.0	4.9	1.3
SINK 1.80	0.0	0.0	1.8	0.6



Specific Gravity
  of Separation = 1.291
Probable Error = 0.013
Imperfection = 0.043
Error Area = 8.

Figure A-9. Distribution Curve, Test No. 73 (9 x 14)

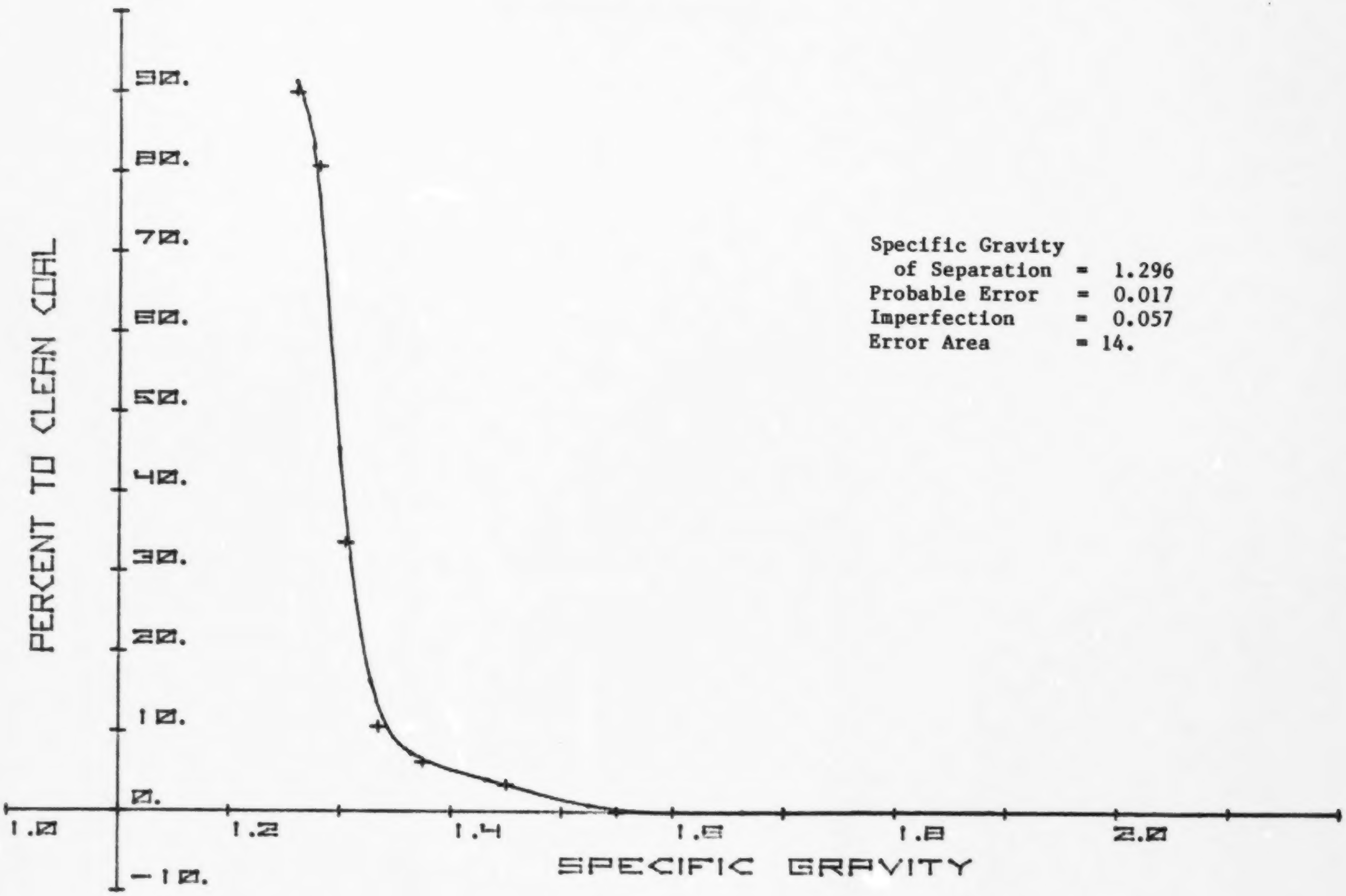
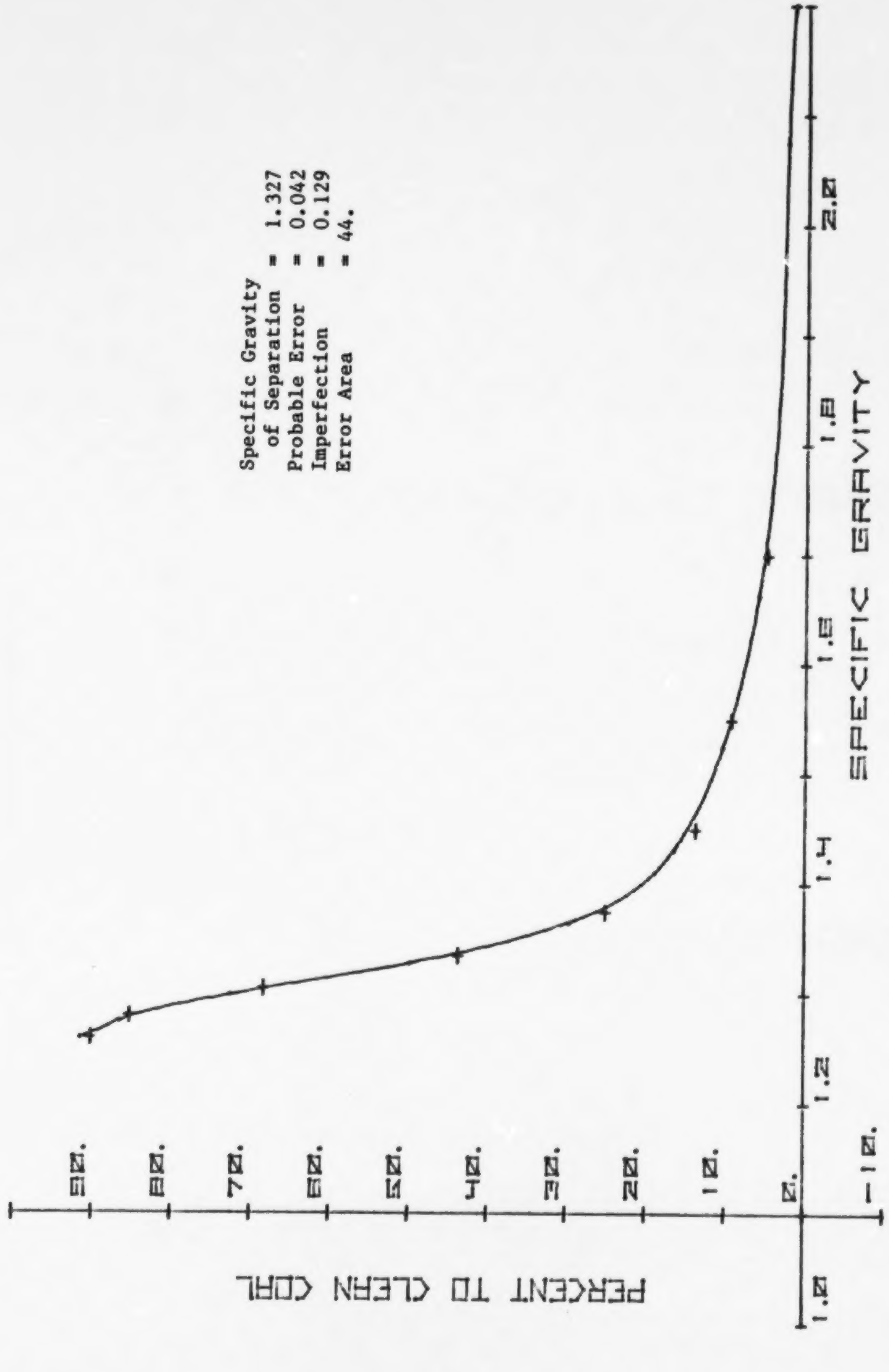


Figure A-10. Distribution Curve, Test No. 73 (14 x 28)



Specific Gravity
  of Separation = 1.327
Probable Error = 0.042
Imperfection = 0.129
Error Area = 44.

Figure A-11. Distribution Curve, Test No. 73 (28 x 100)

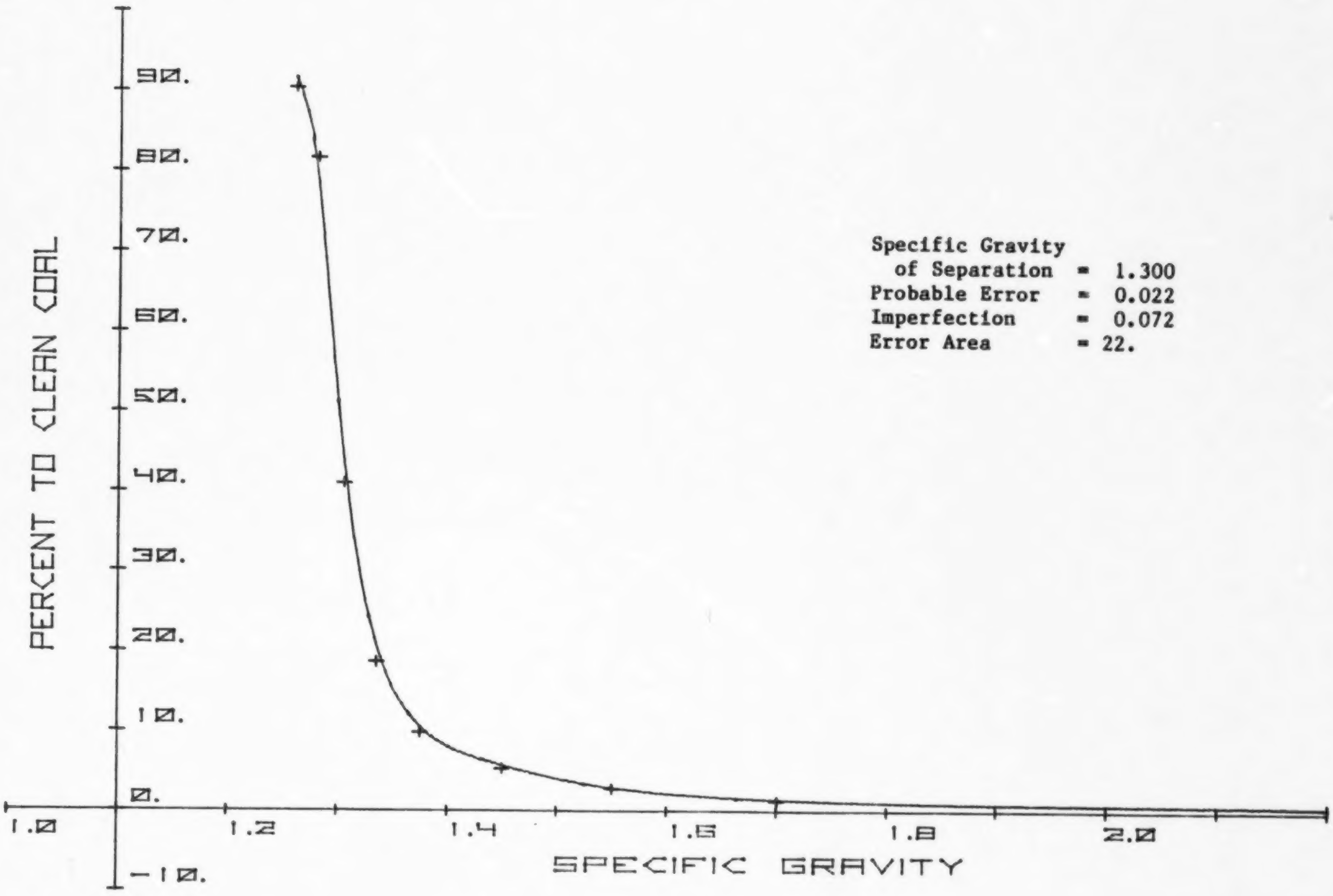


Figure A-12. Distribution Curve, Test No. 73 (9 x 100)

TABLE A-4. TEST 74 DATA SUMMARY

	9x14	14x28	28x100	9x100
SCREEN ANALYSIS, PERCENT:				
FEED	31.1	30.1	34.1	95.3
CLEAN COAL	24.4	31.8	38.7	95.0
REFUSE	35.3	29.0	31.2	95.5
ASH, PERCENT:				
FEED	26.1	24.6	29.2	26.8
CLEAN COAL	4.0	4.0	4.8	4.3
REFUSE	35.7	38.8	48.3	40.8
TOTAL SULFUR, PERCENT:				
FEED	3.11	3.84	5.40	4.16
CLEAN COAL	1.09	1.02	1.18	1.10
REFUSE	3.99	5.80	8.71	6.08
WEIGHT RECOVERY (YIELD)	30.3	40.8	43.9	38.5
THEORETICAL WEIGHT RECOVERY	40.6	52.2	55.0	50.8
WEIGHT RECOVERY EFFICIENCY	74.8	78.2	79.7	75.8
ASH ERROR	0.5	1.1	1.7	1.3
FLOAT IN REFUSE...% OF PRODUCT	10.0	16.5	19.6	14.7
SINK IN CLEAN COAL	15.7	10.6	9.6	15.2
TOTAL MISPLACED MATERIAL..% OF FEED	11.7	14.1	15.2	14.9
NEAR GRAVITY MATERIAL	57.0	63.1	57.7	58.7
SPECIFIC GRAVITY OF SEPARATION	1.300	1.310	1.364	1.313
PROBABLE ERROR	0.020	0.028	0.091	0.032
IMPERFECTION	0.067	0.090	0.250	0.103
ERROR AREA	13.	25.	58.	33.
DISTRIBUTION, PERCENT TO CLEANED COAL (SPECIFIC GRAVITY FRACTION):				
FLOAT 1.27	89.3	86.0	84.0	86.0
1.27-1.29	75.6	75.4	80.9	77.2
1.29-1.32	43.8	65.6	74.9	59.6
1.32-1.35	9.0	23.1	59.5	29.0
1.35-1.40	4.3	15.3	39.0	17.5
1.40-1.50	0.0	8.8	34.7	15.5
1.50-1.60	0.0	4.5	4.6	3.1
1.60-1.80	0.0	0.0	7.5	2.3
SINK 1.80	0.0	0.0	1.8	0.7

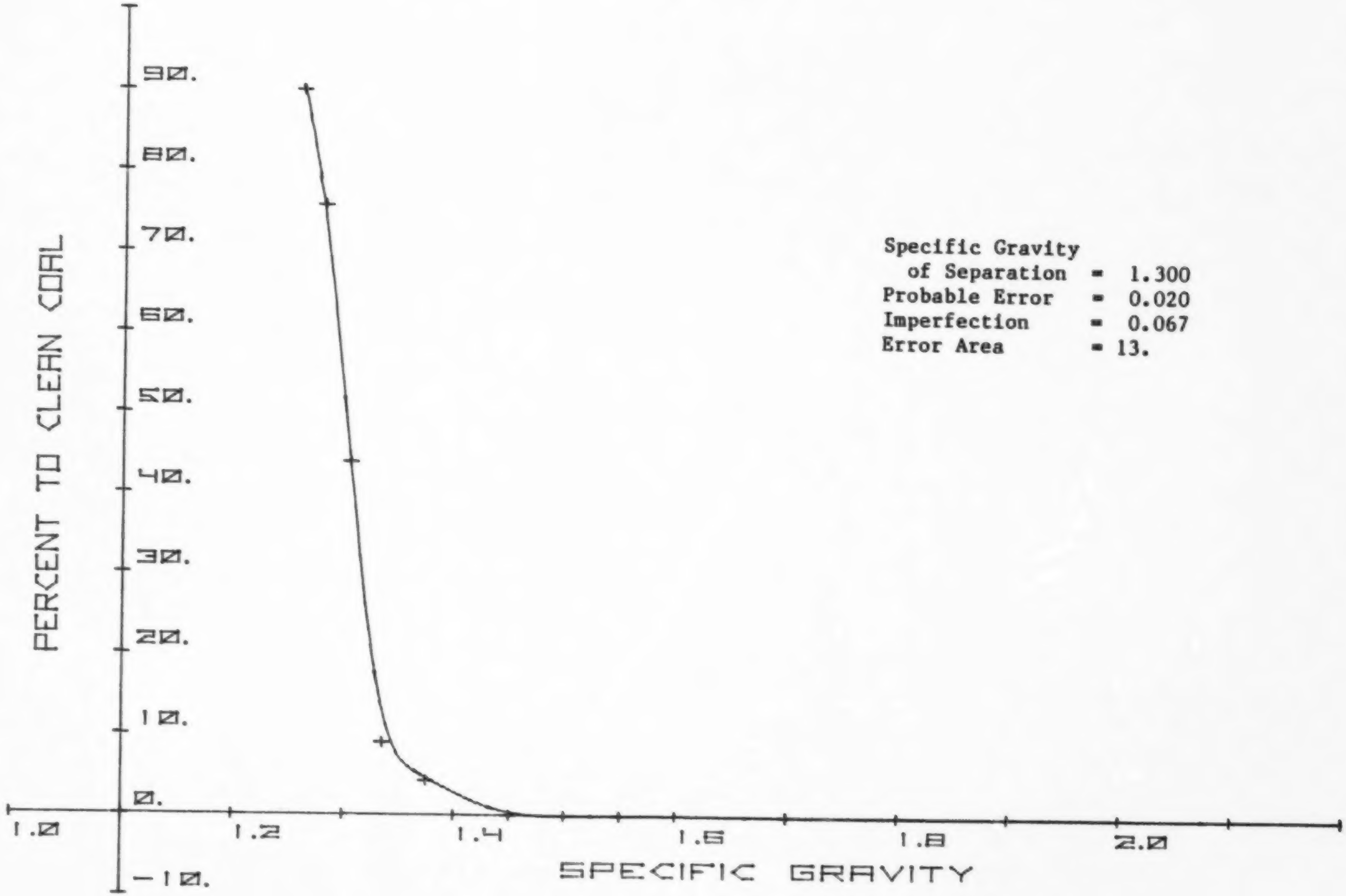
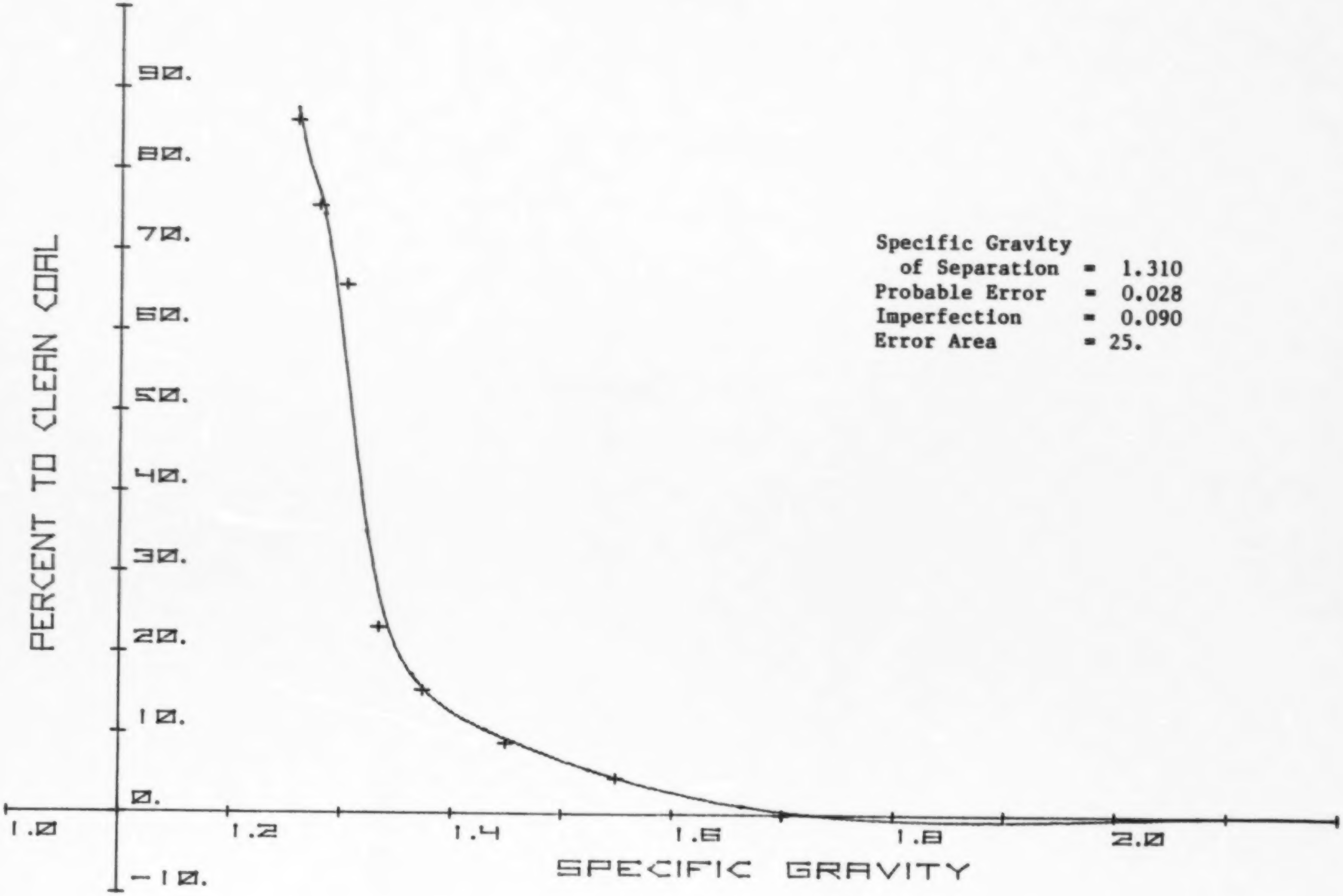


Figure A-13. Distribution Curve, Test No. 74 (9 x 14)



Specific Gravity
  of Separation = 1.310
Probable Error = 0.028
Imperfection = 0.090
Error Area = 25.

Figure A-14. Distribution Curve, Test No. 74 (14 x 28)

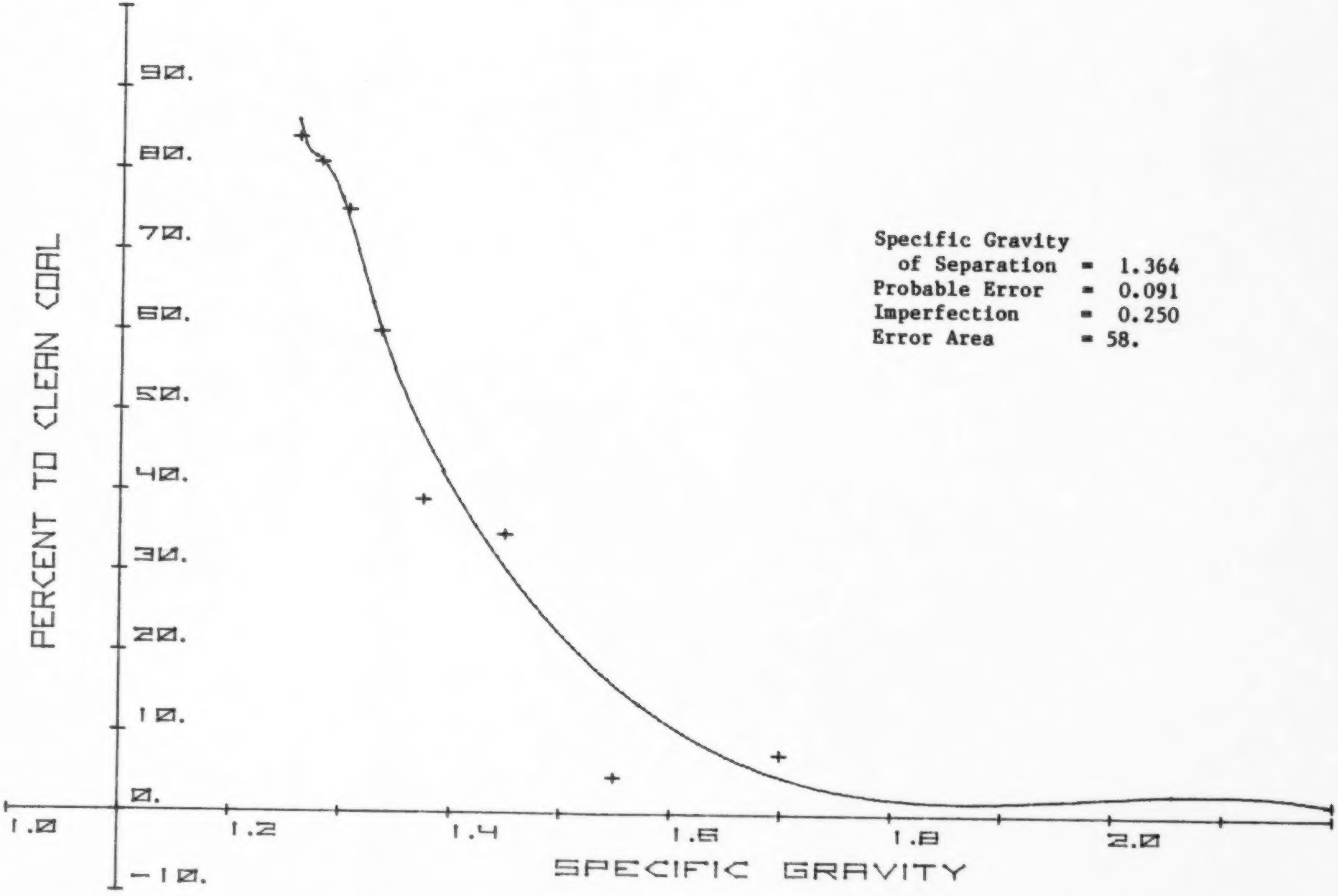


Figure A-15. Distribution Curve, Test No. 74 (28 x 100)

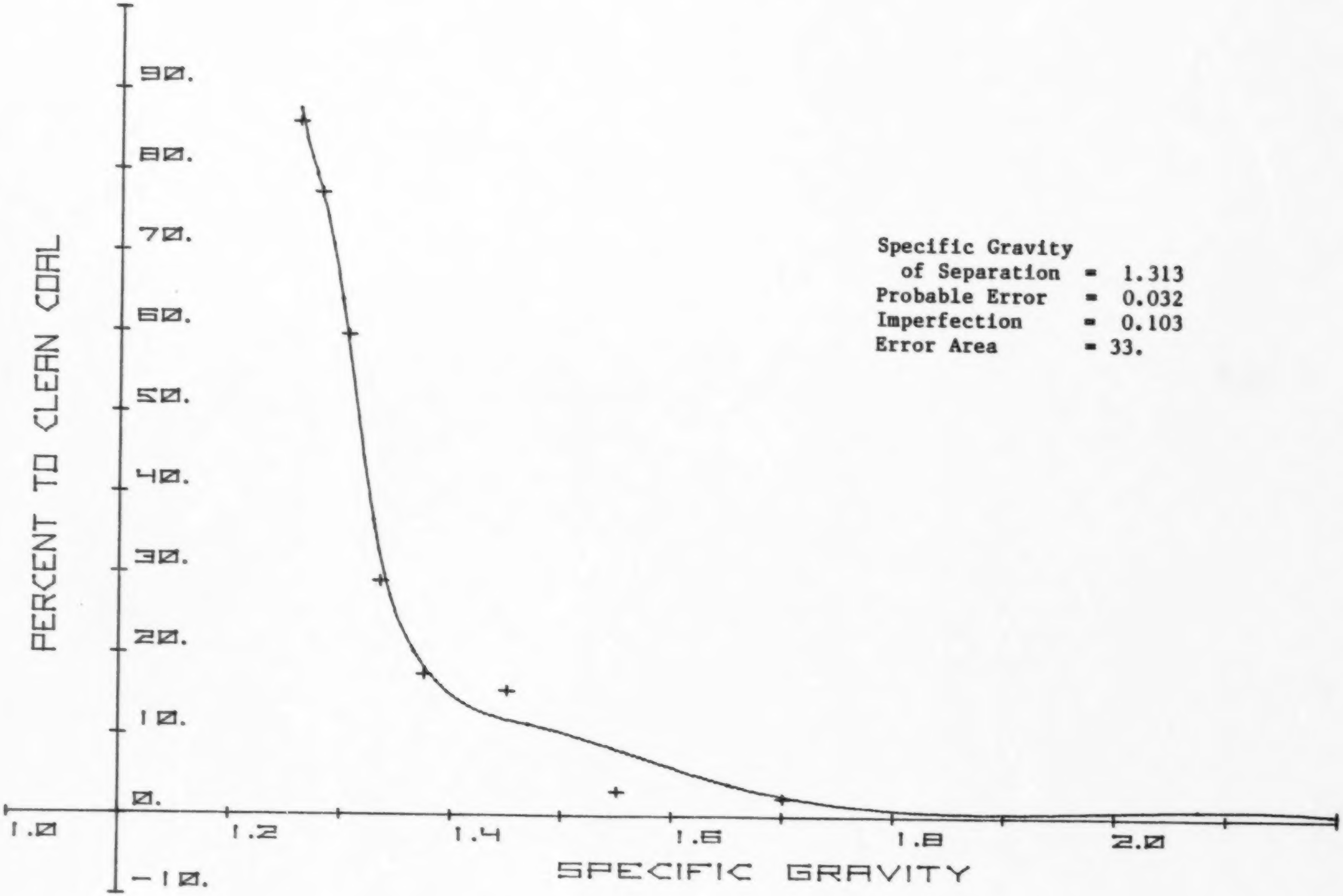
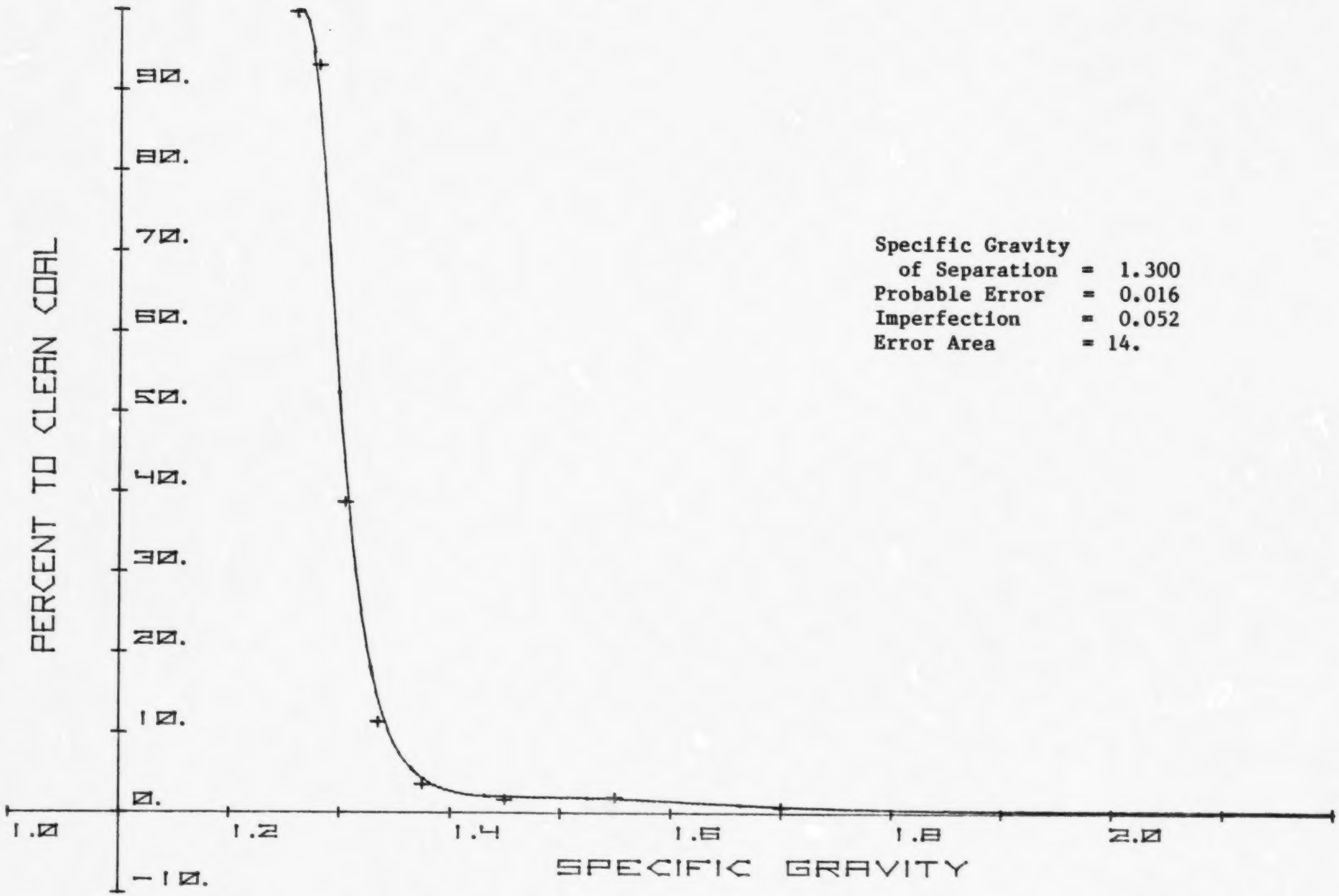


Figure A-16. Distribution Curve, Test No. 74 (9 x 100)

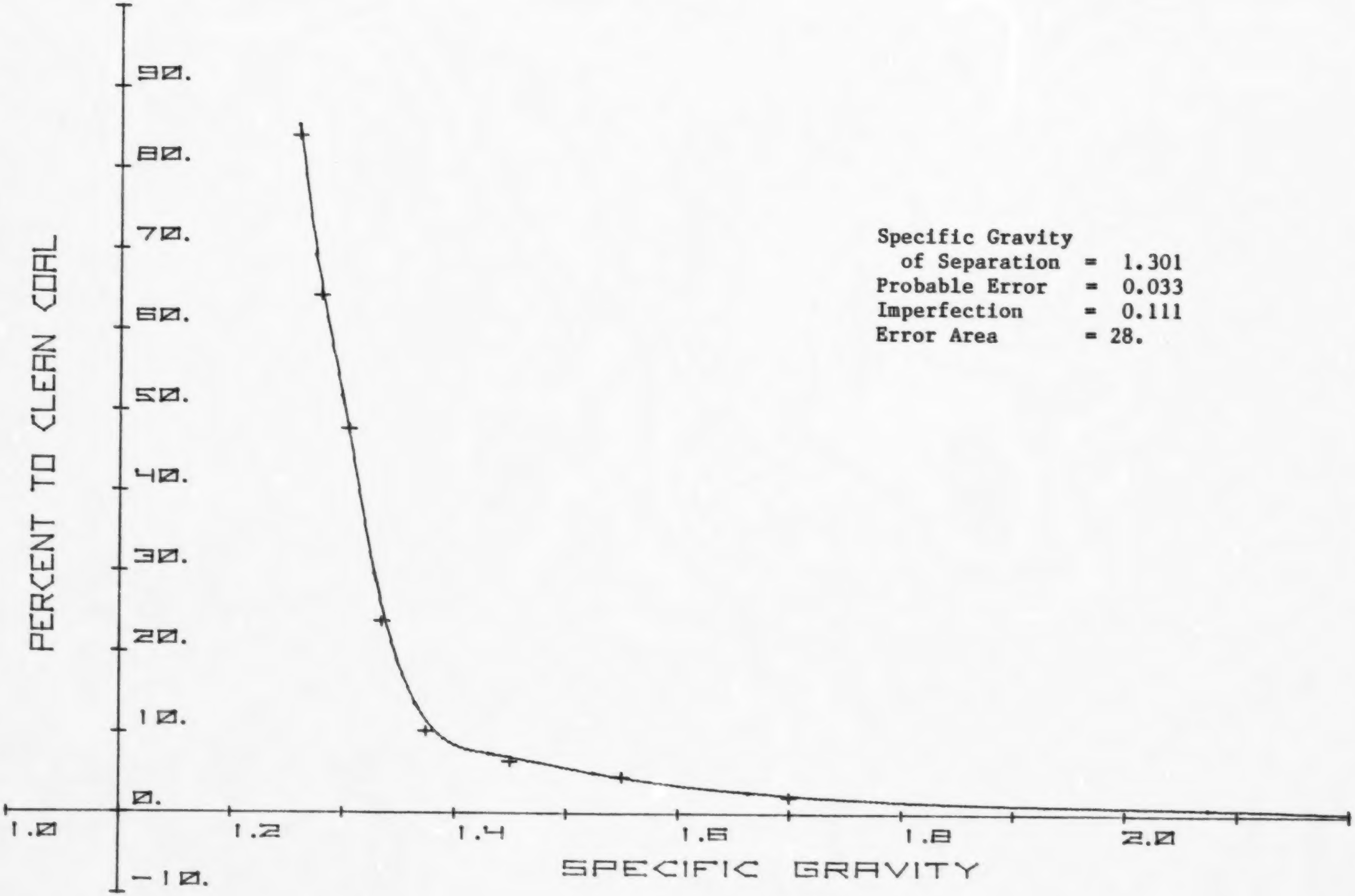
TABLE A-5. TEST 75 DATA SUMMARY

	9x14	14x28	28x100	9x100
SCREEN ANALYSIS, PERCENT:				
FEED..............................	25.6	29.1	39.2	94.0
CLEAN COAL........................	17.7	28.6	46.1	92.4
REFUSE............................	30.3	29.5	35.1	94.9
ASH, PERCENT:				
FEED..............................	33.3	26.4	27.5	28.8
CLEAN COAL........................	3.5	3.5	5.1	4.3
REFUSE............................	43.7	39.7	45.2	43.0
TOTAL SULFUR, PERCENT:				
FEED..............................	3.26	3.41	4.98	4.02
CLEAN COAL........................	0.96	0.46	1.00	0.83
REFUSE............................	4.07	5.13	8.10	5.89
WEIGHT RECOVERY (YIELD)............	25.9	36.7	44.0	36.8
THEORETICAL WEIGHT RECOVERY........	26.6	47.8	56.2	46.4
WEIGHT RECOVERY EFFICIENCY.........	97.3	76.8	78.3	79.3
ASH ERROR..........................	0.1	1.1	1.3	1.0
FLOAT IN REFUSE...% OF PRODUCT.....	3.2	16.1	22.9	13.8
SINK IN CLEAN COAL.................	13.7	16.1	12.1	17.3
TOTAL MISPLACED MATERIAL..% OF FEED	5.9	16.1	18.1	15.1
NEAR GRAVITY MATERIAL..............	45.0	60.1	60.7	55.9
SPECIFIC GRAVITY OF SEPARATION.....	1.300	1.301	1.333	1.308
PROBABLE ERROR.....................	0.016	0.033	0.103	0.035
IMPERFECTION.......................	0.052	0.111	0.310	0.114
ERROR AREA.........................	14.	28.	80.	43.
DISTRIBUTION, PERCENT TO CLEANED COAL (SPECIFIC GRAVITY FRACTION):				
FLOAT 1.27	99.7	84.1	83.0	86.0
1.27-1.29	93.1	64.2	68.9	71.8
1.29-1.32	38.7	47.7	69.7	52.9
1.32-1.35	11.3	23.8	48.7	28.0
1.35-1.40	3.6	10.2	34.6	17.2
1.40-1.50	1.7	6.5	26.8	13.0
1.50-1.60	2.0	4.6	21.3	11.0
1.60-1.80	0.8	2.1	16.4	6.3
SINK 1.80	0.2	0.4	0.1	0.2



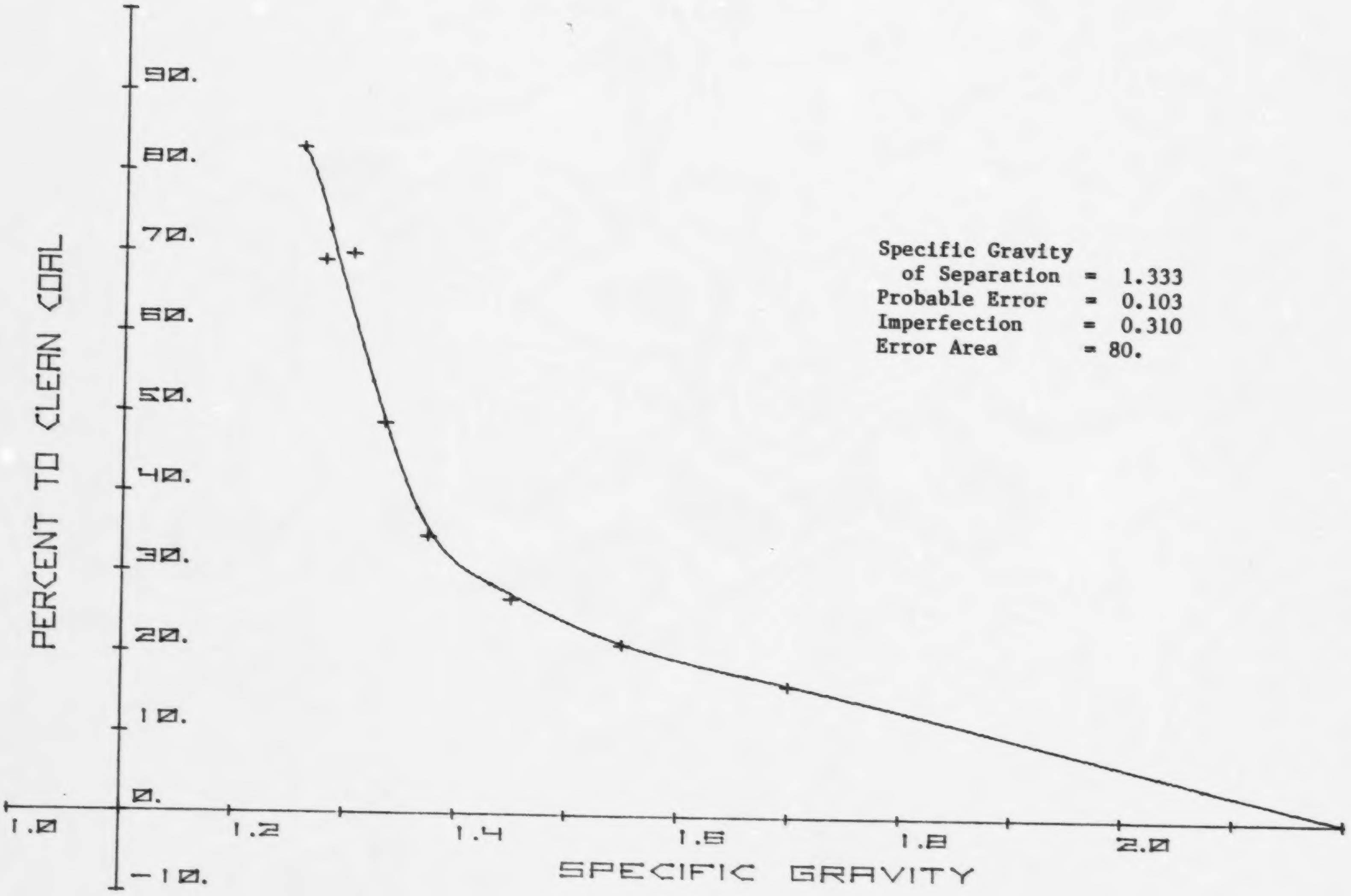
Specific Gravity
  of Separation = 1.300
Probable Error = 0.016
Imperfection = 0.052
Error Area = 14.

Figure A-17. Distribution Curve, Test No. 75 (9 x 14)



Specific Gravity
of Separation = 1.301
Probable Error = 0.033
Imperfection = 0.111
Error Area = 28.

Figure A-18. Distribution Curve, Test No. 75 (14 x 28)



Specific Gravity
of Separation = 1.333
Probable Error = 0.103
Imperfection = 0.310
Error Area = 80.

Figure A-19. Distribution Curve, Test No. 75 (28 x 100)

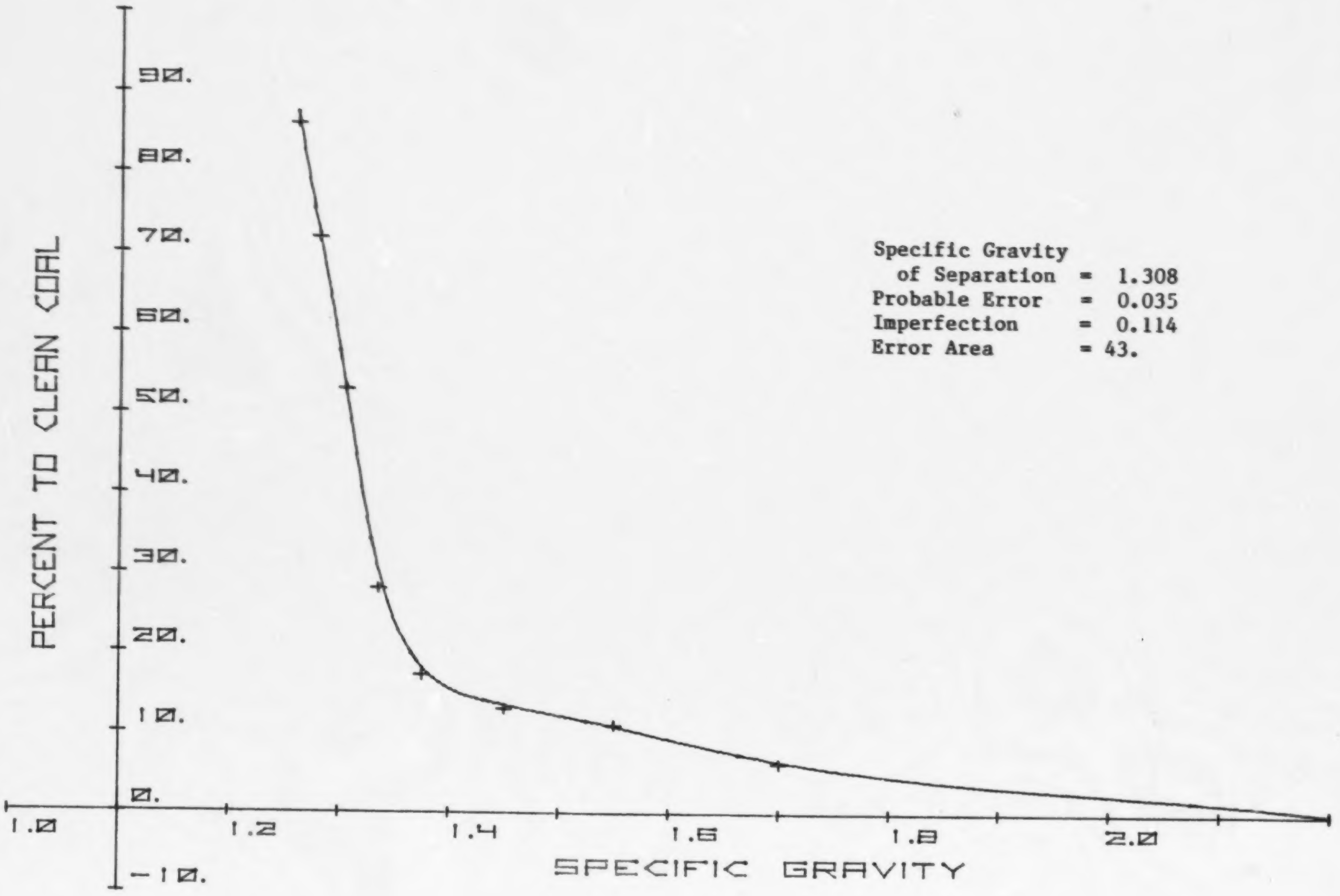
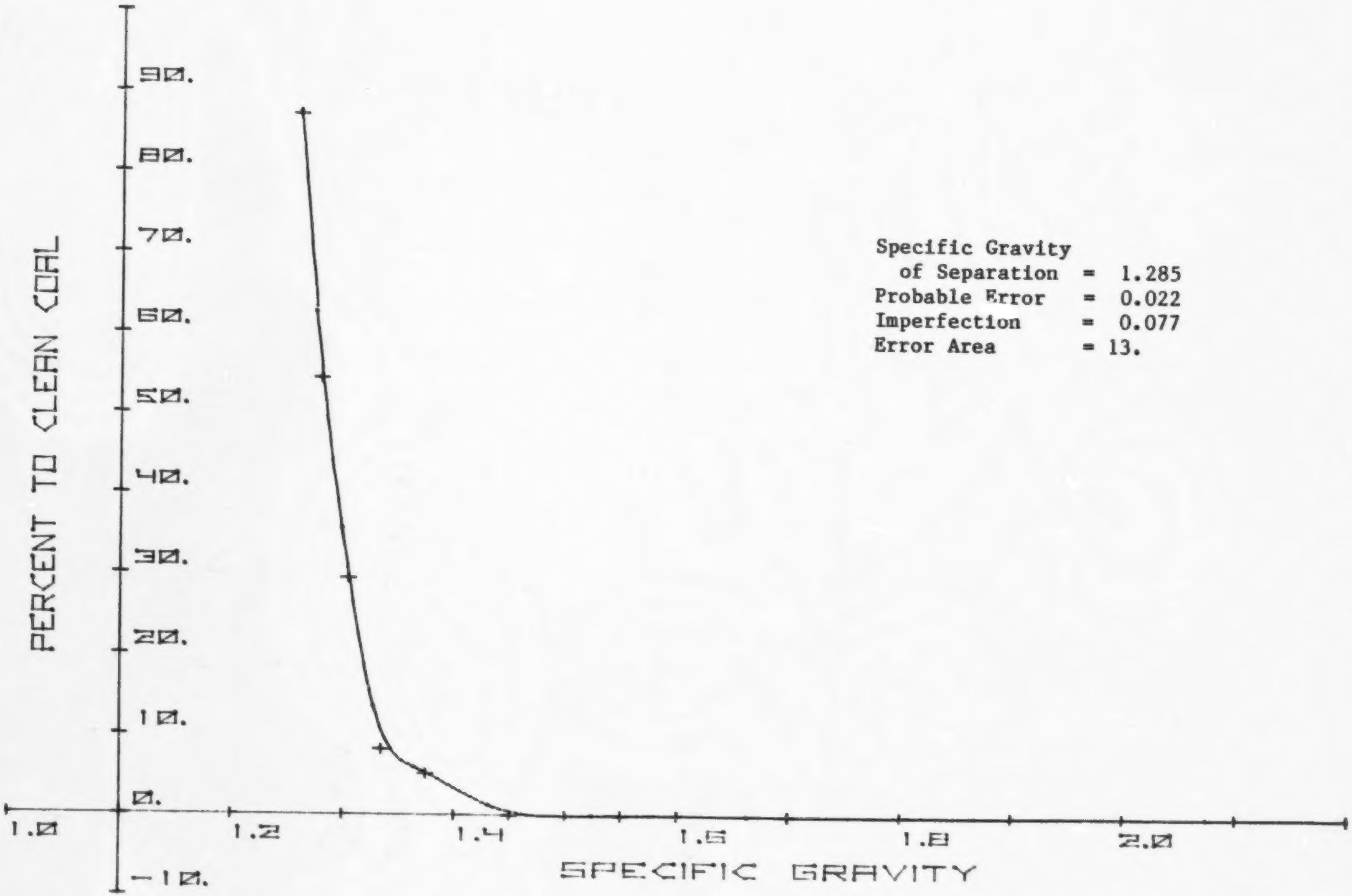


Figure A-20. Distribution Curve, Test No. 75 (9 x 100)

TABLE A-6. TEST 76 DATA SUMMARY

	9x14	14x28	28x100	9x100
SCREEN ANALYSIS, PERCENT:				
FEED	33.7	29.1	32.3	95.1
CLEAN COAL	24.0	29.7	39.7	93.4
REFUSE	39.1	28.8	28.1	96.0
ASH, PERCENT:				
FEED	29.3	27.5	27.5	28.1
CLEAN COAL	3.4	3.5	4.6	3.9
REFUSE	38.2	41.3	45.7	41.3
TOTAL SULFUR, PERCENT:				
FEED	3.51	3.58	5.45	4.19
CLEAN COAL	0.92	0.89	1.15	1.01
REFUSE	4.40	5.13	8.86	5.92
WEIGHT RECOVERY (YIELD)	25.6	36.6	44.2	35.3
THEORETICAL WEIGHT RECOVERY	34.5	48.5	60.3	49.7
WEIGHT RECOVERY EFFICIENCY	74.1	75.5	73.3	71.0
ASH ERROR	0.7	0.8	2.0	1.4
FLOAT IN REFUSE...% OF PRODUCT	8.1	14.7	20.6	13.3
SINK IN CLEAN COAL	20.4	12.4	13.5	17.2
TOTAL MISPLACED MATERIAL..% OF FEED	11.2	13.8	17.4	14.7
NEAR GRAVITY MATERIAL	51.3	59.6	60.1	56.8
SPECIFIC GRAVITY OF SEPARATION	1.285	1.299	1.332	1.298
PROBABLE ERROR	0.022	0.030	0.084	0.033
IMPERFECTION	0.077	0.100	0.252	0.110
ERROR AREA	13.	18.	60.	28.
DISTRIBUTION, PERCENT TO CLEANED COAL (SPECIFIC GRAVITY FRACTION):				
FLOAT 1.27	87.1	84.9	85.2	85.6
1.27-1.29	54.3	66.4	68.7	63.1
1.29-1.32	29.4	44.7	64.4	45.3
1.32-1.35	8.2	20.6	48.7	22.9
1.35-1.40	5.2	11.8	35.3	15.7
1.40-1.50	0.0	0.0	23.4	6.4
1.50-1.60	0.0	0.0	15.8	4.9
1.60-1.80	0.0	0.0	6.4	1.4
SINK 1.80	0.0	0.0	2.3	0.8



Specific Gravity
of Separation = 1.285
Probable Error = 0.022
Imperfection = 0.077
Error Area = 13.

Figure A-21. Distribution Curve, Test No. 76 (9 x 14)

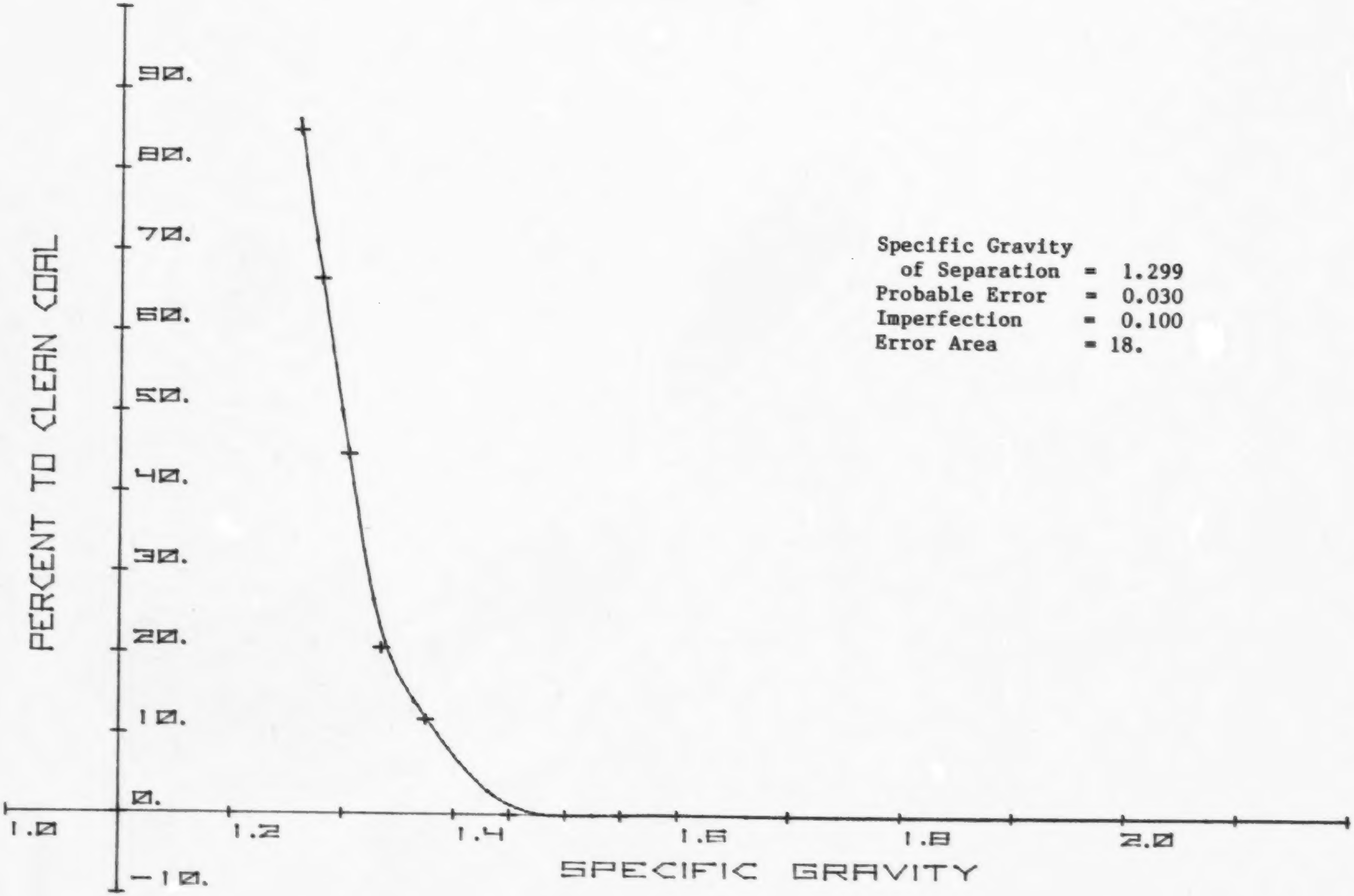


Figure A-22. Distribution Curve, Test No. 76 (14 x 28)

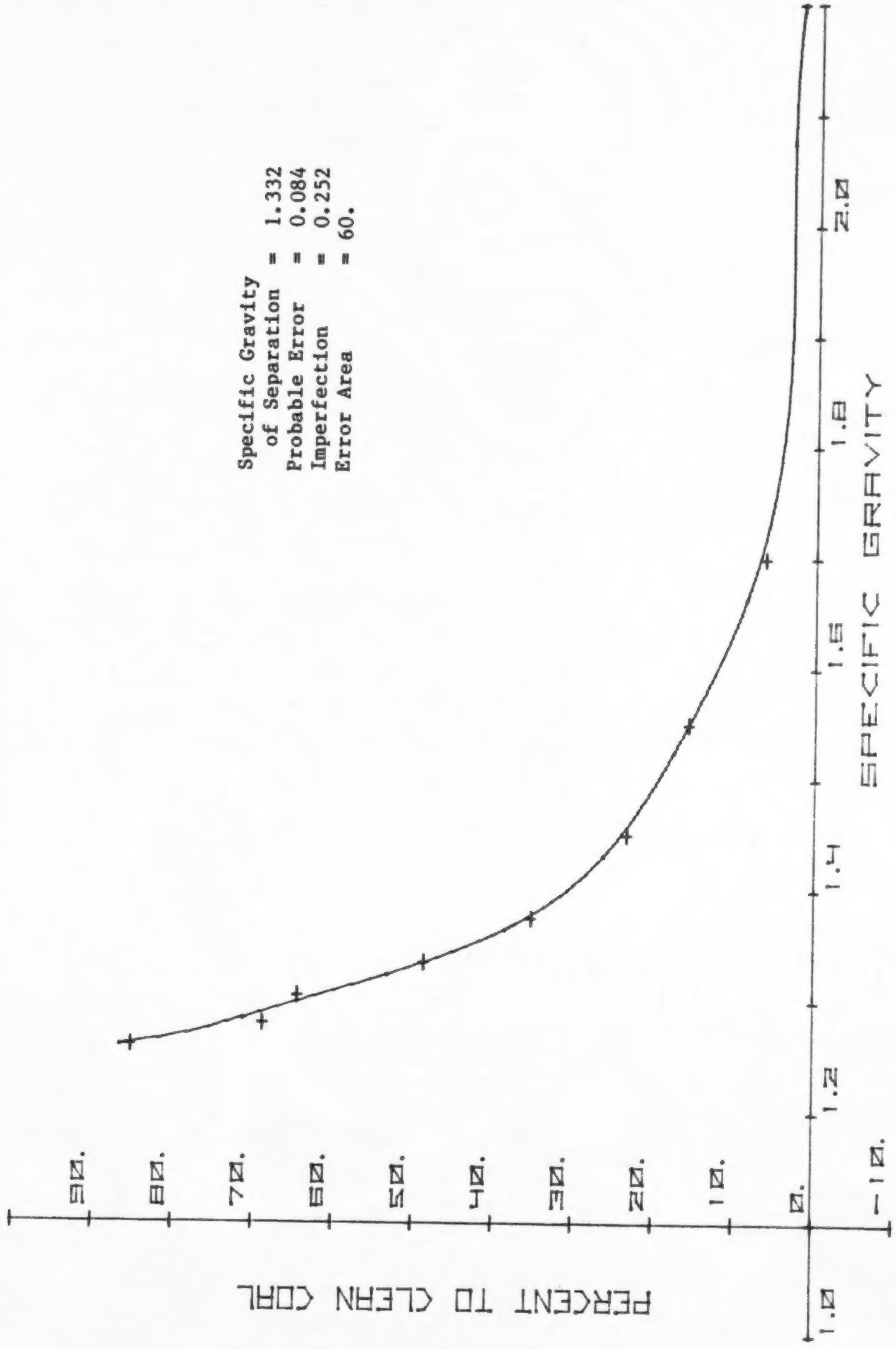


Figure A-23. Distribution Curve, Test No. 76 (28 x 100)

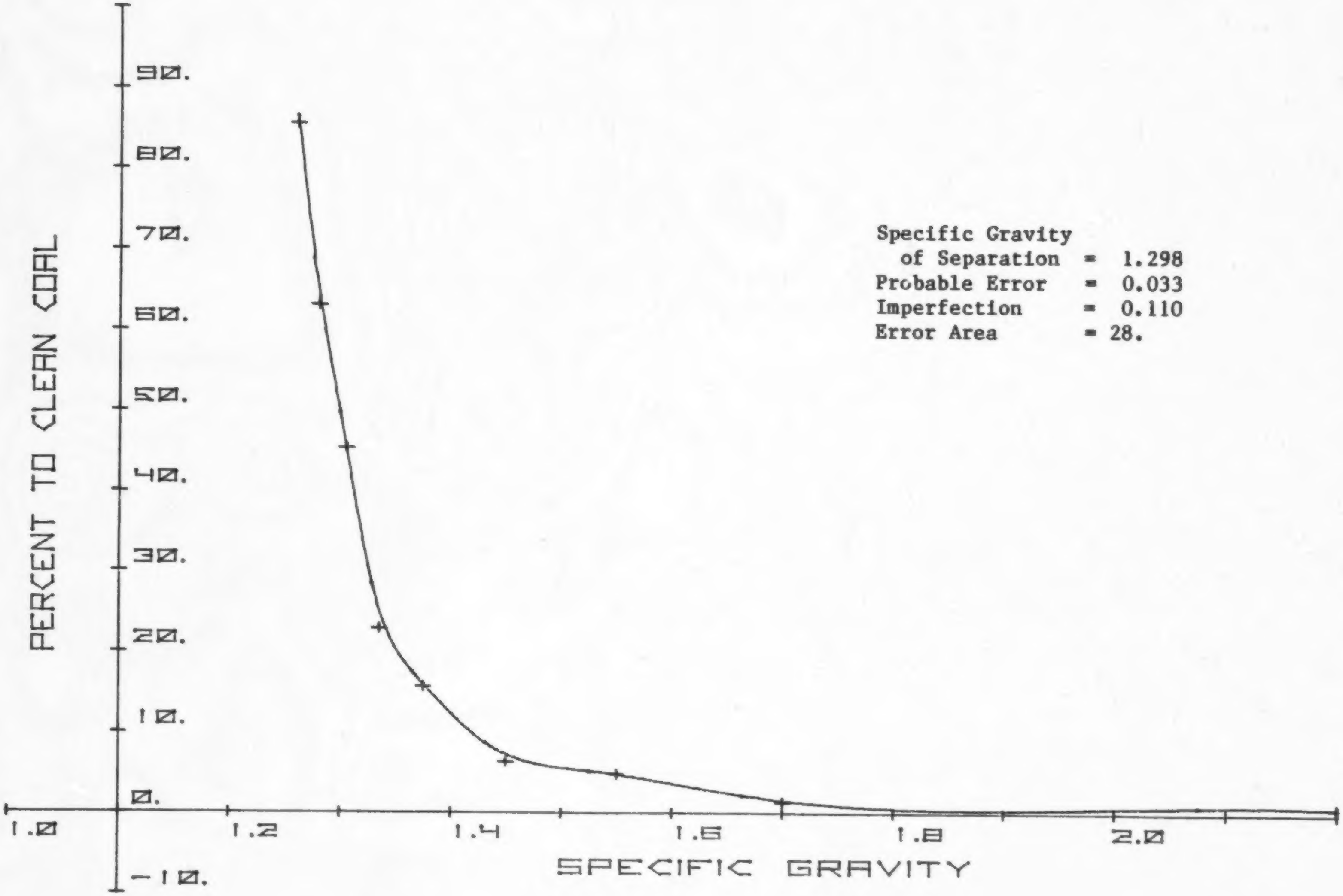
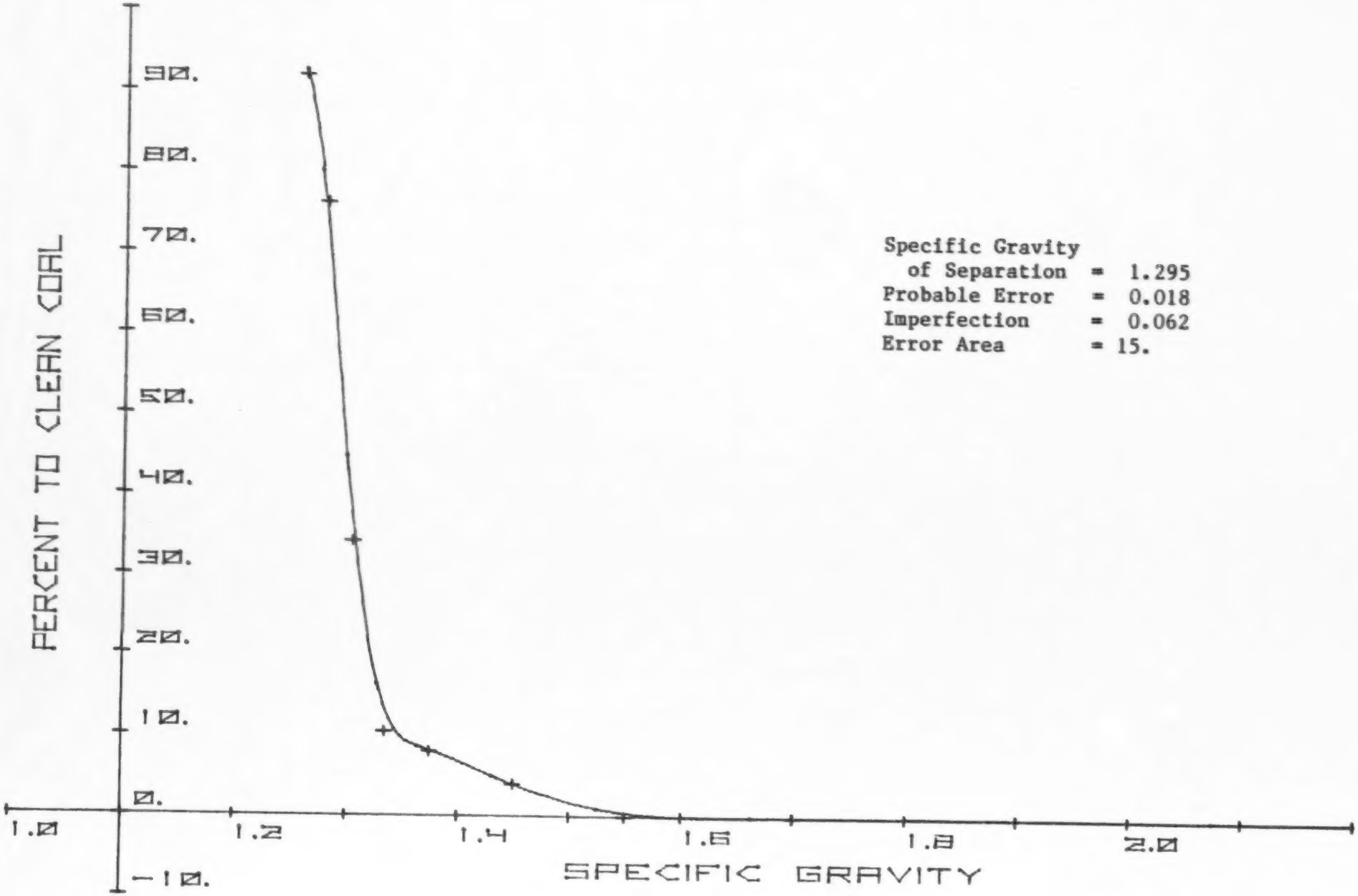


Figure A-24. Distribution Curve, Test No. 76 (9 x 100)

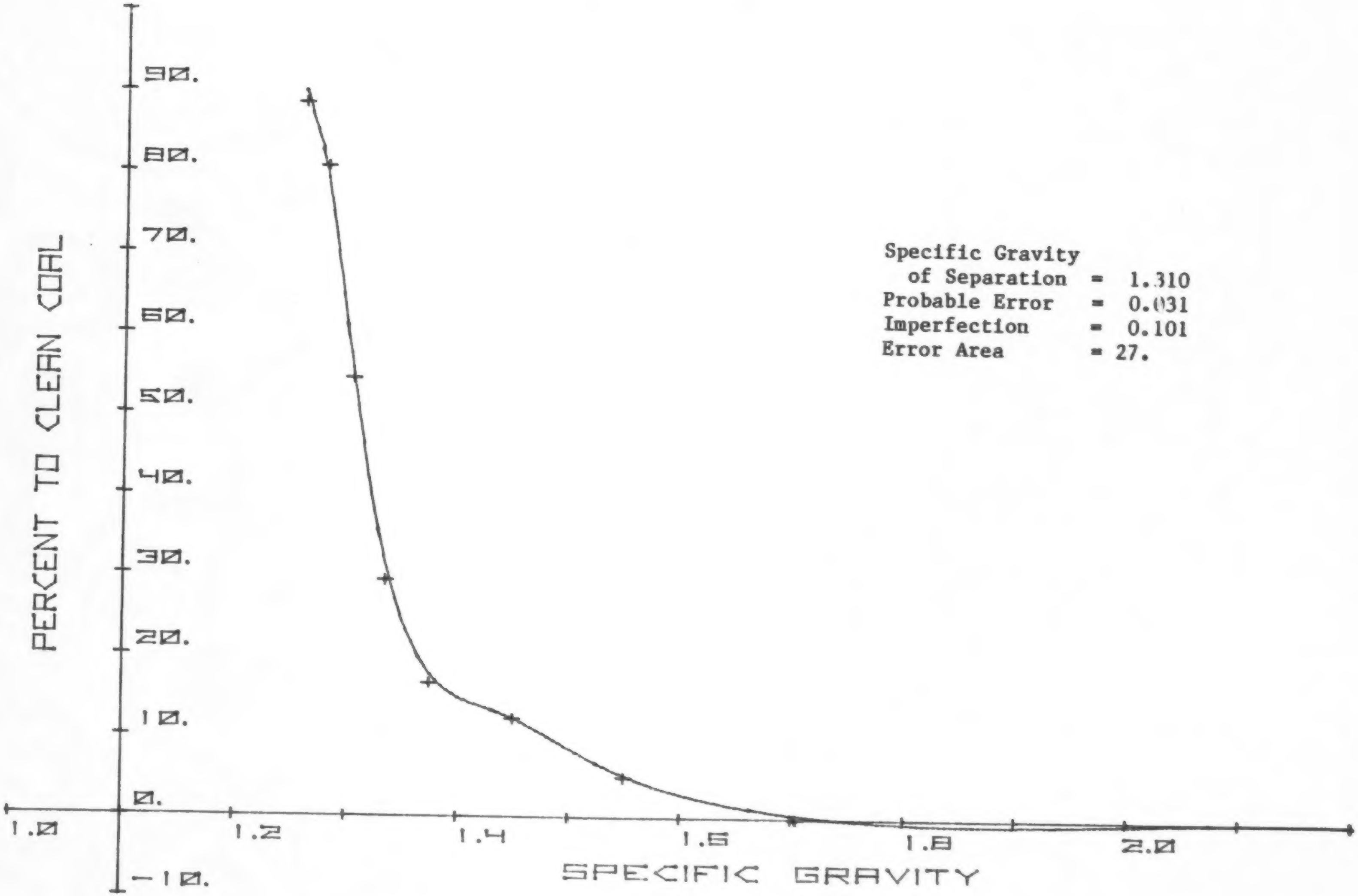
TABLE A-7. TEST 77 DATA SUMMARY

	9x14	14x28	28x100	9x100
SCREEN ANALYSIS, PERCENT:				
FEED	34.1	29.5	31.4	95.1
CLEAN COAL	24.3	30.6	38.6	93.6
REFUSE	40.7	28.8	26.6	96.2
ASH, PERCENT:				
FEED	33.0	26.1	27.6	29.1
CLEAN COAL	4.2	3.9	5.7	4.7
REFUSE	44.5	41.9	49.0	45.0
TOTAL SULFUR, PERCENT:				
FEED	3.70	3.57	5.05	4.11
CLEAN COAL	0.94	0.96	1.09	1.01
REFUSE	4.80	5.43	8.91	6.13
WEIGHT RECOVERY (YIELD)	28.6	41.6	49.4	39.5
THEORETICAL WEIGHT RECOVERY	36.2	48.7	61.4	50.3
WEIGHT RECOVERY EFFICIENCY	78.9	85.4	80.4	78.5
ASH ERROR	0.7	0.6	2.0	1.3
FLOAT IN REFUSE...% OF PRODUCT	6.0	13.5	19.0	11.5
SINK IN CLEAN COAL	11.8	12.4	9.6	15.2
TOTAL MISPLACED MATERIAL..% OF FEED	7.6	13.0	14.4	12.9
NEAR GRAVITY MATERIAL	48.7	61.0	27.1	56.1
SPECIFIC GRAVITY OF SEPARATION	1.295	1.310	1.375	1.310
PROBABLE ERROR	0.018	0.031	0.095	0.040
IMPERFECTION	0.062	0.101	0.253	0.130
ERROR AREA	15.	27.	65.	35.
DISTRIBUTION, PERCENT TO CLEANED COAL (SPECIFIC GRAVITY FRACTION):				
FLOAT 1.27	91.8	88.6	89.7	89.9
1.27-1.29	76.0	80.7	82.9	79.9
1.29-1.32	34.1	54.4	77.9	54.1
1.32-1.35	10.4	29.3	62.6	30.4
1.35-1.40	8.0	16.6	49.4	23.1
1.40-1.50	4.0	12.2	30.8	15.0
1.50-1.60	0.0	5.0	18.4	7.8
1.60-1.80	0.0	0.0	7.1	1.1
SINK 1.80	0.0	0.0	3.4	1.2



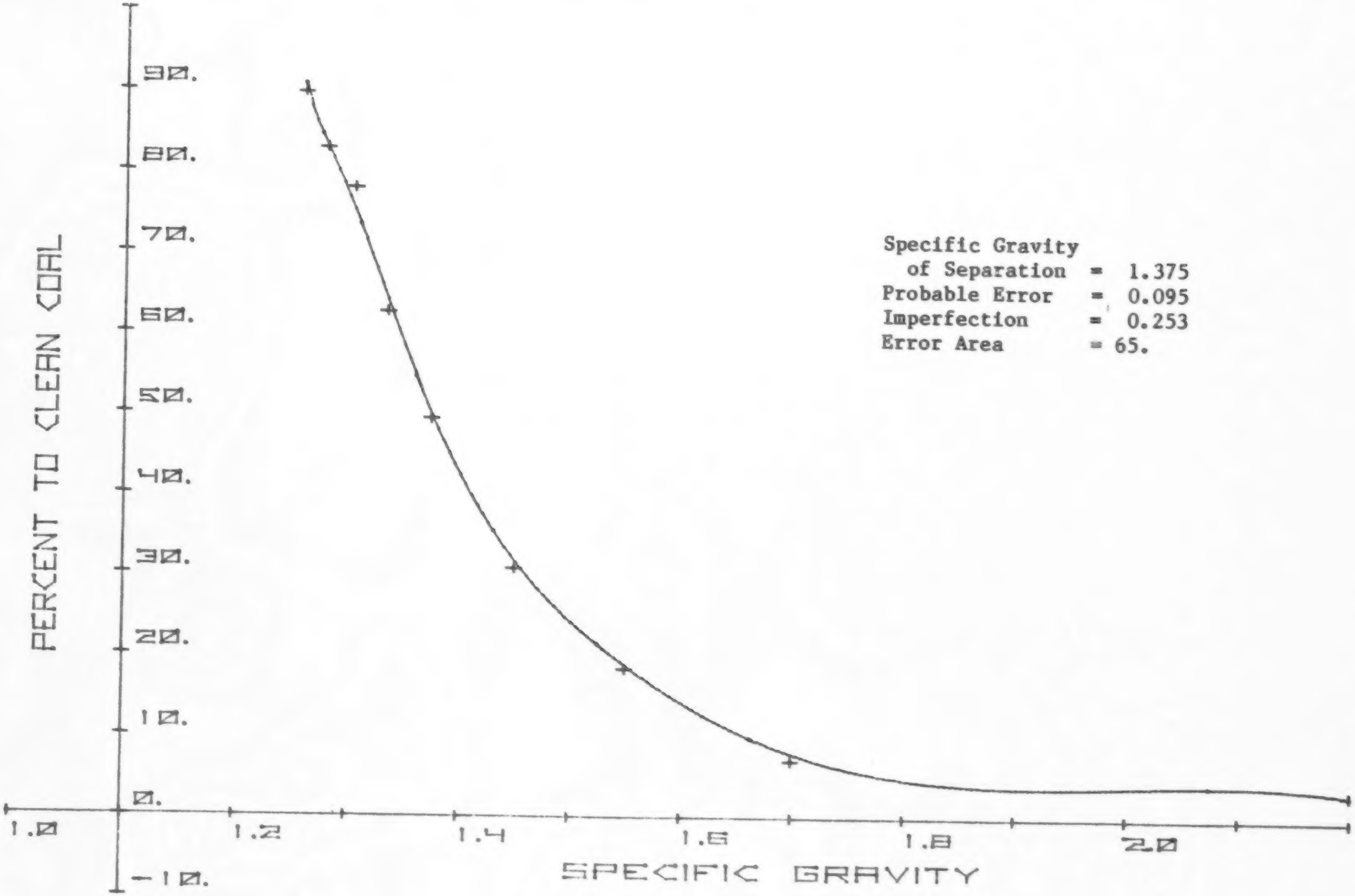
Specific Gravity
  of Separation = 1.295
Probable Error = 0.018
Imperfection = 0.062
Error Area = 15.

Figure A-25. Distribution Curve, Test No. 77 (9 x 14)



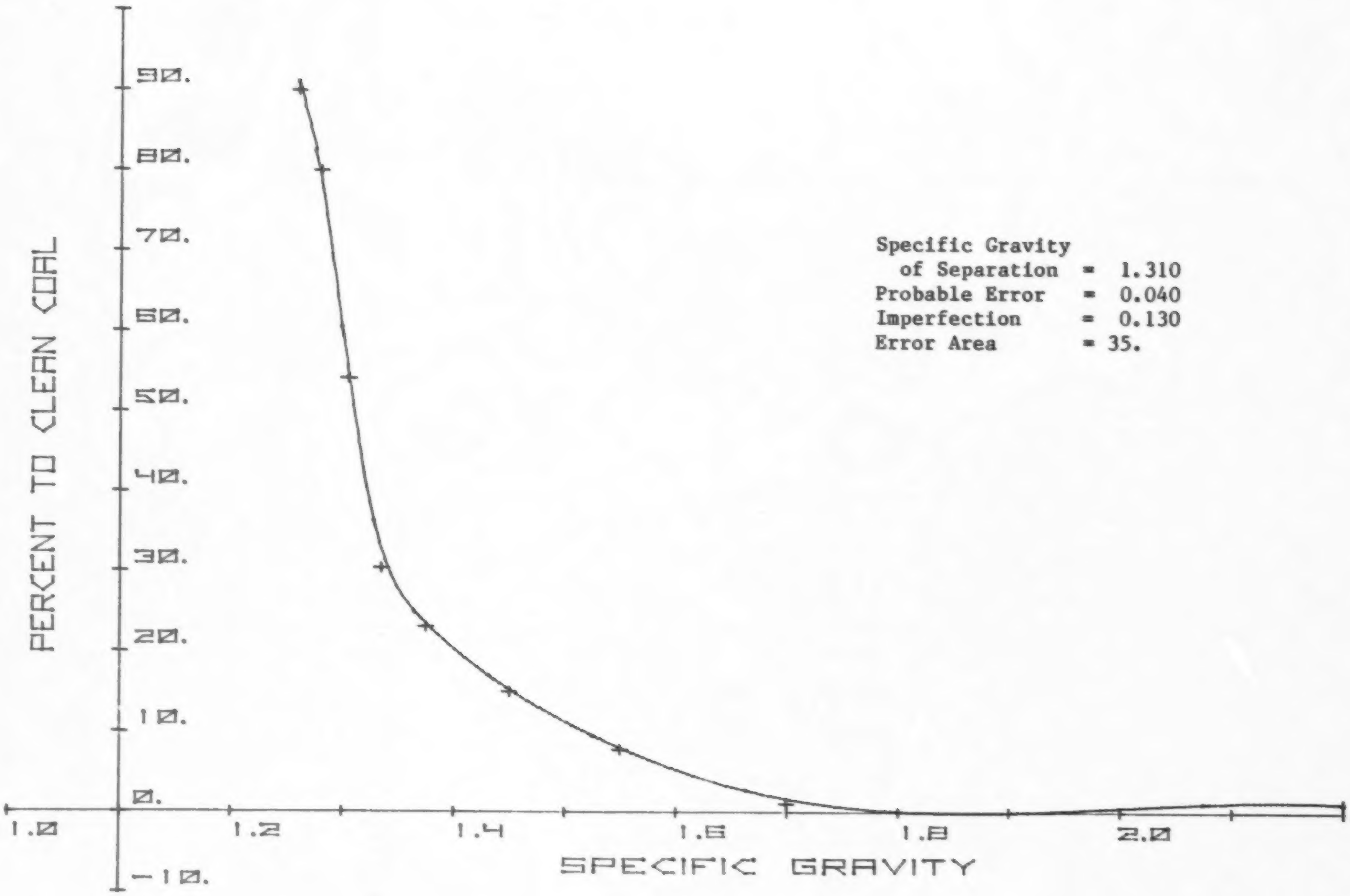
Specific Gravity
of Separation = 1.310
Probable Error = 0.031
Imperfection = 0.101
Error Area = 27.

Figure A-26. Distribution Curve, Test No. 77 (14 x 28)



Specific Gravity
  of Separation = 1.375
Probable Error = 0.095
Imperfection = 0.253
Error Area = 65.

Figure A-27. Distribution Curve, Test No. 77 (28 x 100)



Specific Gravity
  of Separation = 1.310
Probable Error = 0.040
Imperfection = 0.130
Error Area = 35.

Figure A-28. Distribution Curve, Test No. 77 (9 x 100)

TABLE A-8. TEST 78 DATA SUMMARY

	9x14	14x28	28x100	9x100
SCREEN ANALYSIS, PERCENT:				
FEED	29.0	30.0	35.4	94.4
CLEAN COAL	21.4	30.0	41.8	93.2
REFUSE	34.2	30.0	31.0	95.2
ASH, PERCENT:				
FEED	27.5	20.9	24.5	24.3
CLEAN COAL	3.1	3.2	4.7	3.8
REFUSE	37.9	33.0	42.7	37.9
TOTAL SULFUR, PERCENT:				
FEED	3.71	3.08	4.73	3.89
CLEAN COAL	1.03	0.90	1.10	1.02
REFUSE	4.86	4.56	8.06	5.81
WEIGHT RECOVERY (YIELD)	29.9	40.5	47.9	40.0
THEORETICAL WEIGHT RECOVERY	33.6	48.0	61.6	51.5
WEIGHT RECOVERY EFFICIENCY	88.7	84.4	77.7	77.7
ASH ERROR	0.2	0.6	1.7	1.1
FLOAT IN REFUSE...% OF PRODUCT	9.2	15.3	20.3	13.5
SINK IN CLEAN COAL	12.5	11.1	11.3	15.2
TOTAL MISPLACED MATERIAL..% OF FEED	10.2	13.6	16.0	14.2
NEAR GRAVITY MATERIAL	58.9	67.1	64.1	62.9
SPECIFIC GRAVITY OF SEPARATION	1.287	1.297	1.327	1.299
PROBABLE ERROR	0.018	0.025	0.063	0.034
IMPERFECTION	0.063	0.085	0.192	0.113
ERROR AREA	12.	18.	51.	30.
DISTRIBUTION, PERCENT TO CLEANED COAL (SPECIFIC GRAVITY FRACTION):				
FLOAT 1.27	87.8	86.0	86.4	86.6
1.27-1.29	62.5	66.2	75.5	68.1
1.29-1.32	23.7	41.7	69.0	44.8
1.32-1.35	4.6	13.5	44.5	17.8
1.35-1.40	6.5	7.5	33.3	16.6
1.40-1.50	0.0	6.1	17.8	9.6
1.50-1.60	0.0	0.0	12.7	4.5
1.60-1.80	0.0	0.0	5.3	1.7
SINK 1.80	0.0	0.0	1.7	0.6

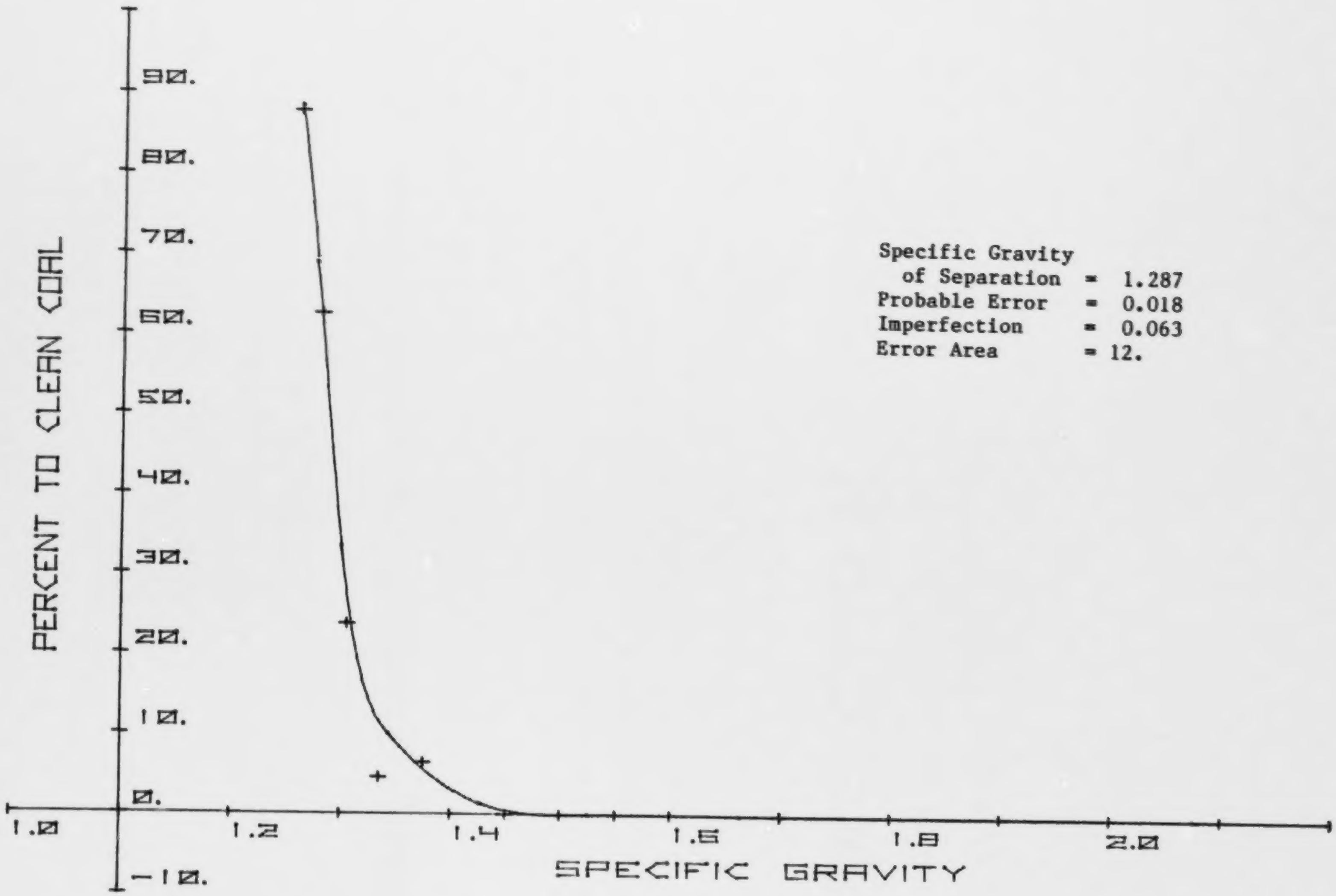
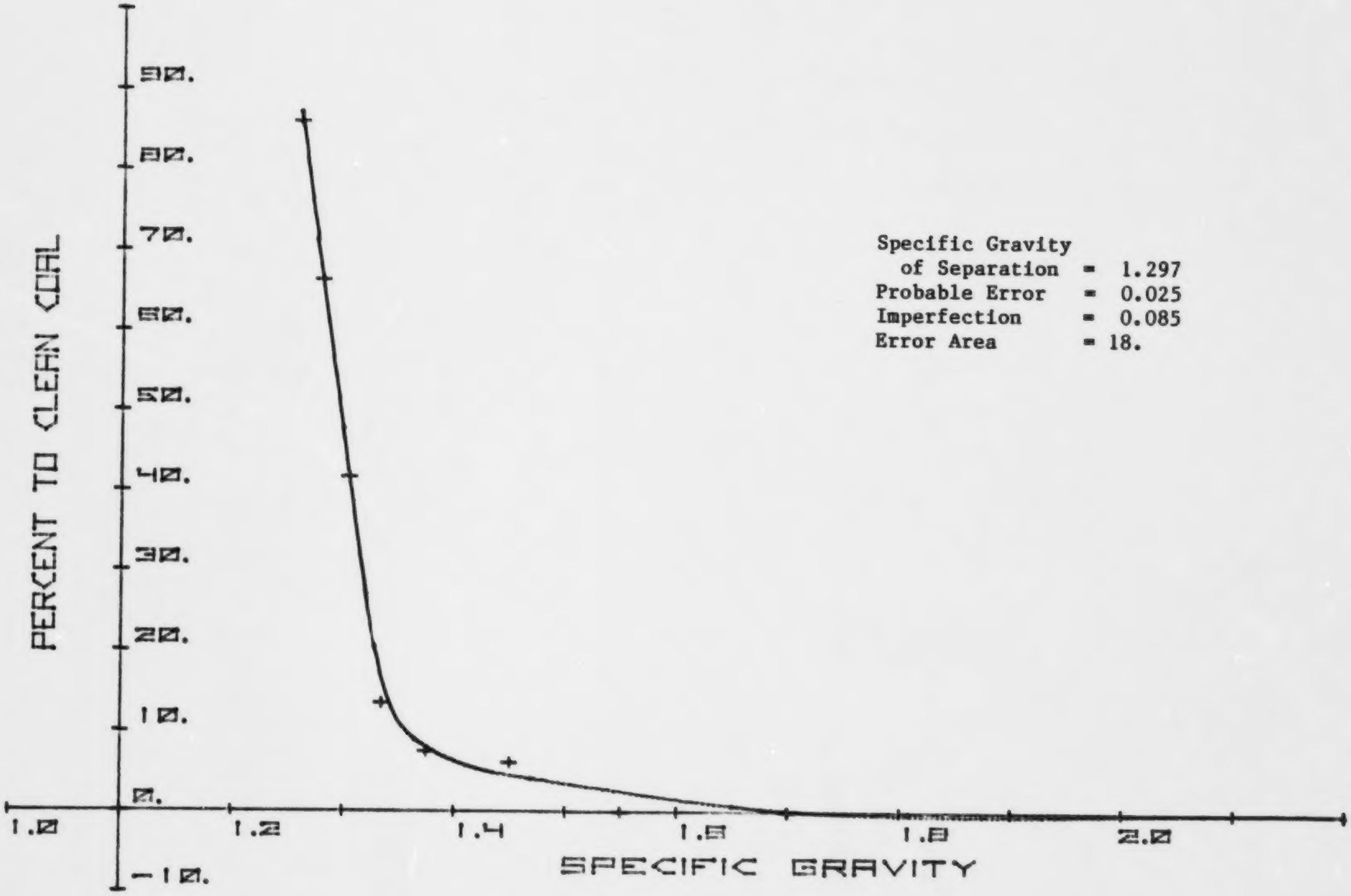


Figure A-29. Distribution Curve, Test No. 78 (9 x 14)



Specific Gravity
  of Separation = 1.297
Probable Error = 0.025
Imperfection = 0.085
Error Area = 18.

Figure A-30. Distribution Curve, Test No. 78 (14 x 28)

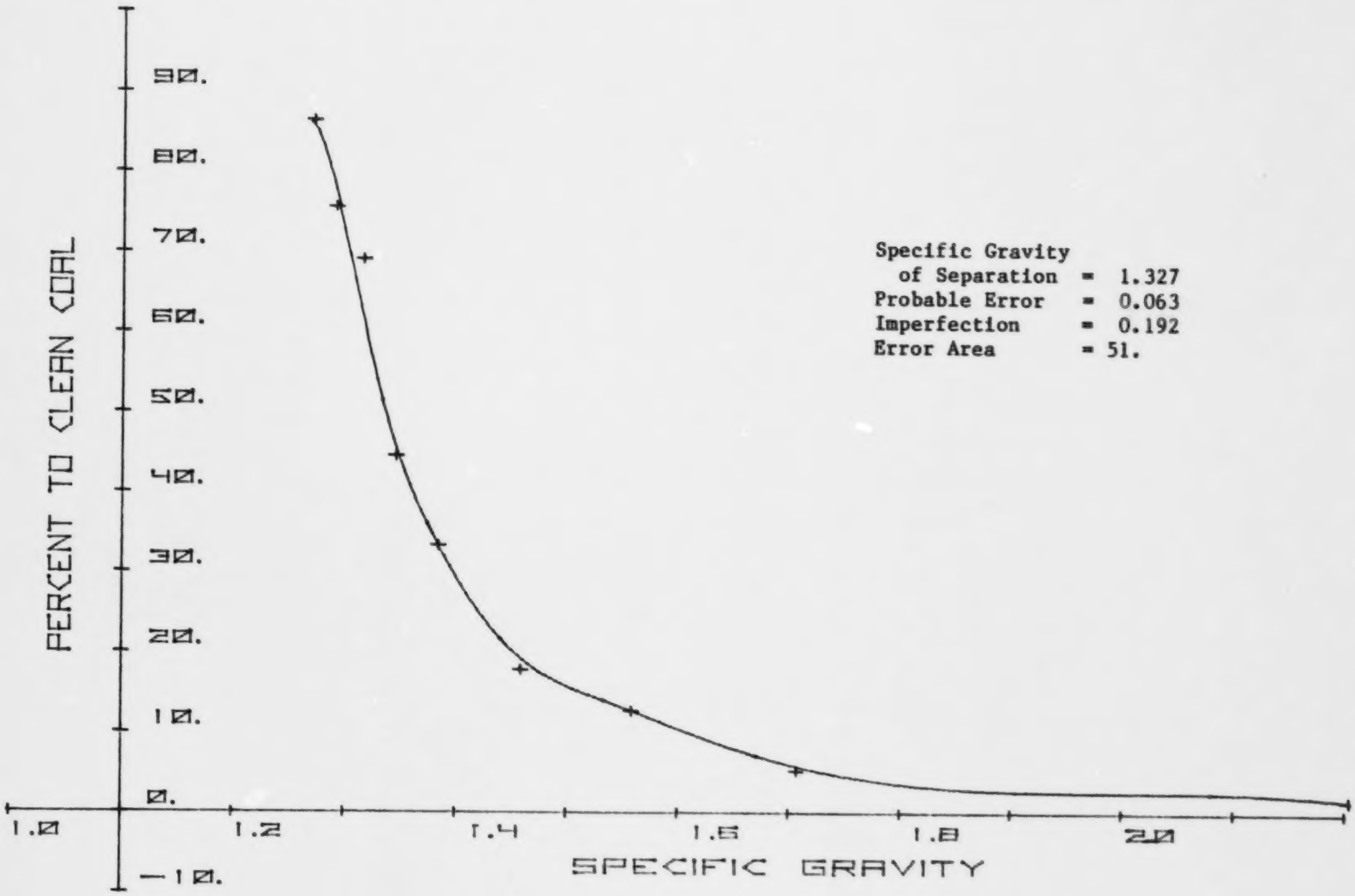
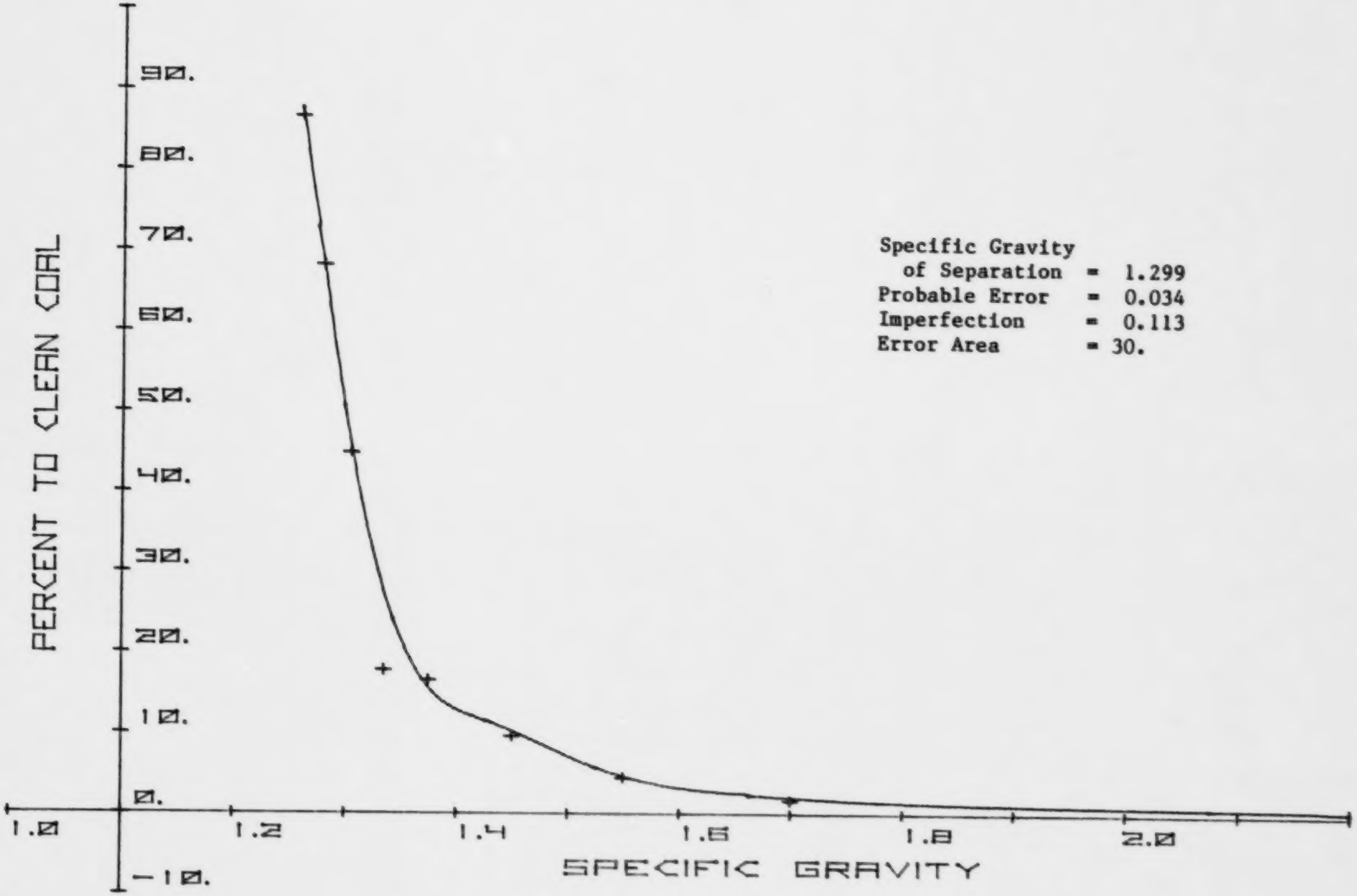


Figure A-31. Distribution Curve, Test No. 78 (28 x 100)

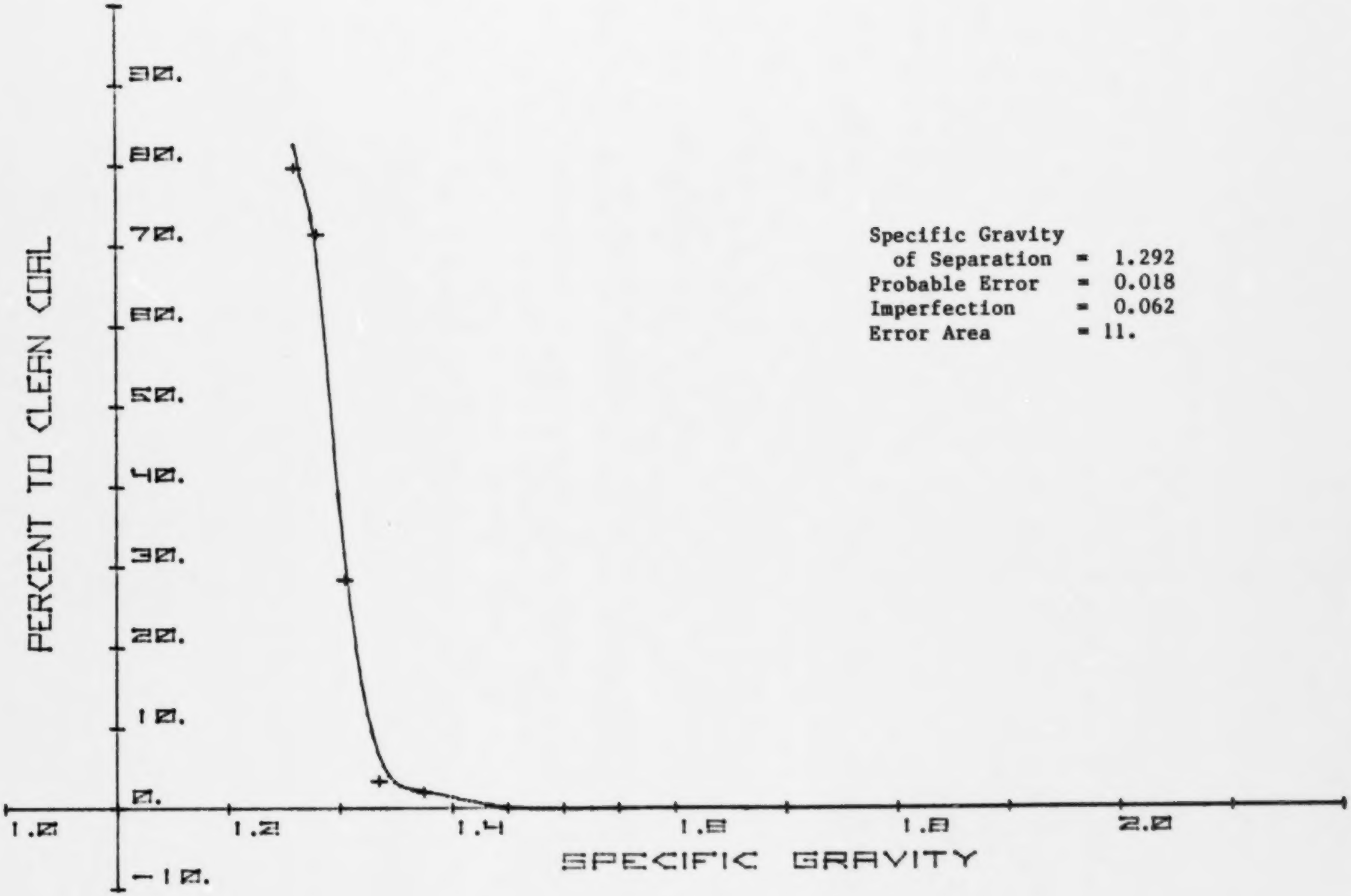


Specific Gravity
  of Separation = 1.299
Probable Error = 0.034
Imperfection = 0.113
Error Area = 30.

Figure A-32. Distribution Curve, Test No. 78 (9 x 100)

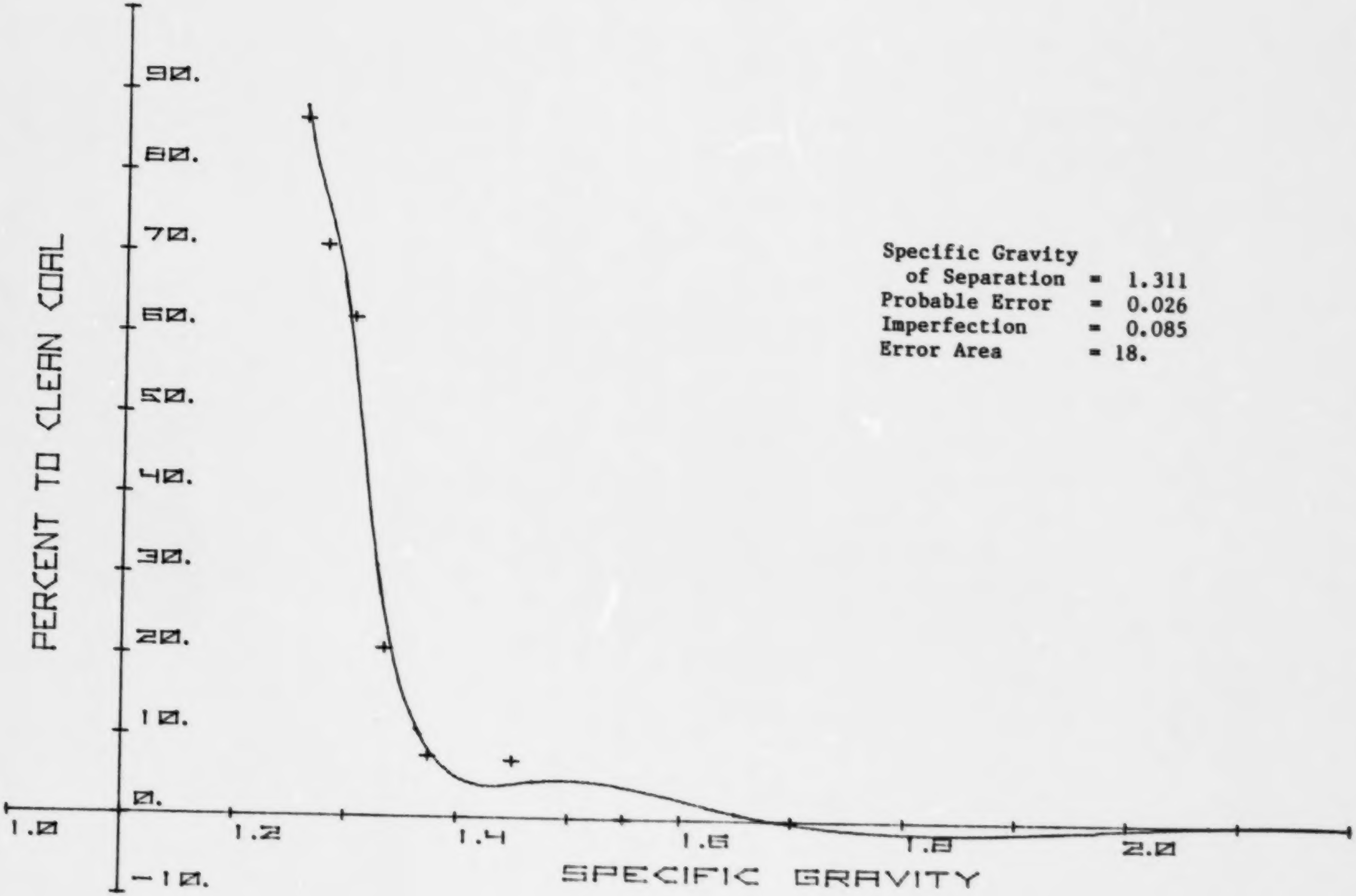
TABLE A-9. TEST 79 DATA SUMMARY

	9x14	14x28	28x100	9x100
SCREEN ANALYSIS, PERCENT:				
FEED	31.4	30.6	32.7	94.8
CLEAN COAL	17.4	32.9	42.1	92.4
REFUSE	40.1	29.2	27.0	96.3
ASH, PERCENT:				
FEED	31.5	23.9	26.8	27.5
CLEAN COAL	3.6	3.4	5.5	4.4
REFUSE	39.0	38.1	47.3	41.0
TOTAL SULFUR, PERCENT:				
FEED	3.75	3.46	5.71	4.34
CLEAN COAL	0.91	0.89	1.12	1.00
REFUSE	4.51	5.24	10.12	6.31
WEIGHT RECOVERY (YIELD)	21.1	40.9	49.0	37.1
THEORETICAL WEIGHT RECOVERY	27.9	49.5	61.9	48.9
WEIGHT RECOVERY EFFICIENCY	75.5	82.6	79.1	75.8
ASH ERROR	0.6	0.7	1.8	1.3
FLOAT IN REFUSE...% OF PRODUCT	7.3	16.2	19.4	12.9
SINK IN CLEAN COAL	12.2	9.8	7.9	14.9
TOTAL MISPLACED MATERIAL..% OF FEED	8.3	13.5	13.8	13.6
NEAR GRAVITY MATERIAL	49.7	65.1	60.9	58.5
SPECIFIC GRAVITY OF SEPARATION	1.292	1.311	1.361	1.309
PROBABLE ERROR	0.018	0.026	0.080	0.028
IMPERFECTION	0.062	0.085	0.221	0.091
ERROR AREA	11.	18.	60.	30.
DISTRIBUTION, PERCENT TO CLEANED COAL (SPECIFIC GRAVITY FRACTION):				
FLOAT 1.27	79.7	86.5	88.7	86.0
1.27-1.29	71.4	70.8	81.4	74.4
1.29-1.32	28.4	61.9	80.4	56.5
1.32-1.35	3.4	20.9	57.3	22.2
1.35-1.40	2.0	7.6	43.7	14.6
1.40-1.50	0.0	7.1	25.7	10.4
1.50-1.60	0.0	0.0	14.7	6.0
1.60-1.80	0.0	0.0	7.4	2.1
SINK 1.80	0.0	0.0	1.9	0.6



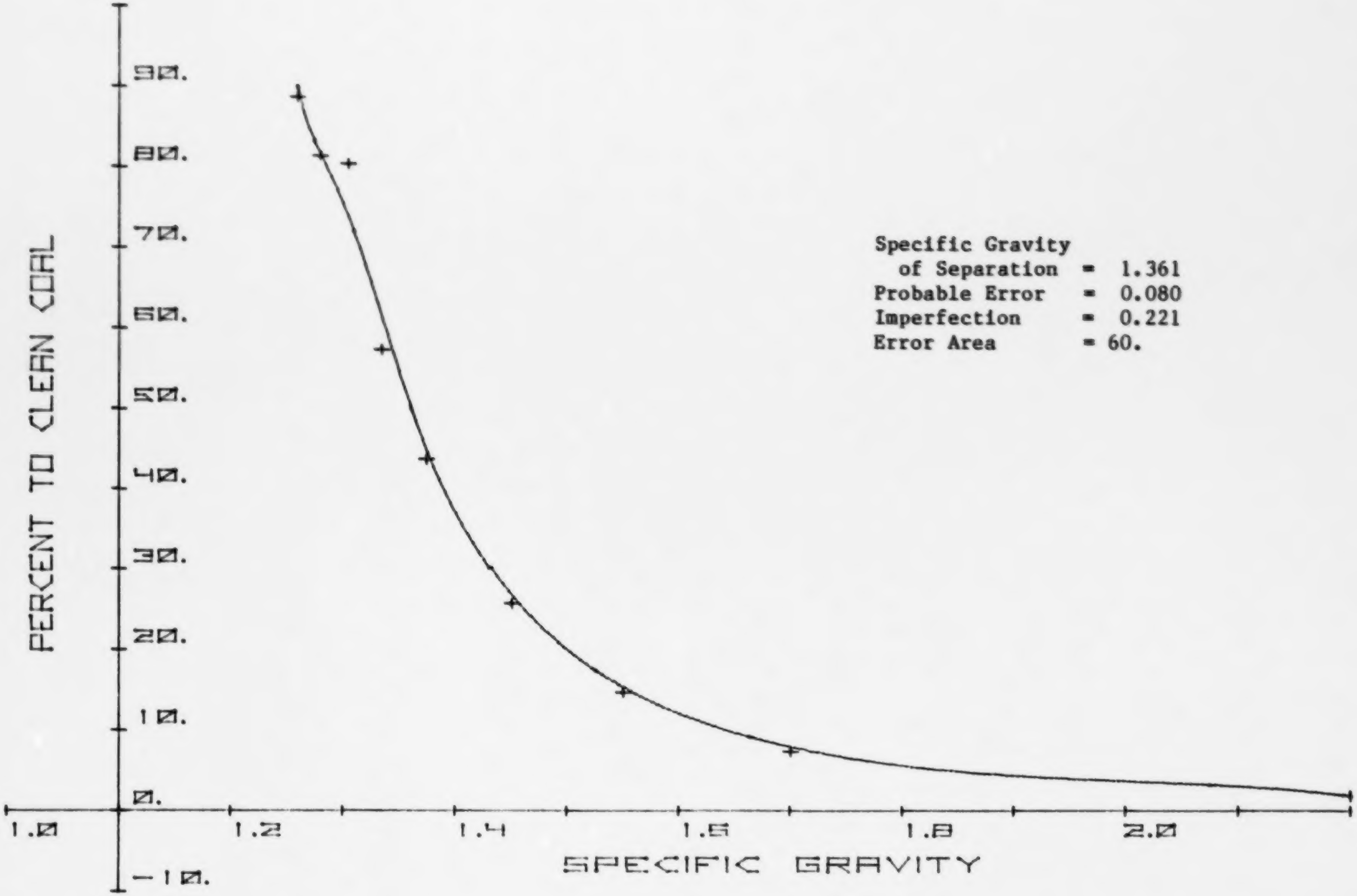
Specific Gravity
  of Separation = 1.292
Probable Error = 0.018
Imperfection = 0.062
Error Area = 11.

Figure A-33. Distribution Curve, Test No. 79 (9 x 14)



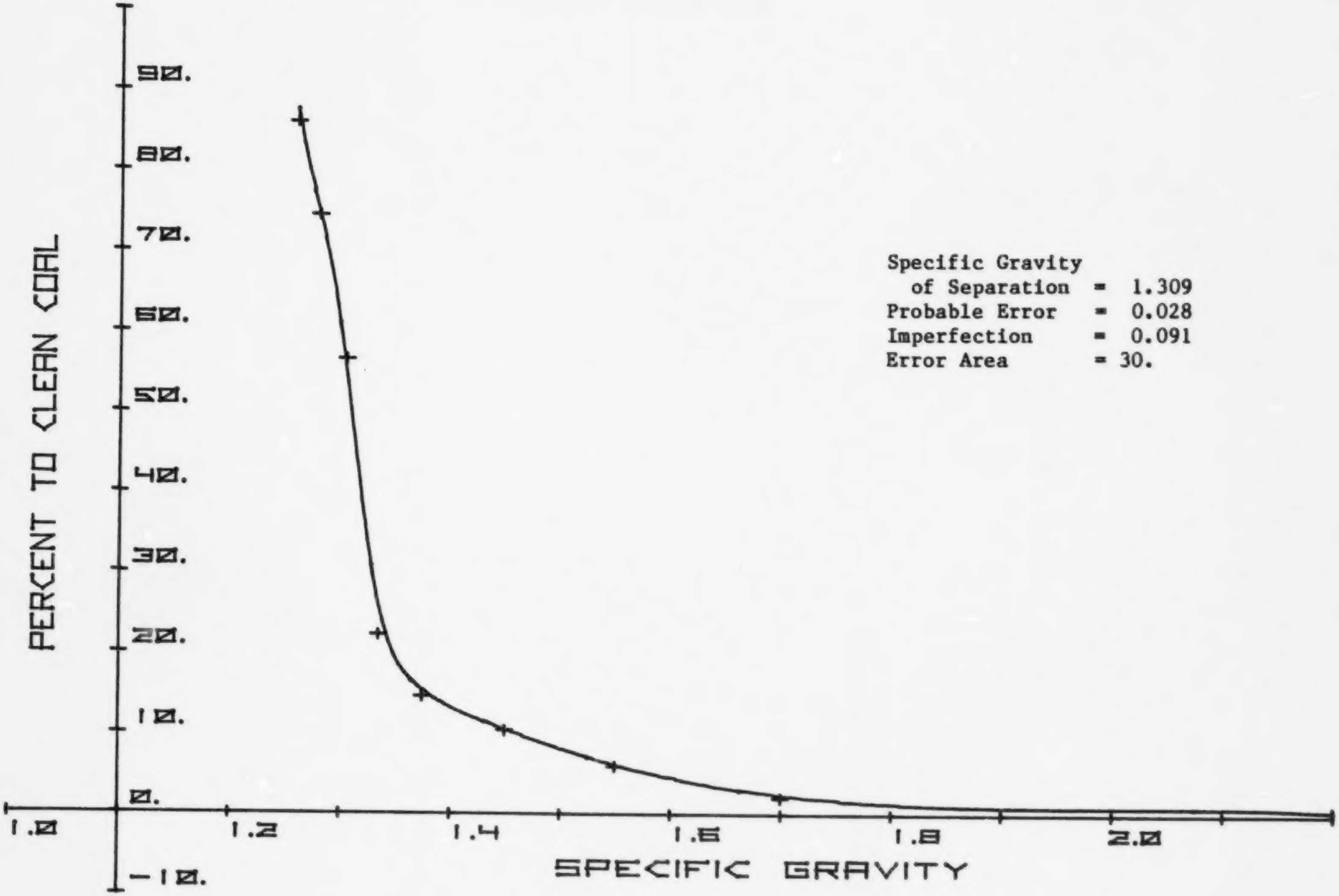
Specific Gravity
of Separation = 1.311
Probable Error = 0.026
Imperfection = 0.085
Error Area = 18.

Figure A-34. Distribution Curve, Test No. 79 (14 x 28)



Specific Gravity
of Separation = 1.361
Probable Error = 0.080
Imperfection = 0.221
Error Area = 60.

Figure A-35. Distribution Curve, Test No. 79 (28 x 100)

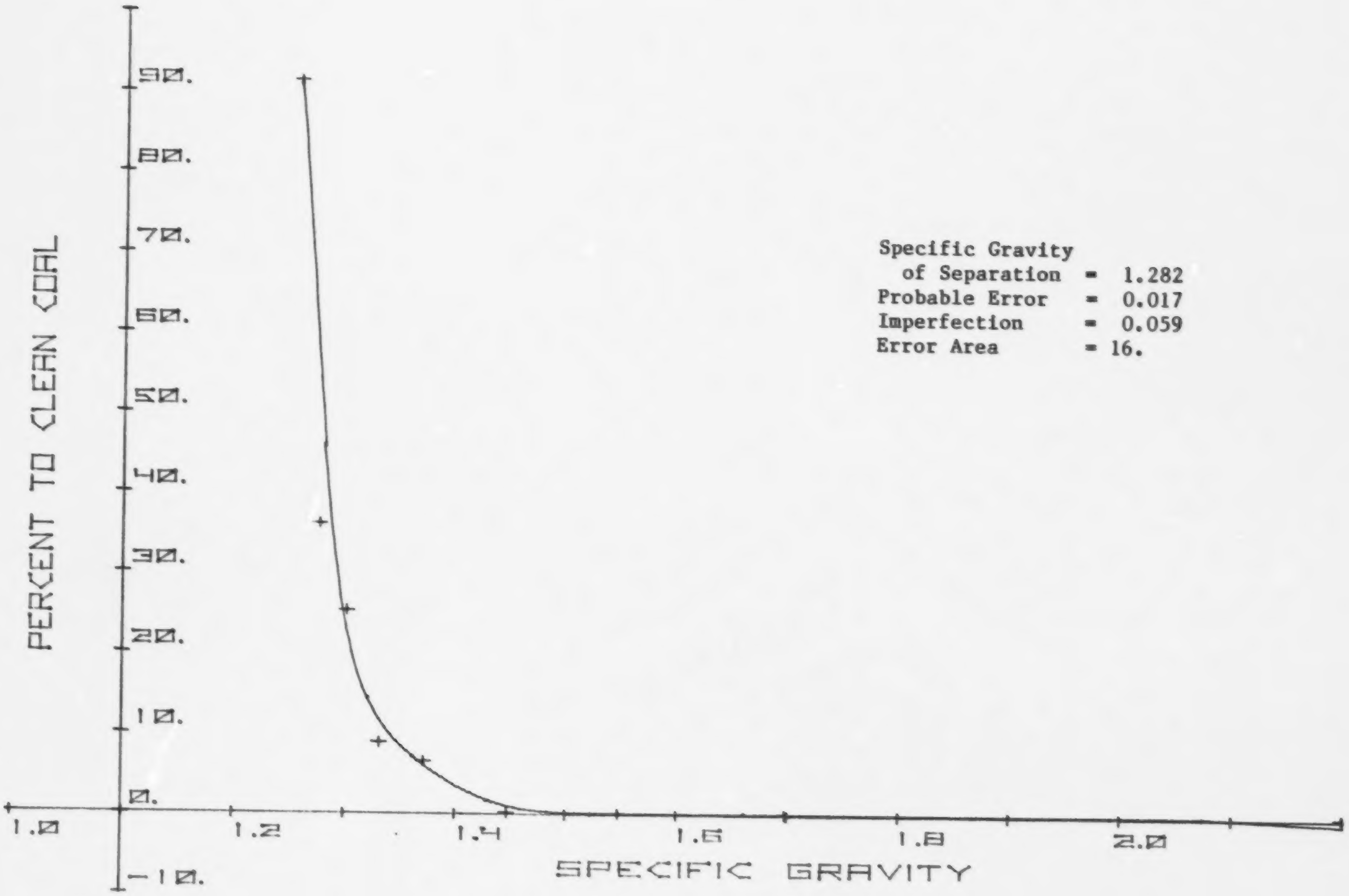


Specific Gravity
of Separation = 1.309
Probable Error = 0.028
Imperfection = 0.091
Error Area = 30.

Figure A-36. Distribution Curve, Test No. 79 (9 x 100)

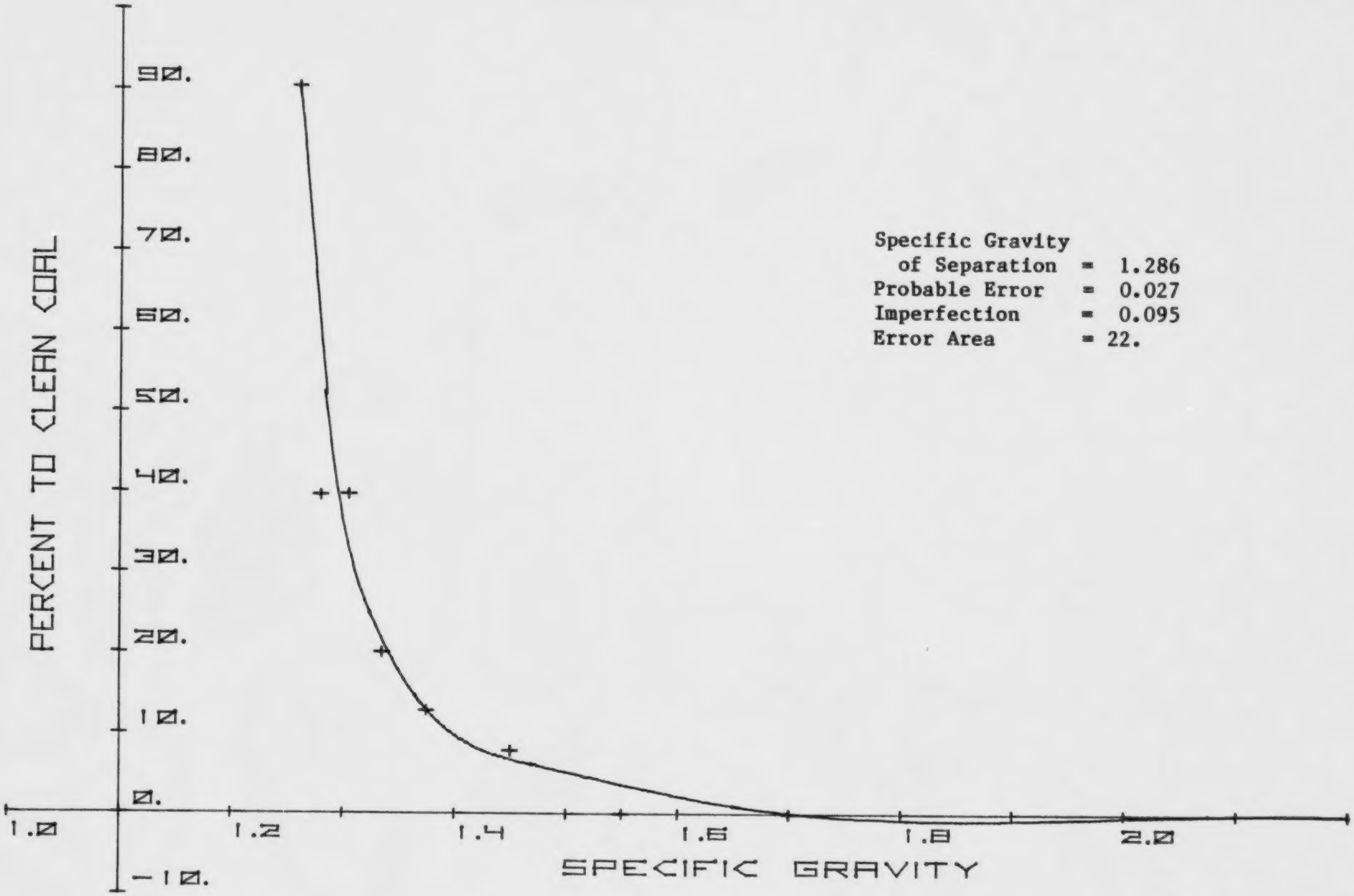
TABLE A-10. TEST 80 DATA SUMMARY

	9x14	14x28	28x100	9x100
SCREEN ANALYSIS, PERCENT:				
FEED	33.2	28.6	32.9	94.7
CLEAN COAL	22.5	29.0	41.4	92.8
REFUSE	39.1	28.4	28.2	95.8
ASH, PERCENT:				
FEED	32.5	27.8	24.2	28.2
CLEAN COAL	3.8	6.7	6.5	5.9
REFUSE	41.5	39.5	38.5	40.0
TOTAL SULFUR, PERCENT:				
FEED	3.88	3.99	5.18	4.36
CLEAN COAL	0.88	0.99	1.22	1.07
REFUSE	4.82	5.67	8.37	6.12
WEIGHT RECOVERY (YIELD)	24.0	35.8	44.6	34.7
THEORETICAL WEIGHT RECOVERY	32.3	59.3	69.3	56.0
WEIGHT RECOVERY EFFICIENCY	74.2	60.4	64.4	62.0
ASH ERROR	0.8	3.3	2.1	2.2
FLOAT IN REFUSE...% OF PRODUCT	2.3	7.5	20.7	7.2
SINK IN CLEAN COAL	32.6	28.4	25.7	31.7
TOTAL MISPLACED MATERIAL..% OF FEED	9.5	15.0	22.9	15.7
NEAR GRAVITY MATERIAL	49.6	59.6	65.1	57.6
SPECIFIC GRAVITY OF SEPARATION	1.282	1.286	1.335	1.285
PROBABLE ERROR	0.017	0.027	0.117	0.029
IMPERFECTION	0.059	0.095	0.350	0.100
ERROR AREA	16.	22.	76.	35.
DISTRIBUTION, PERCENT TO CLEANED COAL (SPECIFIC GRAVITY FRACTION):				
FLOAT 1.27	91.2	90.4	87.9	89.5
1.27-1.29	35.9	39.6	44.3	40.0
1.29-1.32	25.1	39.7	44.8	36.4
1.32-1.35	8.7	20.0	36.1	21.1
1.35-1.40	6.4	12.8	43.0	19.4
1.40-1.50	0.0	7.8	40.3	11.6
1.50-1.60	0.0	0.0	21.0	7.0
1.60-1.80	0.0	0.0	10.7	3.4
SINK 1.80	0.0	0.0	2.6	0.8



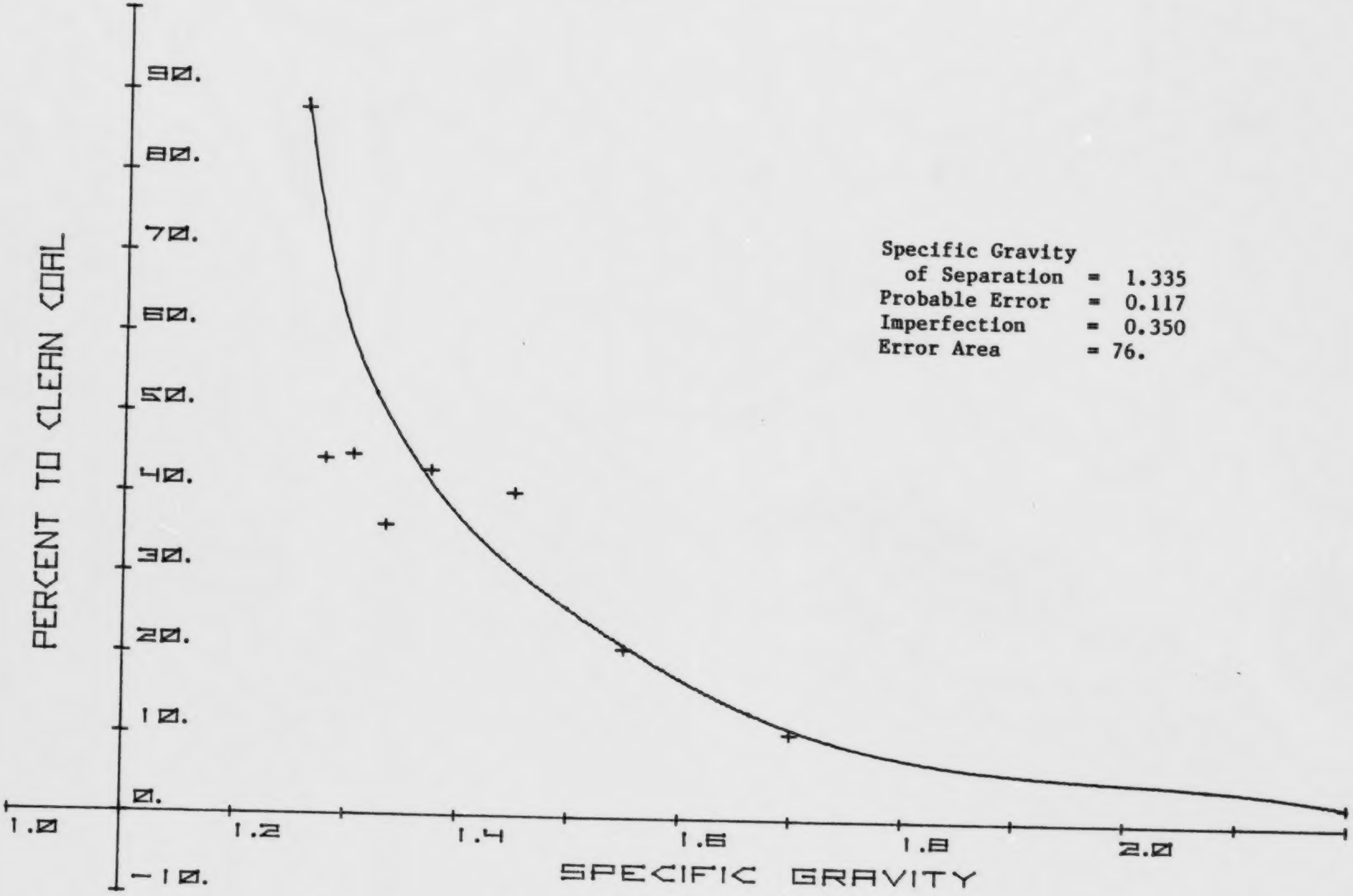
Specific Gravity
of Separation = 1.282
Probable Error = 0.017
Imperfection = 0.059
Error Area = 16.

Figure A-37. Distribution Curve, Test No. 80 (9 x 14)



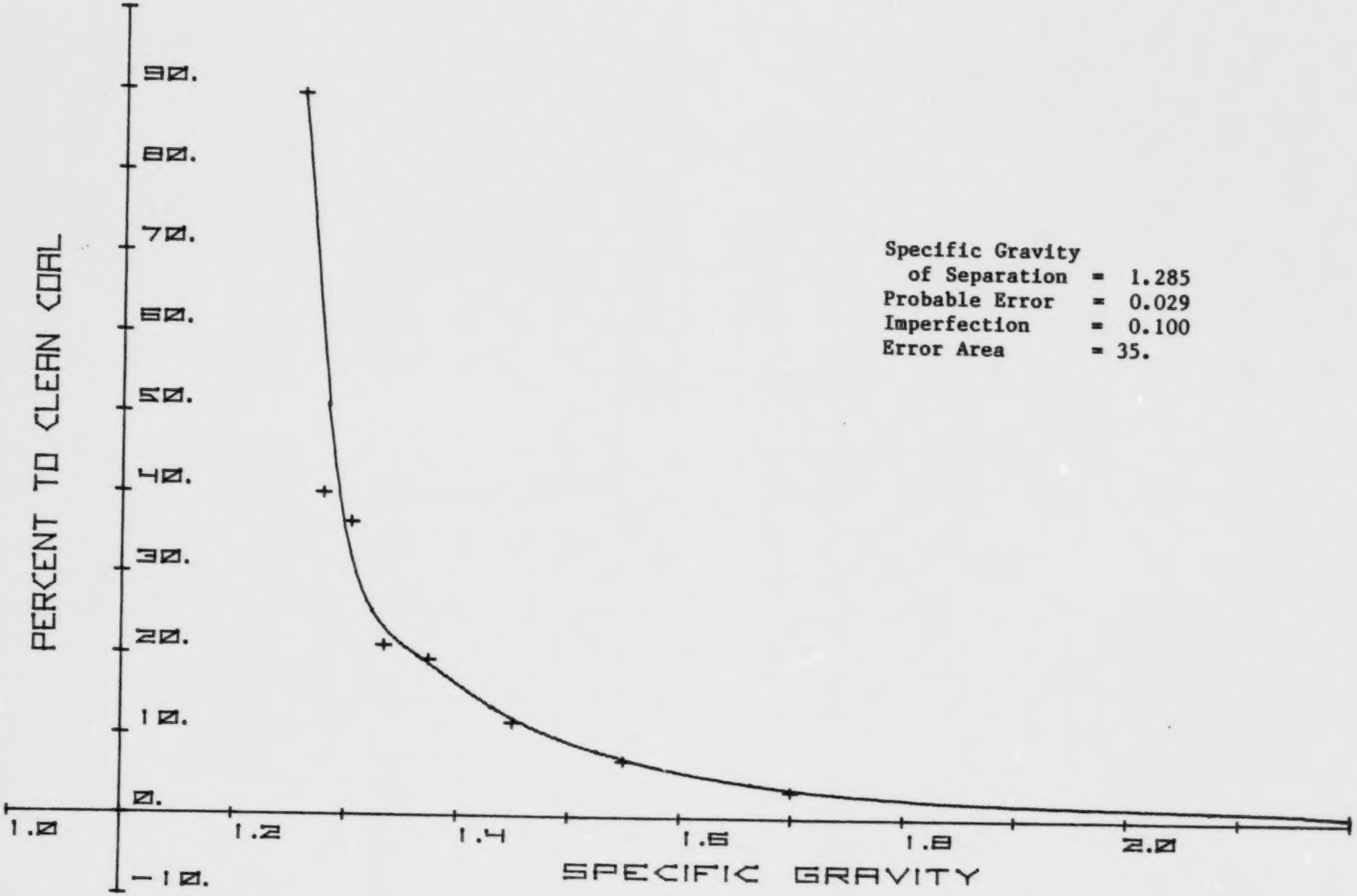
Specific Gravity
  of Separation = 1.286
Probable Error = 0.027
Imperfection = 0.095
Error Area = 22.

Figure A-38. Distribution Curve, Test No. 80 (14 x 28)



Specific Gravity
  of Separation = 1.335
Probable Error = 0.117
Imperfection = 0.350
Error Area = 76.

Figure A-39. Distribution Curve, Test No. 80 (28 x 100)

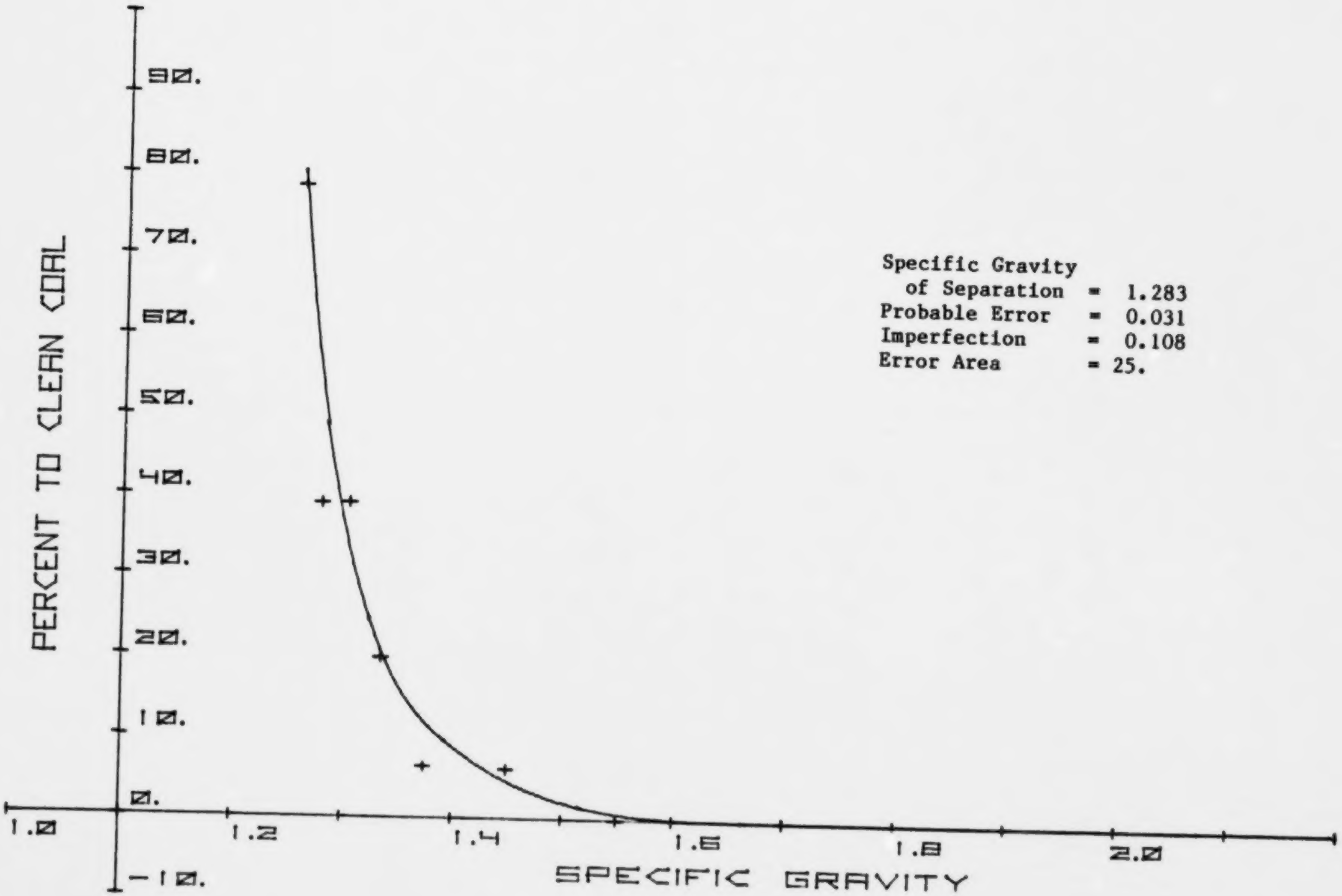


Specific Gravity
  of Separation = 1.285
Probable Error = 0.029
Imperfection = 0.100
Error Area = 35.

Figure A-40. Distribution Curve, Test No. 80 (9 x 100)

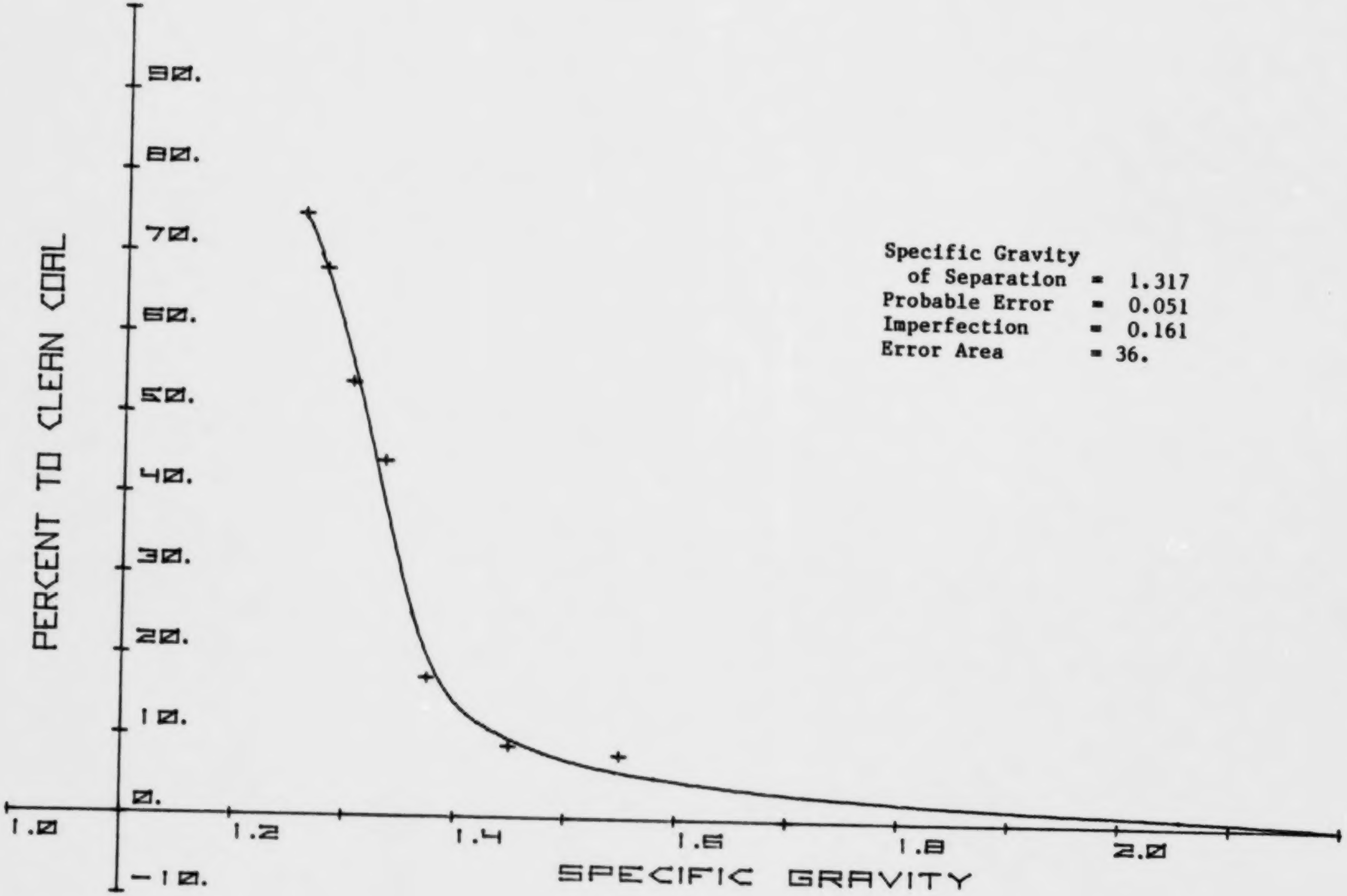
TABLE A-11. TEST 81 DATA SUMMARY

	9x14	14x28	28x100	9x100
SCREEN ANALYSIS, PERCENT:				
FEED	34.1	29.4	31.3	94.8
CLEAN COAL	26.6	29.9	36.8	93.3
REFUSE	39.0	29.0	27.6	95.7
ASH, PERCENT:				
FEED	28.0	24.1	26.5	26.3
CLEAN COAL	4.0	4.0	6.2	4.9
REFUSE	38.6	37.7	44.2	39.9
TOTAL SULFUR, PERCENT:				
FEED	3.40	3.54	5.09	4.00
CLEAN COAL	1.00	0.86	1.32	1.08
REFUSE	4.47	5.34	8.38	5.86
WEIGHT RECOVERY (YIELD)	30.8	40.2	46.5	38.9
THEORETICAL WEIGHT RECOVERY	37.1	52.8	63.1	54.2
WEIGHT RECOVERY EFFICIENCY	82.9	76.2	73.7	71.8
ASH ERROR	0.5	1.1	2.7	1.6
FLOAT IN REFUSE...% OF PRODUCT	9.8	26.5	26.3	23.7
SINK IN CLEAN COAL	30.6	10.9	8.7	14.8
TOTAL MISPLACED MATERIAL..% OF FEED	16.2	20.2	18.1	20.3
NEAR GRAVITY MATERIAL	56.6	65.3	21.1	62.2
SPECIFIC GRAVITY OF SEPARATION	1.283	1.317	1.361	1.317
PROBABLE ERROR	0.031	0.051	0.097	0.053
IMPERFECTION	0.108	0.161	0.268	0.166
ERROR AREA	25.	36.	72.	41.
DISTRIBUTION, PERCENT TO CLEANED COAL (SPECIFIC GRAVITY FRACTION):				
FLOAT 1.27	78.6	74.7	73.4	75.7
1.27-1.29	39.1	67.9	85.5	66.6
1.29-1.32	39.2	53.9	74.2	53.8
1.32-1.35	19.9	44.2	81.2	45.3
1.35-1.40	6.5	17.3	43.3	19.3
1.40-1.50	6.2	8.9	30.3	14.5
1.50-1.60	0.0	7.9	19.4	8.1
1.60-1.80	0.0	0.0	11.0	2.6
SINK 1.80	0.0	0.0	3.9	1.5



Specific Gravity
  of Separation = 1.283
Probable Error = 0.031
Imperfection = 0.108
Error Area = 25.

Figure A-41. Distribution Curve, Test No. 81 (9 x 14)



Specific Gravity
  of Separation = 1.317
Probable Error = 0.051
Imperfection = 0.161
Error Area = 36.

Figure A-42. Distribution Curve, Test No. 81 (14 x 28)

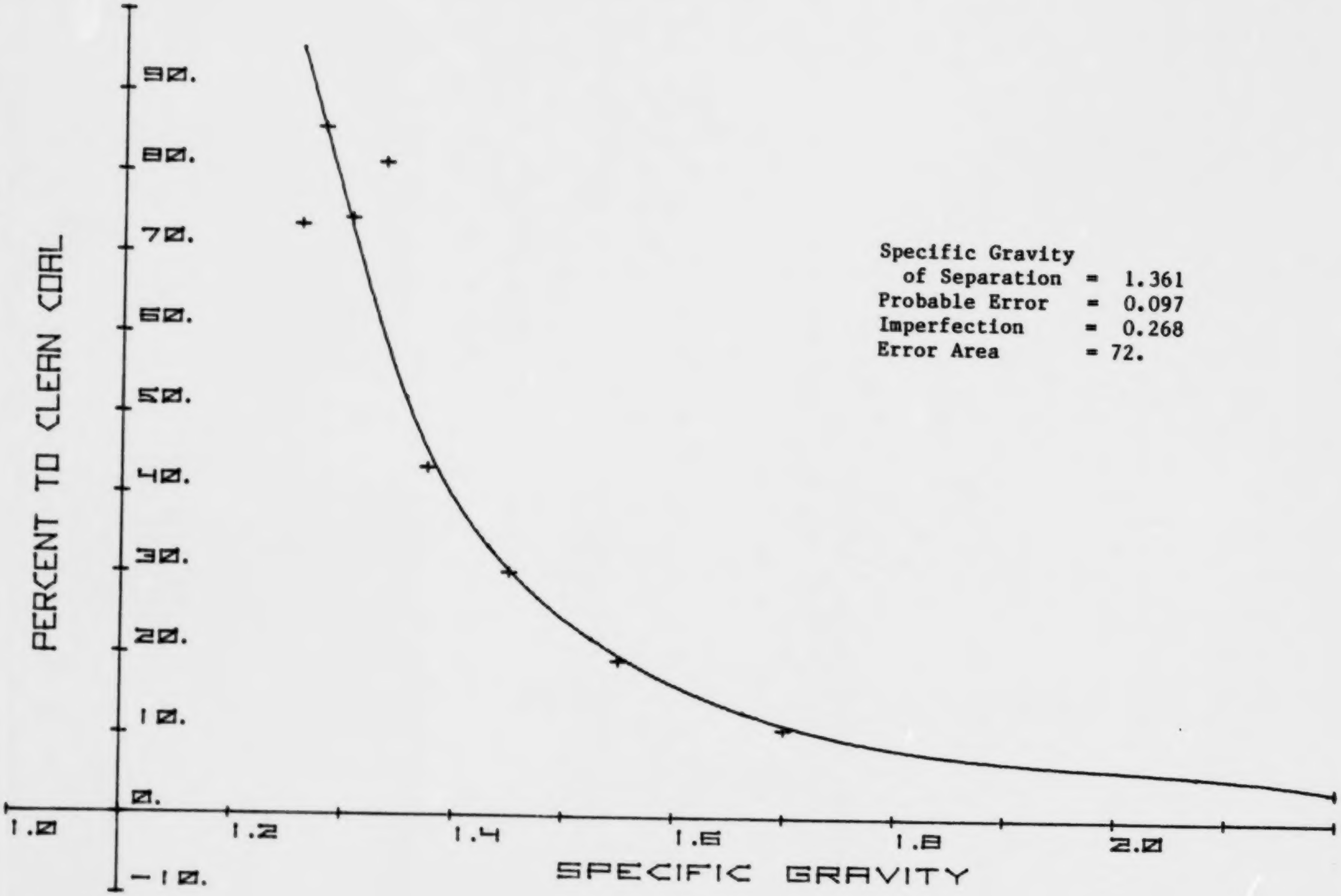


Figure A-43. Distribution Curve, Test No. 81 (28 x 100)

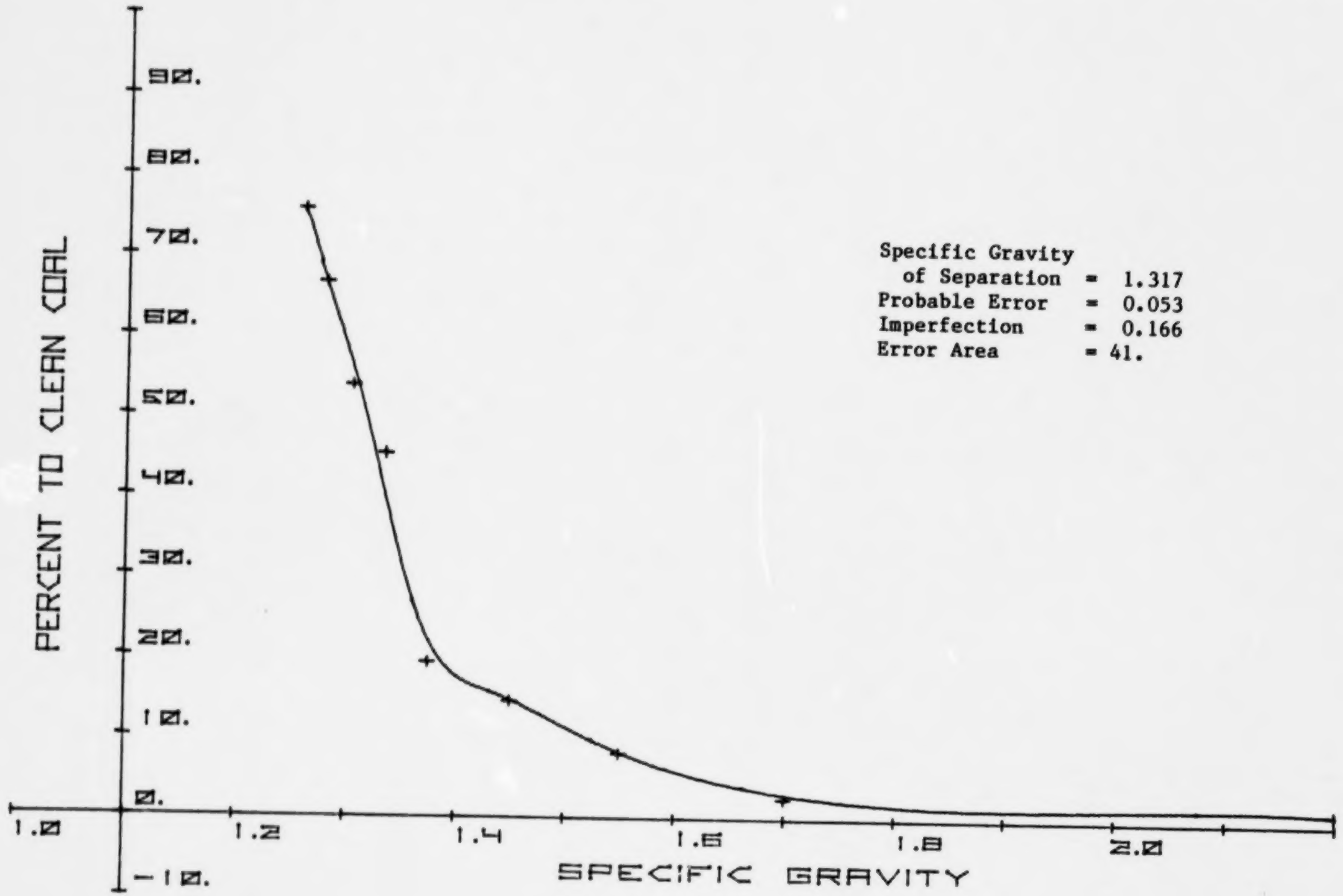


Figure A-44. Distribution Curve, Test No. 81 (9 x 100)

TABLE A-12. TEST 82 DATA SUMMARY

	9x14	14x28	28x100	9x100
SCREEN ANALYSIS, PERCENT:				
FEED.............................	29.7	29.5	35.1	94.3
CLEAN COAL.......................	20.3	28.8	43.4	92.5
REFUSE...........................	35.3	29.9	30.1	95.4
ASH, PERCENT:				
FEED.............................	26.8	23.0	24.8	24.9
CLEAN COAL.......................	3.2	3.1	4.1	3.6
REFUSE...........................	35.0	34.6	42.9	37.3
TOTAL SULFUR, PERCENT:				
FEED.............................	3.29	3.42	4.62	3.83
CLEAN COAL.......................	0.73	0.70	1.00	0.85
REFUSE...........................	4.18	5.01	7.78	5.58
WEIGHT RECOVERY (YIELD)............	25.8	36.8	46.6	37.0
THEORETICAL WEIGHT RECOVERY........	35.2	48.9	58.9	51.0
WEIGHT RECOVERY EFFICIENCY.........	73.5	75.3	79.1	72.5
ASH ERROR..........................	0.6	0.8	1.5	1.1
FLOAT IN REFUSE...% OF PRODUCT.....	11.0	17.4	22.9	16.1
SINK IN CLEAN COAL.................	20.9	13.9	11.5	16.2
TOTAL MISPLACED MATERIAL..% OF FEED	13.5	16.1	17.6	16.1
NEAR GRAVITY MATERIAL..............	59.9	66.6	64.5	63.5
SPECIFIC GRAVITY OF SEPARATION.....	1.282	1.290	1.331	1.292
PROBABLE ERROR.....................	0.017	0.023	0.054	0.029
IMPERFECTION.......................	0.059	0.079	0.164	0.101
ERROR AREA.........................	11.	25.	65.	36.
DISTRIBUTION, PERCENT TO CLEANED COAL (SPECIFIC GRAVITY FRACTION):				
FLOAT 1.27	81.0	79.7	83.8	81.8
1.27-1.29	55.4	66.4	76.9	66.7
1.29-1.32	14.0	28.7	59.1	33.0
1.32-1.35	7.1	15.8	55.9	22.7
1.35-1.40	3.2	7.5	25.9	11.5
1.40-1.50	0.0	4.2	20.3	7.4
1.50-1.60	0.0	7.3	18.4	9.2
1.60-1.80	0.0	0.0	10.2	4.1
SINK 1.80	0.0	0.0	1.2	0.5

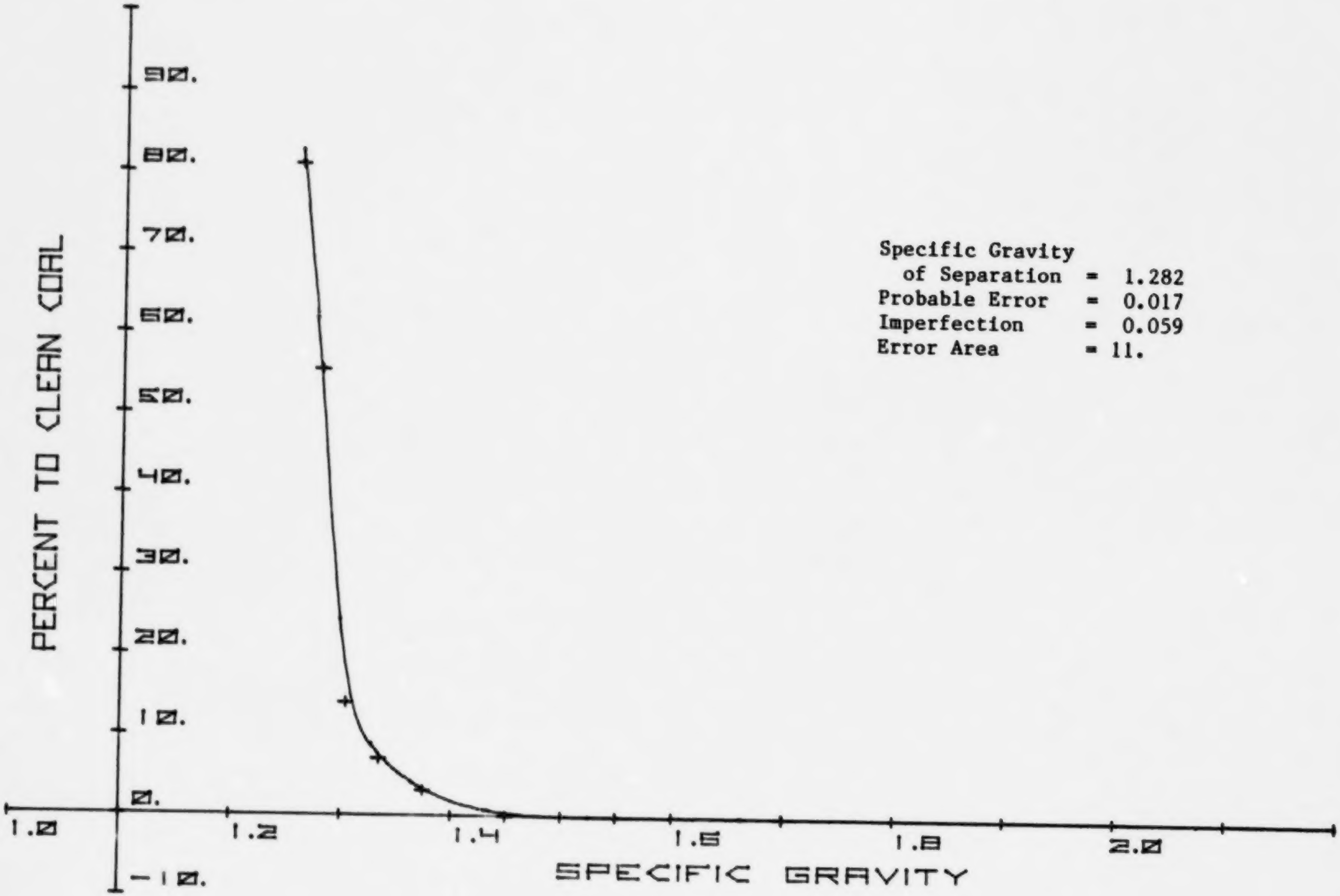
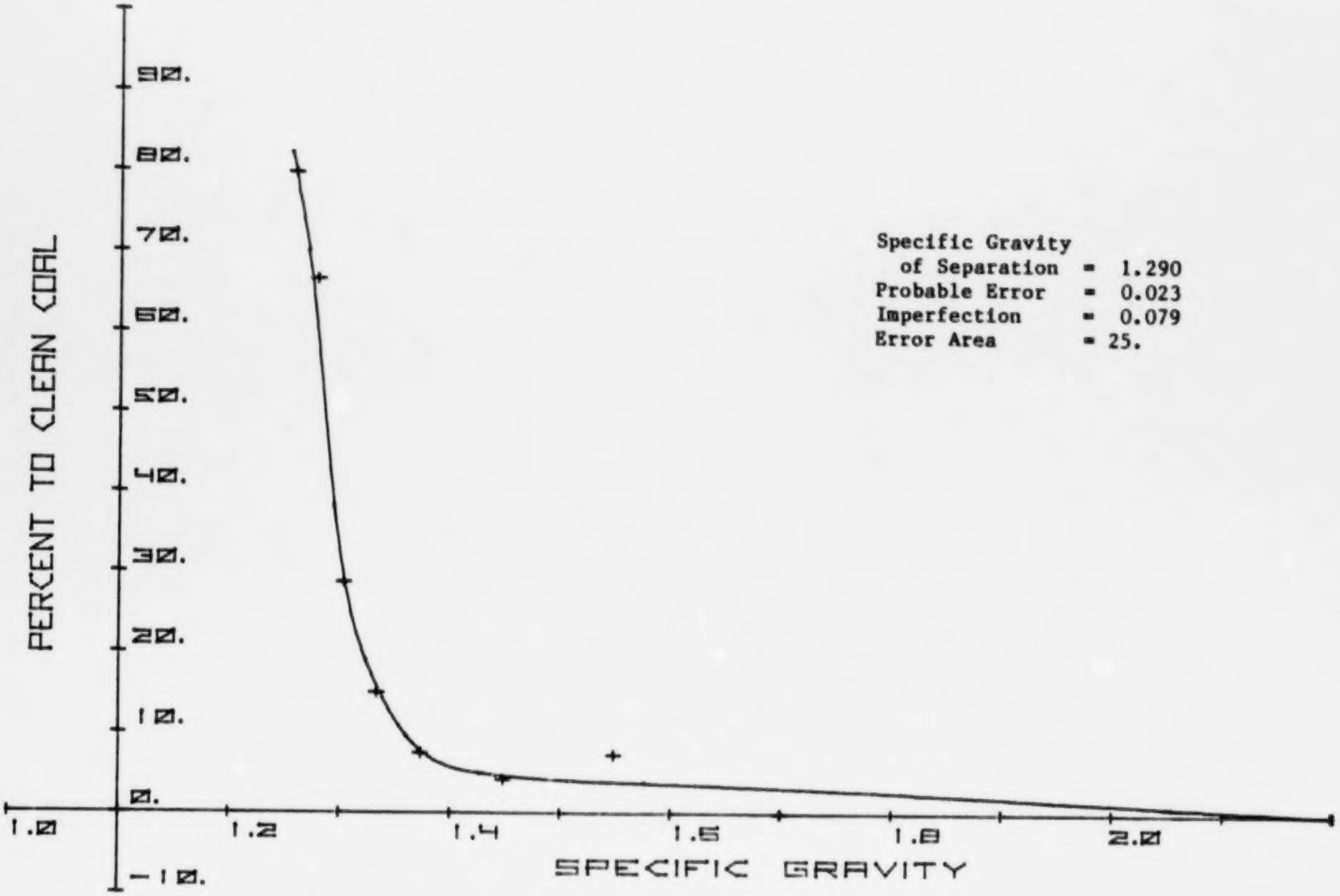


Figure A-45. Distribution Curve, Test No. 82 (9 x 14)



Specific Gravity
of Separation = 1.290
Probable Error = 0.023
Imperfection = 0.079
Error Area = 25.

Figure A-46. Distribution Curve, Test No. 82 (14 x 28)

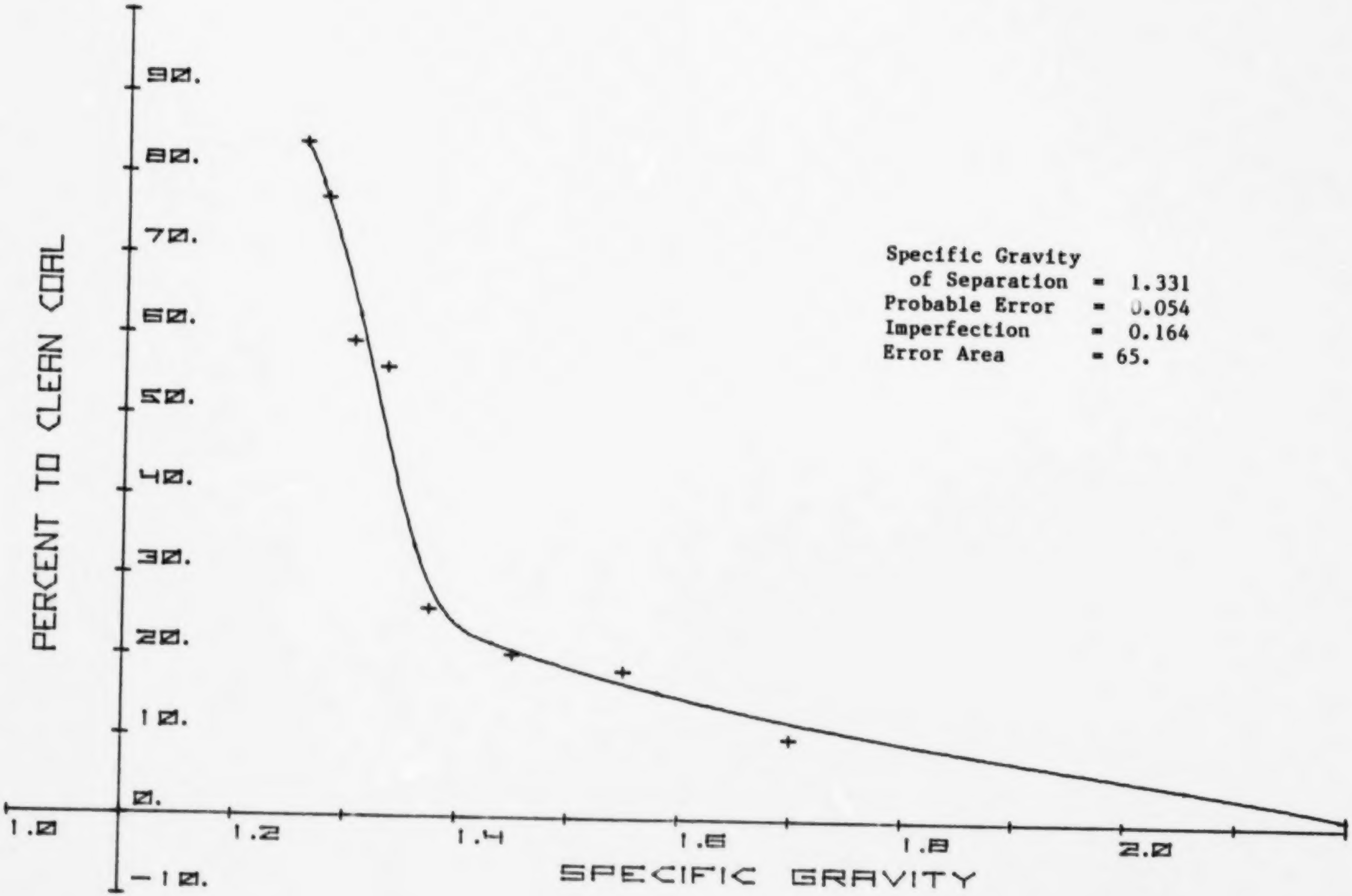
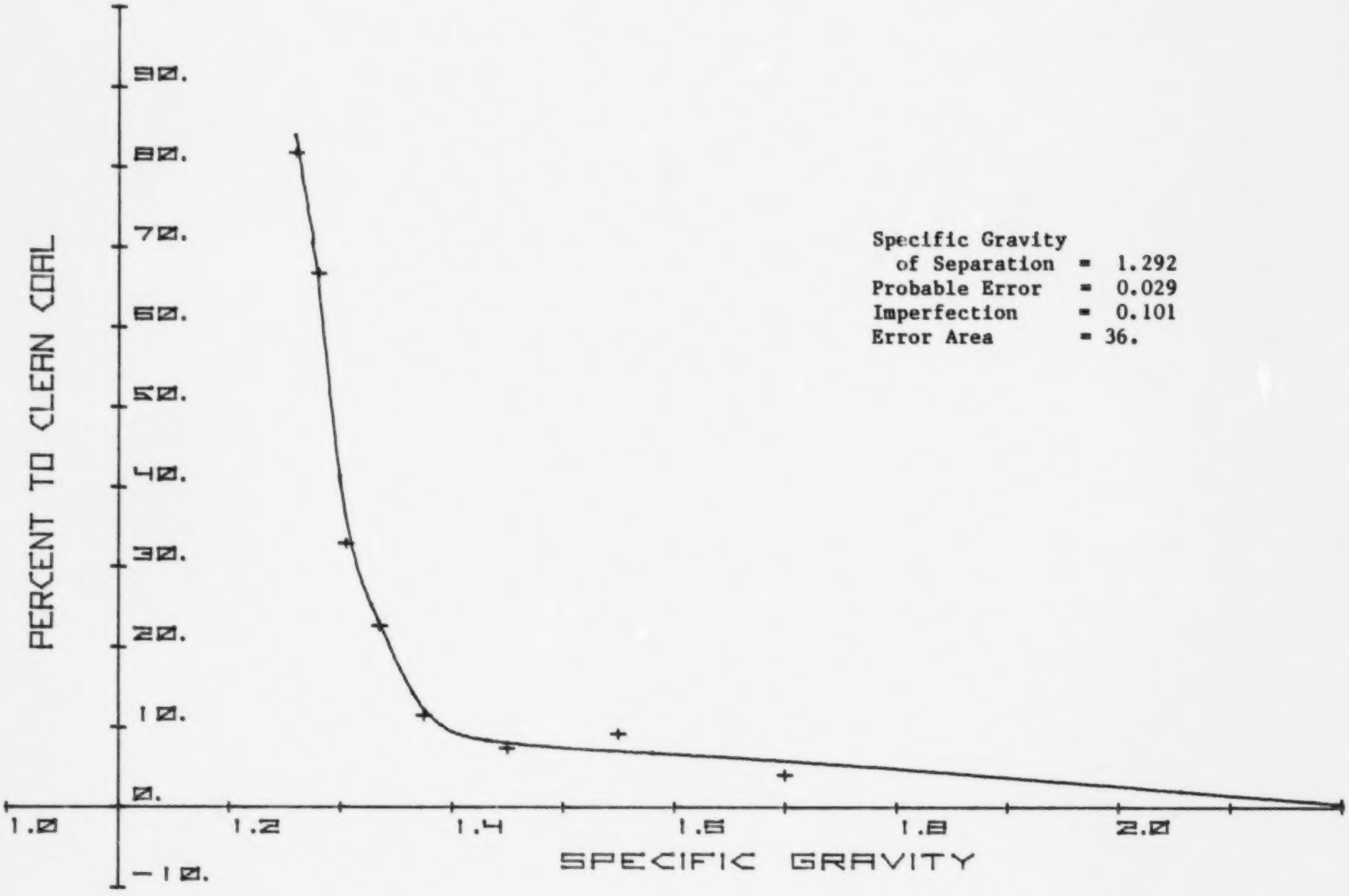


Figure A-47. Distribution Curve, Test No. 82 (28 x 100)

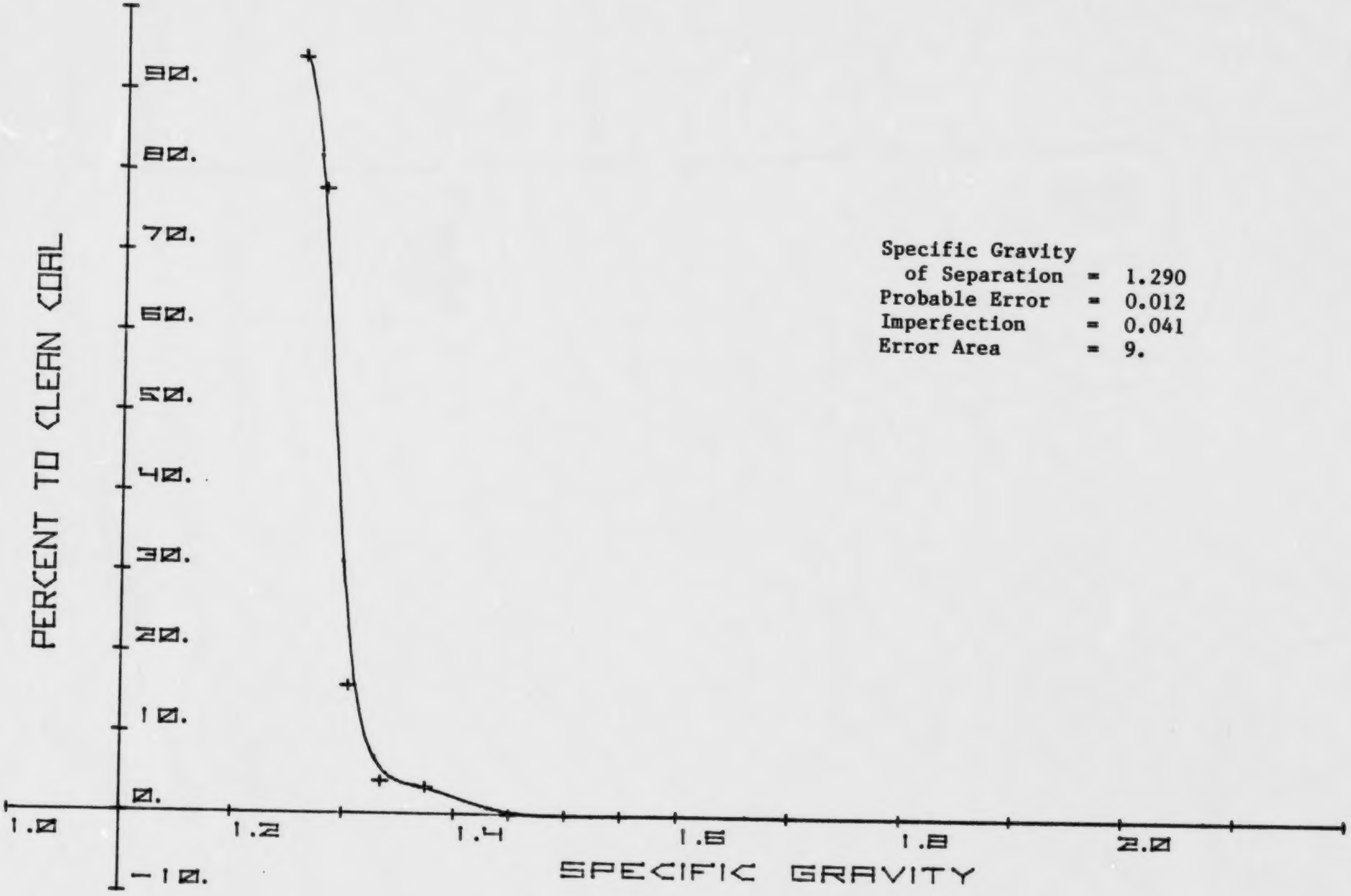


Specific Gravity
of Separation = 1.292
Probable Error = 0.029
Imperfection = 0.101
Error Area = 36.

Figure A-48. Distribution Curve, Test No. 82 (9 x 100)

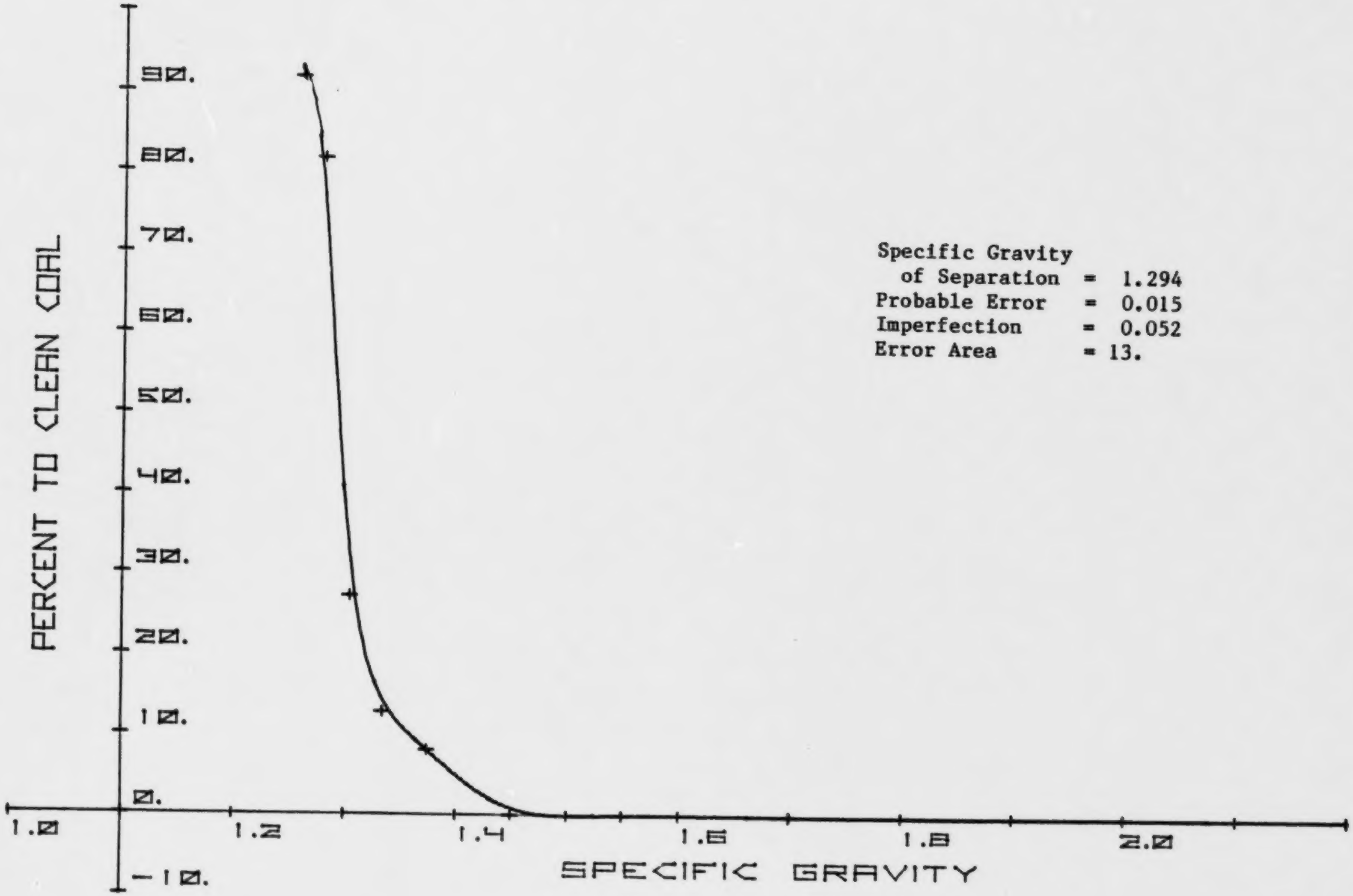
TABLE A-13. TEST 83 DATA SUMMARY

	9x14	14x28	28x100	9x100
SCREEN ANALYSIS, PERCENT:				
FEED	28.6	29.3	36.3	94.3
CLEAN COAL	20.5	29.0	43.5	93.0
REFUSE	34.5	29.6	31.1	95.2
ASH, PERCENT:				
FEED	26.5	24.1	24.2	24.8
CLEAN COAL	3.1	3.0	3.8	3.4
REFUSE	36.6	39.1	44.8	40.1
TOTAL SULFUR, PERCENT:				
FEED	3.17	3.71	4.80	3.97
CLEAN COAL	0.90	1.00	0.87	0.92
REFUSE	4.16	5.65	8.79	6.13
WEIGHT RECOVERY (YIELD)	30.2	41.6	50.3	41.5
THEORETICAL WEIGHT RECOVERY	34.1	49.4	59.2	50.7
WEIGHT RECOVERY EFFICIENCY	88.4	84.3	85.0	81.9
ASH ERROR	0.3	0.6	1.0	0.8
FLOAT IN REFUSE...% OF PRODUCT	5.5	9.8	17.4	10.3
SINK IN CLEAN COAL	7.1	8.4	8.8	11.6
TOTAL MISPLACED MATERIAL..% OF FEED	6.0	9.2	13.0	10.8
NEAR GRAVITY MATERIAL	56.1	65.7	64.6	62.3
SPECIFIC GRAVITY OF SEPARATION	1.290	1.294	1.336	1.297
PROBABLE ERROR	0.012	0.015	0.066	0.022
IMPERFECTION	0.041	0.052	0.196	0.076
ERROR AREA	9.	13.	39.	25.
DISTRIBUTION, PERCENT TO CLEANED COAL (SPECIFIC GRAVITY FRACTION):				
FLOAT 1.27	94.1	91.8	92.0	92.4
1.27-1.29	77.7	81.7	82.7	81.1
1.29-1.32	15.9	27.2	66.3	35.8
1.32-1.35	4.1	12.8	50.0	21.0
1.35-1.40	3.4	8.1	36.0	15.5
1.40-1.50	0.0	0.0	19.5	6.3
1.50-1.60	0.0	0.0	9.7	3.6
1.60-1.80	0.0	0.0	0.0	0.0
SINK 1.80	0.0	0.0	0.0	0.0



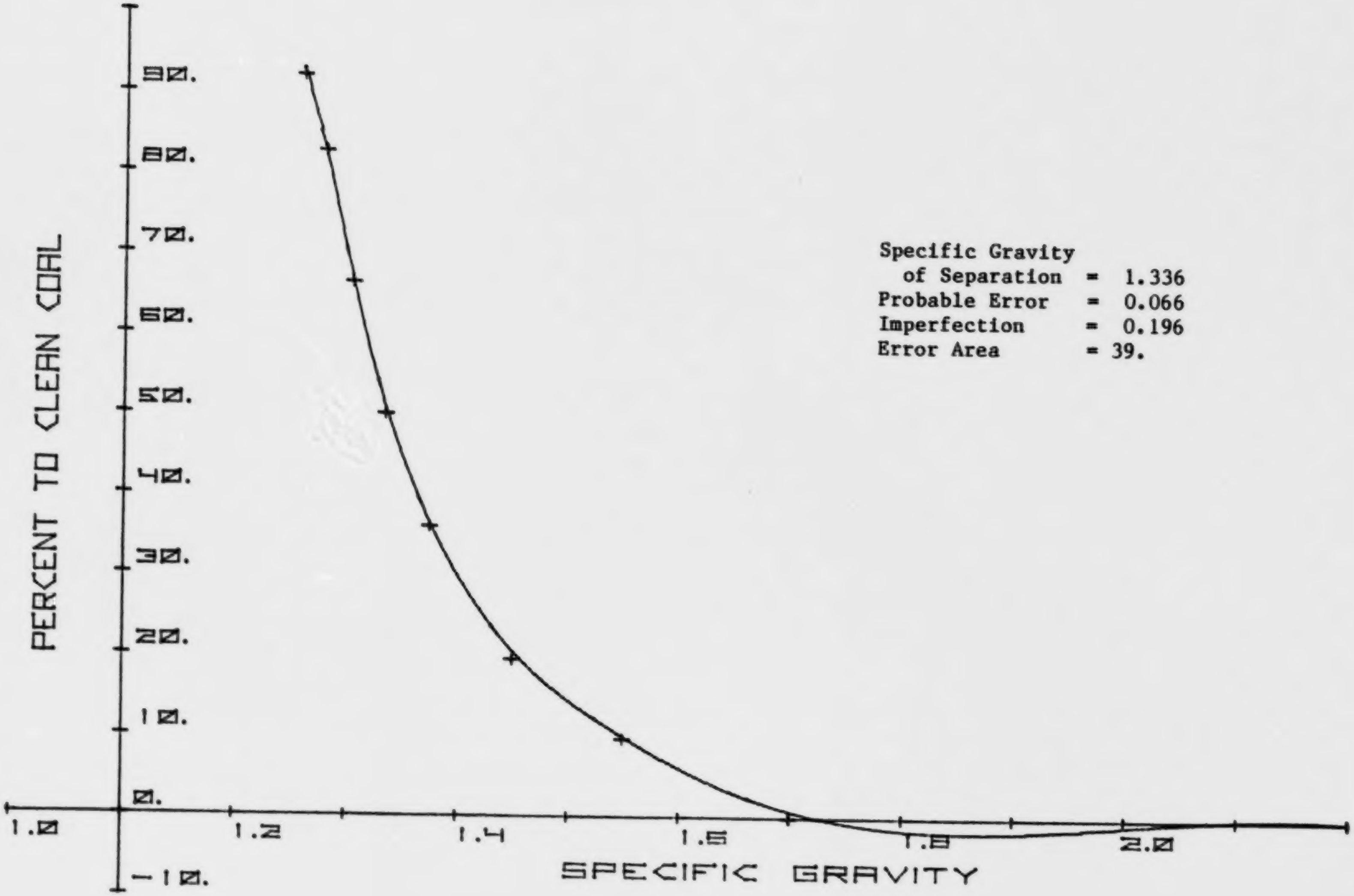
Specific Gravity
  of Separation = 1.290
Probable Error = 0.012
Imperfection = 0.041
Error Area = 9.

Figure A-49. Distribution Curve, Test No. 83 (9 x 14)



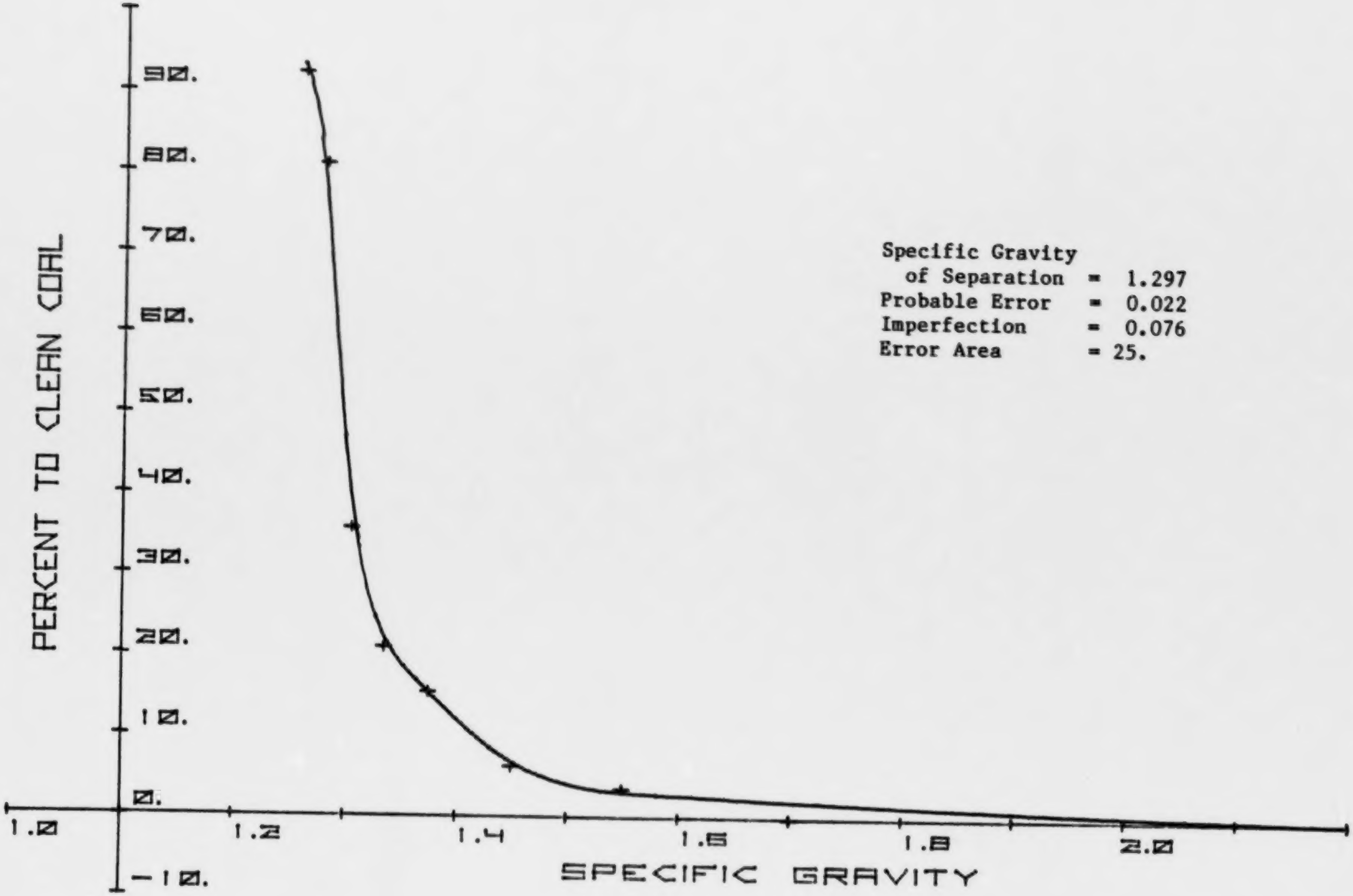
Specific Gravity
  of Separation = 1.294
Probable Error = 0.015
Imperfection = 0.052
Error Area = 13.

Figure A-50. Distribution Curve, Test No. 83 (14 x 28)



Specific Gravity
  of Separation = 1.336
Probable Error = 0.066
Imperfection = 0.196
Error Area = 39.

Figure A-51. Distribution Curve, Test No. 83 (28 x 100)

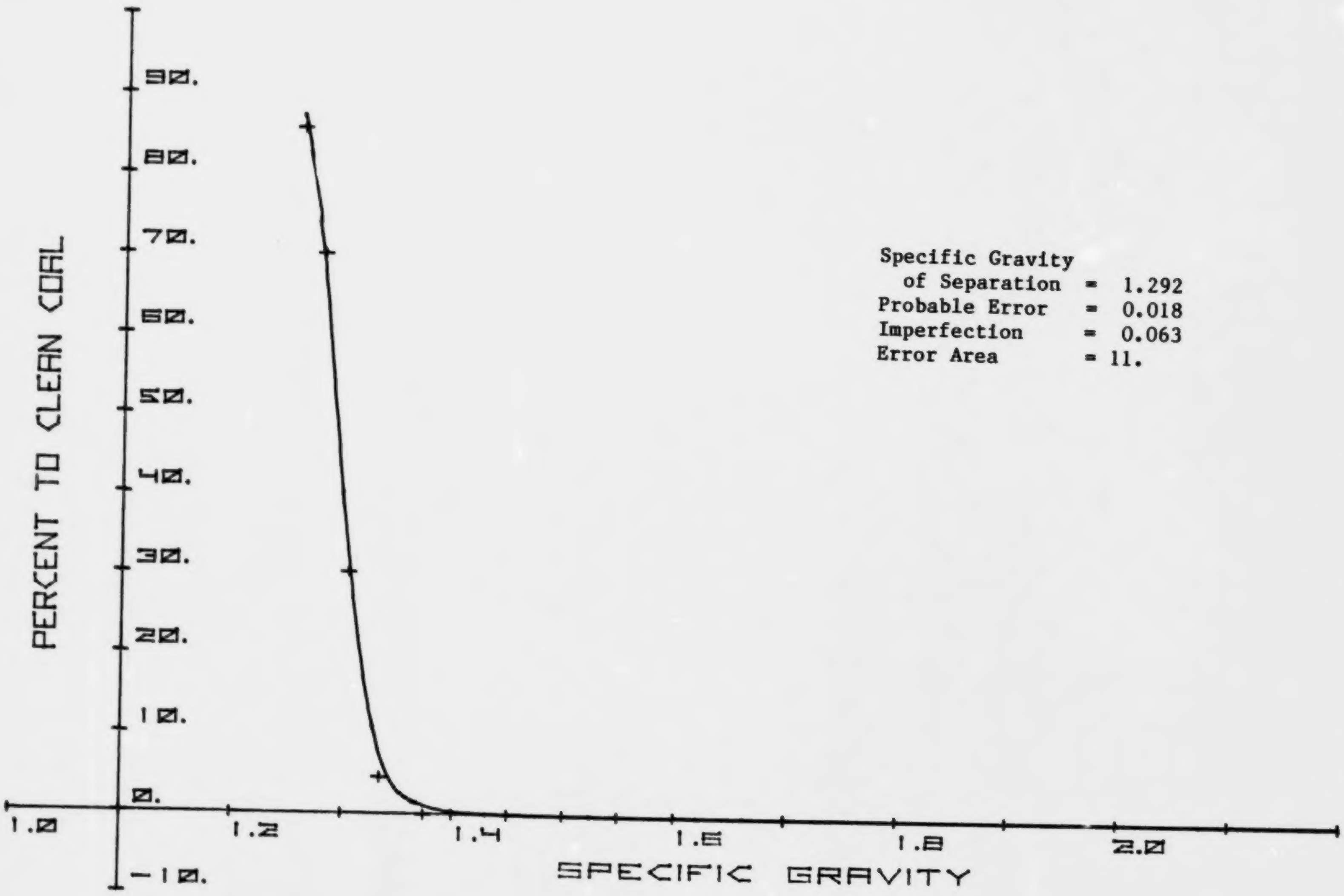


Specific Gravity
  of Separation = 1.297
Probable Error = 0.022
Imperfection = 0.076
Error Area = 25.

Figure A-52. Distribution Curve, Test No. 83 (9 x 100)

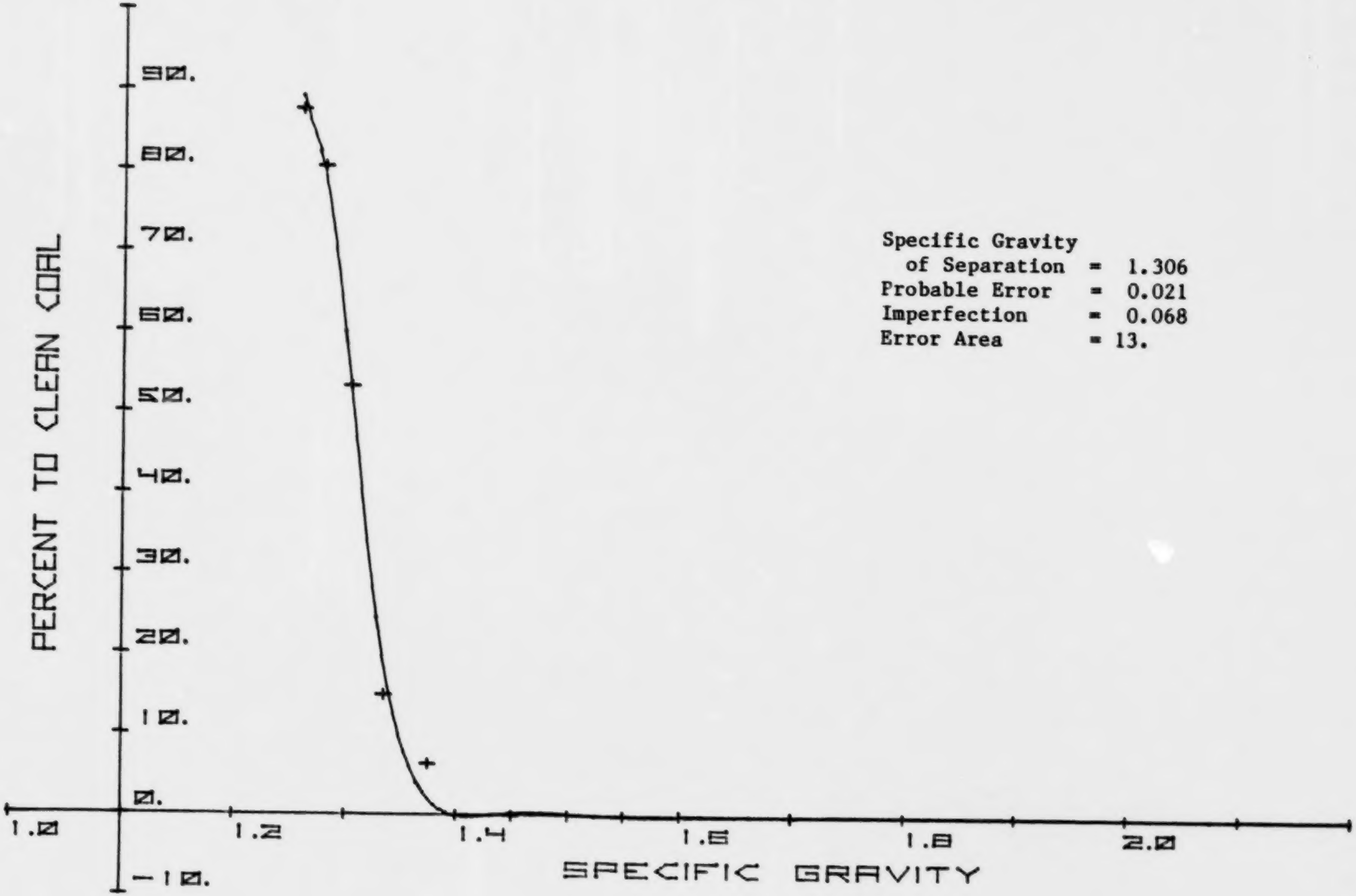
TABLE A-14. TEST 84 DATA SUMMARY

	9x14	14x28	28x100	9x100
SCREEN ANALYSIS, PERCENT:				
FEED............................	28.9	29.8	35.6	94.4
CLEAN COAL......................	19.1	30.5	43.3	92.9
REFUSE..........................	35.9	29.3	30.2	95.4
ASH, PERCENT:				
FEED............................	24.5	24.5	23.6	24.2
CLEAN COAL......................	2.8	3.4	3.7	3.4
REFUSE..........................	32.6	40.2	43.9	38.5
TOTAL SULFUR, PERCENT:				
FEED............................	3.61	3.73	5.08	4.20
CLEAN COAL......................	1.11	1.16	0.90	1.03
REFUSE..........................	4.56	5.63	9.33	6.40
WEIGHT RECOVERY (YIELD)............	27.4	42.6	50.4	40.9
THEORETICAL WEIGHT RECOVERY........	23.3	44.0	57.8	43.2
WEIGHT RECOVERY EFFICIENCY.........	100.0	96.8	87.3	94.7
ASH ERROR..........................	0.0	0.1	0.7	0.2
FLOAT IN REFUSE...% OF PRODUCT.....	9.2	16.1	22.7	15.9
SINK IN CLEAN COAL.................	14.0	9.4	5.9	13.9
TOTAL MISPLACED MATERIAL..% OF FEED	10.5	13.2	14.2	15.1
NEAR GRAVITY MATERIAL..............	59.7	65.7	65.4	64.0
SPECIFIC GRAVITY OF SEPARATION.....	1.292	1.306	1.351	1.308
PROBABLE ERROR.....................	0.018	0.021	0.071	0.027
IMPERFECTION.......................	0.063	0.068	0.203	0.088
ERROR AREA.........................	11.	13.	40.	23.
DISTRIBUTION, PERCENT TO CLEANED COAL (SPECIFIC GRAVITY FRACTION):				
FLOAT 1.27	85.7	87.6	85.2	86.1
1.27-1.29	70.0	80.5	86.8	80.0
1.29-1.32	30.2	53.2	77.3	53.9
1.32-1.35	4.6	14.9	58.7	24.6
1.35-1.40	0.6	0.4	37.2	13.6
1.40-1.50	0.0	0.0	23.4	7.2
1.50-1.60	0.0	0.0	12.1	3.6
1.60-1.80	0.0	0.0	0.0	0.0
SINK 1.80	0.0	0.0	0.0	0.0



Specific Gravity
of Separation = 1.292
Probable Error = 0.018
Imperfection = 0.063
Error Area = 11.

Figure A-53. Distribution Curve, Test No. 84 (9 x 14)



Specific Gravity
of Separation = 1.306
Probable Error = 0.021
Imperfection = 0.068
Error Area = 13.

Figure A-54. Distribution Curve, Test No. 84 (14 x 28)

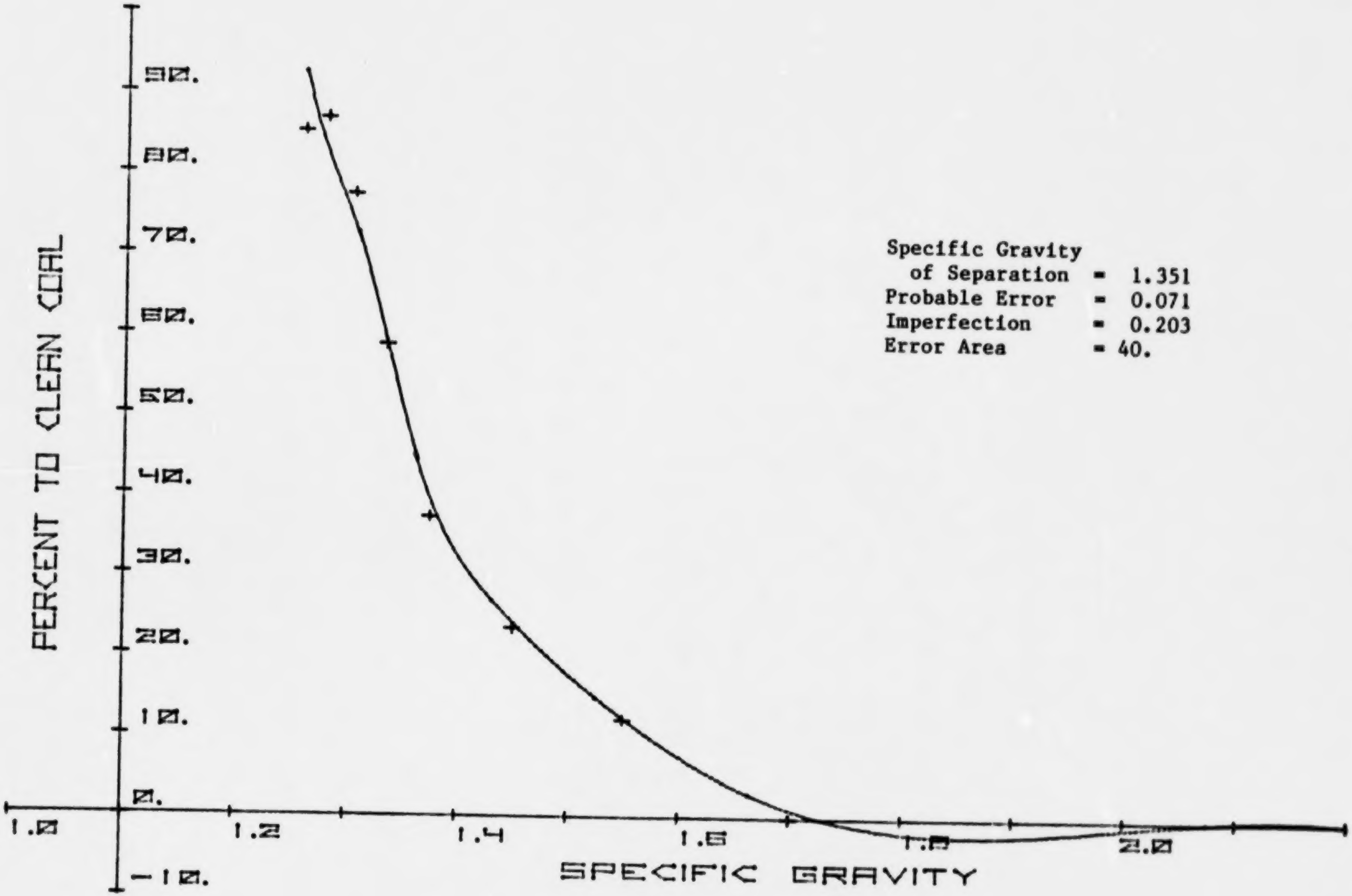
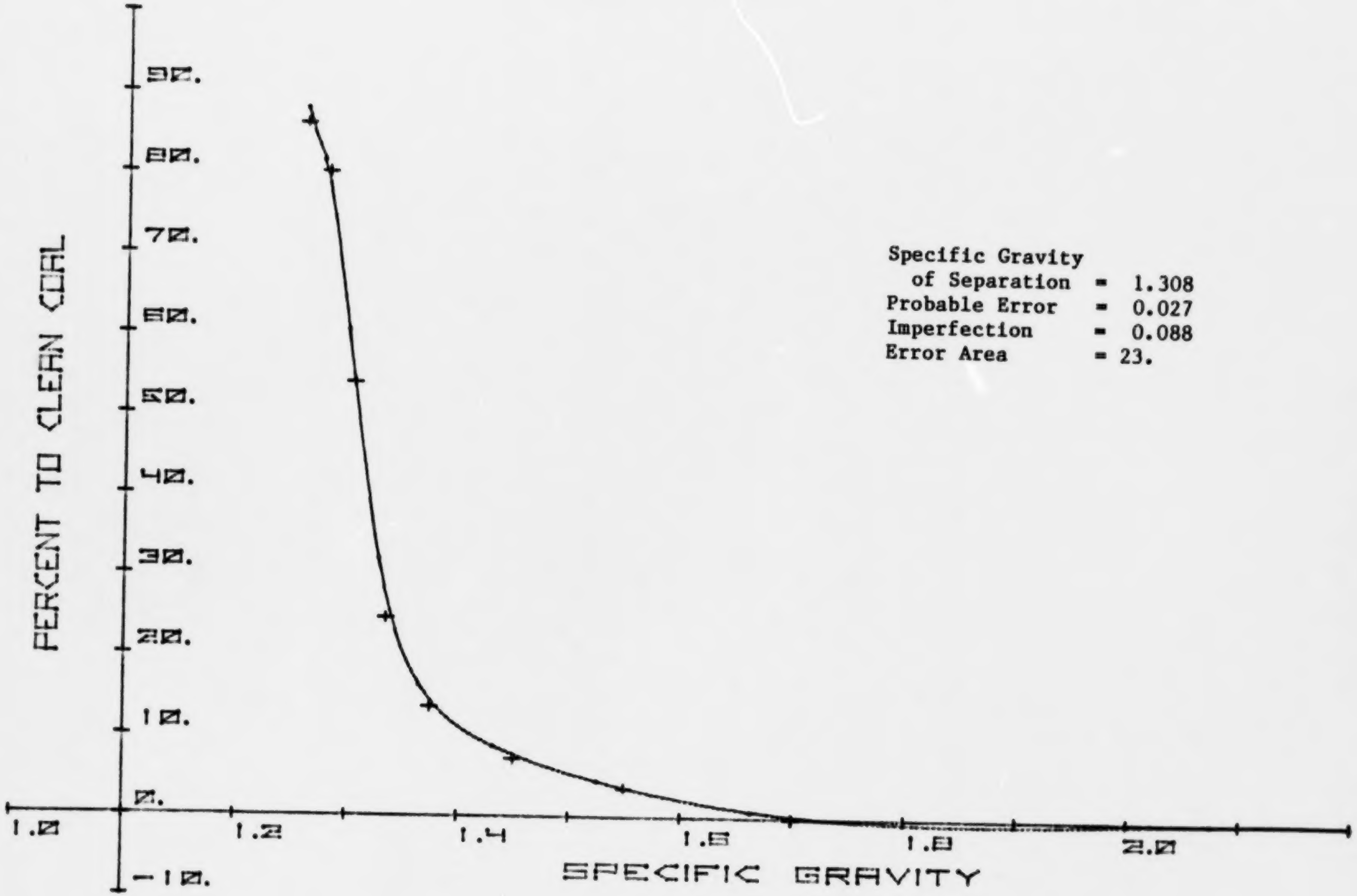


Figure A-55. Distribution Curve, Test No. 84 (28 x 100)

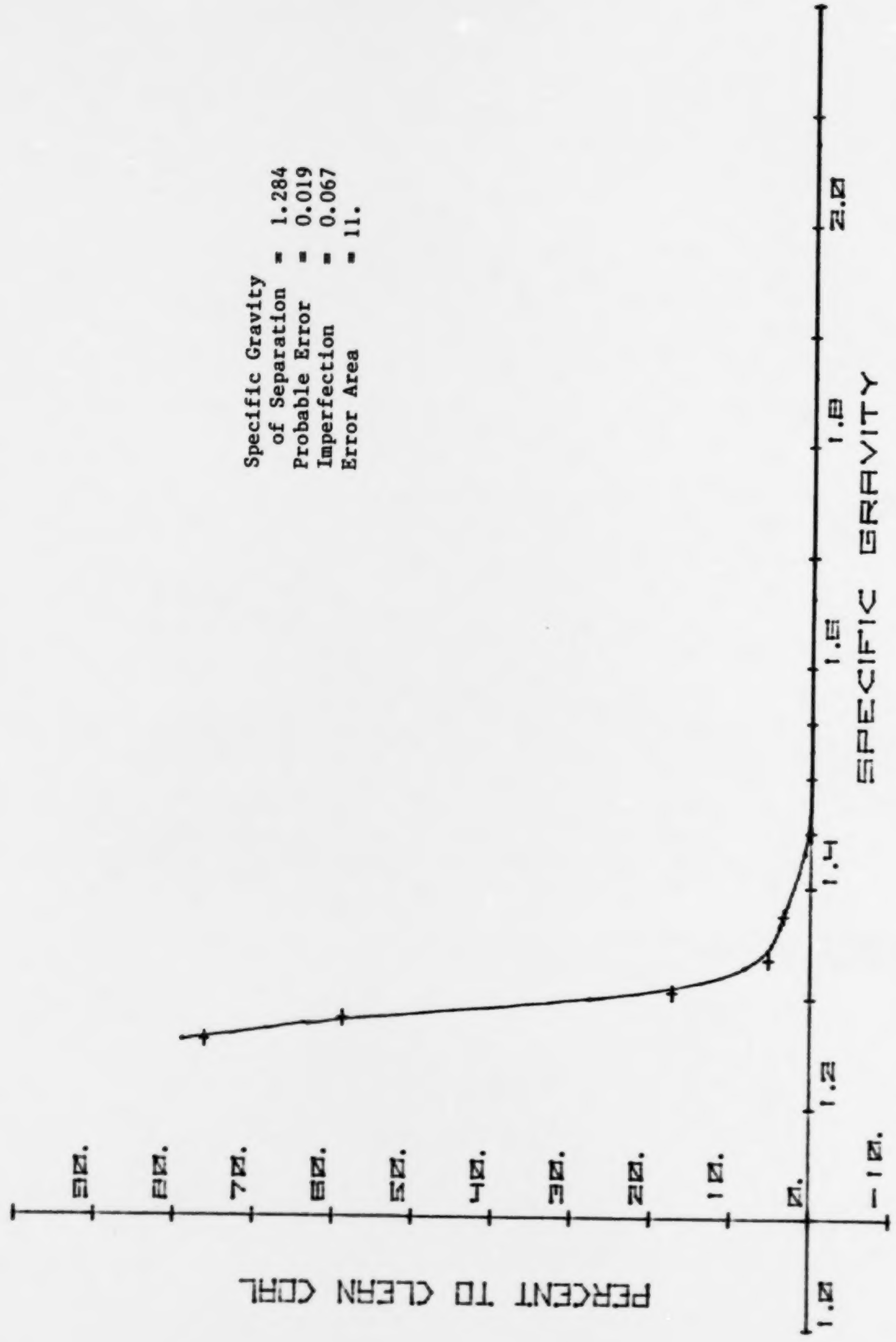


Specific Gravity
of Separation = 1.308
Probable Error = 0.027
Imperfection = 0.088
Error Area = 23.

Figure A-56. Distribution Curve, Test No. 84 (9 x 100)

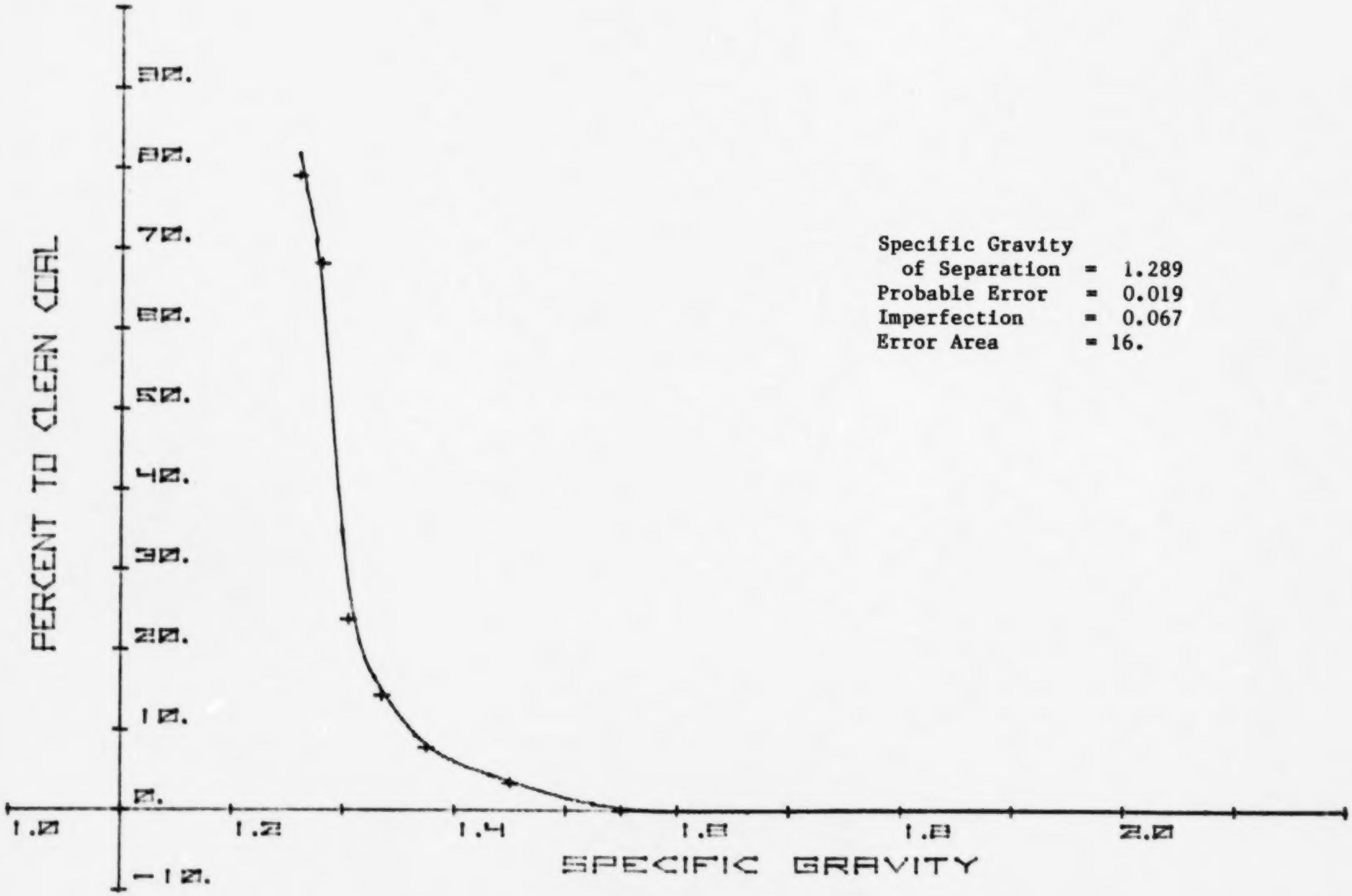
TABLE A-15. TEST 85 DATA SUMMARY

	9x14	14x28	28x100	9x100
SCREEN ANALYSIS, PERCENT:				
FEED..............................	31.1	28.9	34.0	94.1
CLEAN COAL........................	21.6	28.0	41.7	91.3
REFUSE............................	36.8	29.4	29.4	95.7
ASH, PERCENT:				
FEED..............................	27.0	22.1	23.5	24.2
CLEAN COAL........................	3.0	3.0	3.9	3.4
REFUSE............................	35.5	32.9	40.2	36.2
TOTAL SULFUR, PERCENT:				
FEED..............................	3.05	3.11	4.73	3.68
CLEAN COAL........................	0.80	0.80	0.88	0.84
REFUSE............................	3.85	4.42	8.00	5.30
WEIGHT RECOVERY (YIELD)............	26.1	36.3	45.9	36.4
THEORETICAL WEIGHT RECOVERY........	33.8	45.8	60.9	49.6
WEIGHT RECOVERY EFFICIENCY.........	77.0	79.3	75.5	73.3
ASH ERROR..........................	0.4	0.6	1.6	1.1
FLOAT IN REFUSE...% OF PRODUCT.....	12.1	16.0	24.2	16.1
SINK IN CLEAN COAL.................	17.7	13.7	11.8	16.0
TOTAL MISPLACED MATERIAL..% OF FEED	13.6	15.1	18.5	16.1
NEAR GRAVITY MATERIAL..............	58.1	66.1	64.6	62.7
SPECIFIC GRAVITY OF SEPARATION.....	1.284	1.289	1.323	1.292
PROBABLE ERROR.....................	0.019	0.019	0.064	0.026
IMPERFECTION.......................	0.067	0.067	0.198	0.090
ERROR AREA.........................	11.	16.	58.	30.
DISTRIBUTION, PERCENT TO CLEANED COAL (SPECIFIC GRAVITY FRACTION):				
FLOAT 1.27	76.1	79.0	80.3	78.8
1.27-1.29	58.8	68.1	76.4	67.9
1.29-1.32	17.3	23.7	59.3	31.7
1.32-1.35	5.3	14.3	43.9	19.5
1.35-1.40	3.4	7.8	29.1	12.6
1.40-1.50	0.0	3.4	21.0	6.7
1.50-1.60	0.0	0.0	11.3	4.2
1.60-1.80	0.0	0.0	8.4	3.8
SINK 1.80	0.0	0.0	1.8	0.7



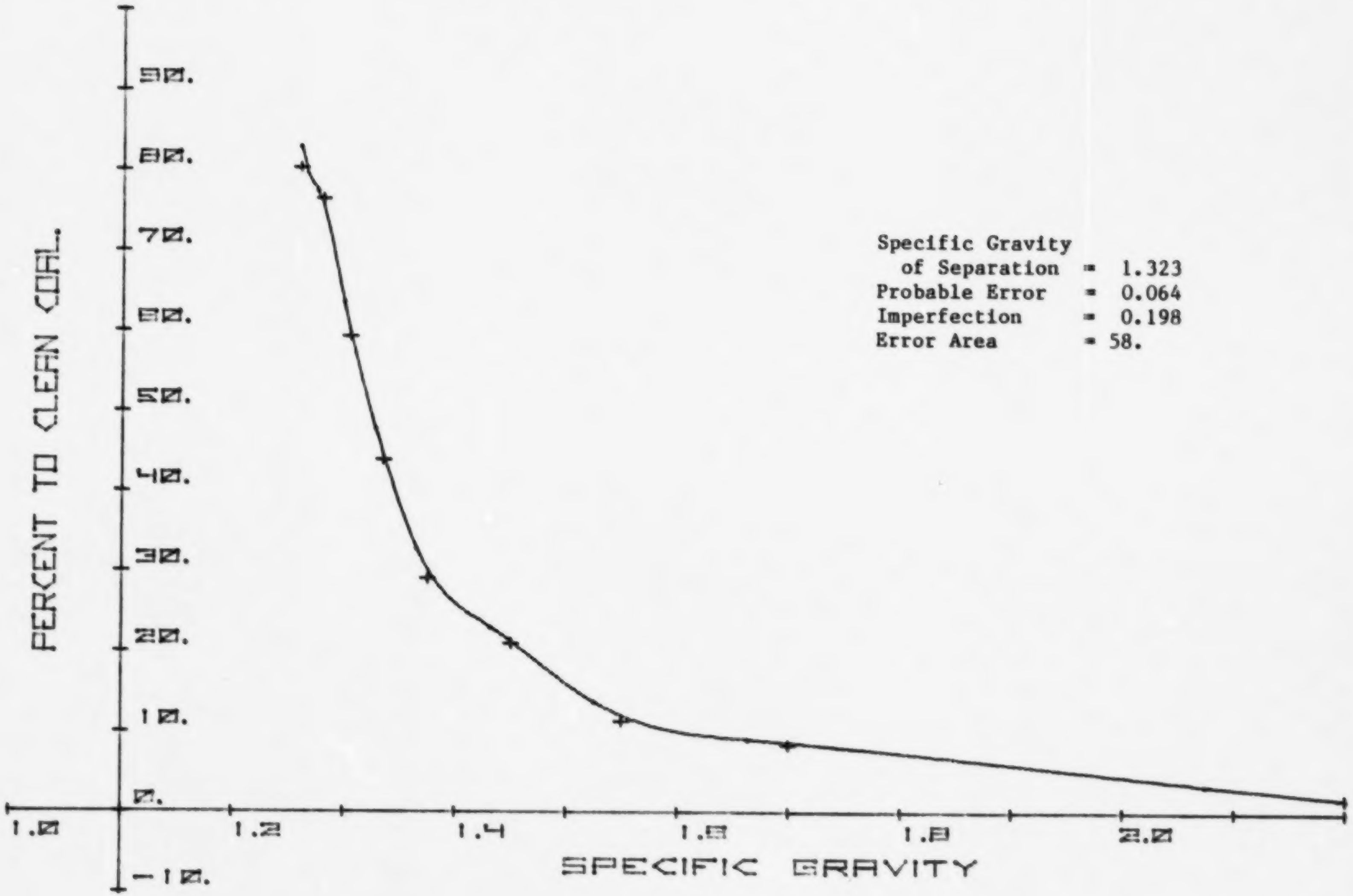
Specific Gravity
of Separation = 1.284
Probable Error = 0.019
Imperfection = 0.067
Error Area = 11.

Figure A-57. Distribution Curve, Test No. 85 (9 x 14)



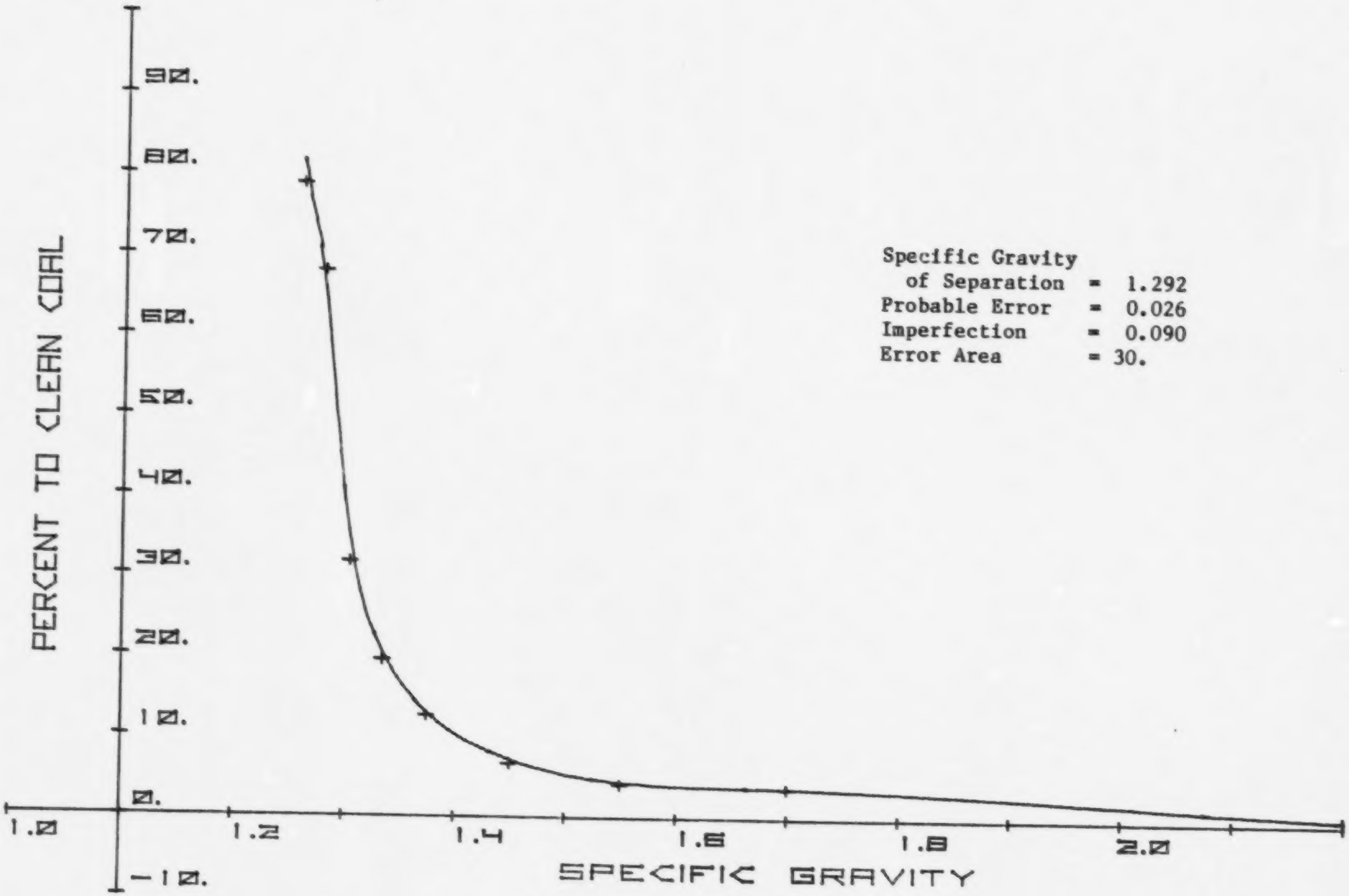
Specific Gravity
  of Separation = 1.289
Probable Error = 0.019
Imperfection = 0.067
Error Area = 16.

Figure A-58. Distribution Curve, Test No. 85 (14 x 28)



Specific Gravity
  of Separation = 1.323
Probable Error = 0.064
Imperfection = 0.198
Error Area = 58.

Figure A-59. Distribution Curve, Test No. 85 (28 x 100)



Specific Gravity
of Separation = 1.292
Probable Error = 0.026
Imperfection = 0.090
Error Area = 30.

Figure A-60. Distribution Curve, Test No. 85 (9 x 100)

TABLE A-16. TEST 86 DATA SUMMARY

	9x14	14x28	28x100	9x100
SCREEN ANALYSIS, PERCENT:				
FEED..............................	33.6	30.0	31.1	94.6
CLEAN COAL........................	25.8	31.0	36.2	93.0
REFUSE............................	38.8	29.4	27.6	95.8
ASH, PERCENT:				
FEED..............................	29.5	24.6	25.1	26.5
CLEAN COAL........................	4.1	4.3	5.0	4.5
REFUSE............................	41.0	39.0	42.9	40.9
TOTAL SULFUR, PERCENT:				
FEED..............................	3.34	3.36	4.46	3.72
CLEAN COAL........................	1.19	0.92	0.91	0.99
REFUSE............................	4.31	5.10	7.62	5.50
WEIGHT RECOVERY (YIELD)............	31.0	41.6	47.0	39.6
THEORETICAL WEIGHT RECOVERY........	35.5	54.3	63.2	53.2
WEIGHT RECOVERY EFFICIENCY.........	87.1	76.6	74.4	74.4
ASH ERROR..........................	0.3	0.8	2.0	1.2
FLOAT IN REFUSE...% OF PRODUCT.....	11.1	16.2	23.1	16.3
SINK IN CLEAN COAL.................	14.8	12.4	12.3	14.8
TOTAL MISPLACED MATERIAL..% OF FEED	12.3	14.7	18.0	15.7
NEAR GRAVITY MATERIAL..............	57.4	64.7	63.4	61.9
SPECIFIC GRAVITY OF SEPARATION.....	1.289	1.297	1.342	1.298
PROBABLE ERROR.....................	0.018	0.031	0.073	0.036
IMPERFECTION.......................	0.064	0.106	0.213	0.123
ERROR AREA.........................	13.	31.	72.	40.
DISTRIBUTION, PERCENT TO CLEANED COAL (SPECIFIC GRAVITY FRACTION):				
FLOAT 1.27	88.5	88.3	84.6	86.9
1.27-1.29	66.0	69.7	73.4	69.4
1.29-1.32	25.7	41.1	66.1	42.1
1.32-1.35	12.3	25.4	53.5	28.3
1.35-1.40	6.9	13.0	35.8	17.4
1.40-1.50	0.0	8.4	19.9	9.3
1.50-1.60	0.0	8.0	19.8	9.8
1.60-1.80	0.0	0.0	9.4	2.6
SINK 1.80	0.0	0.0	2.5	0.8

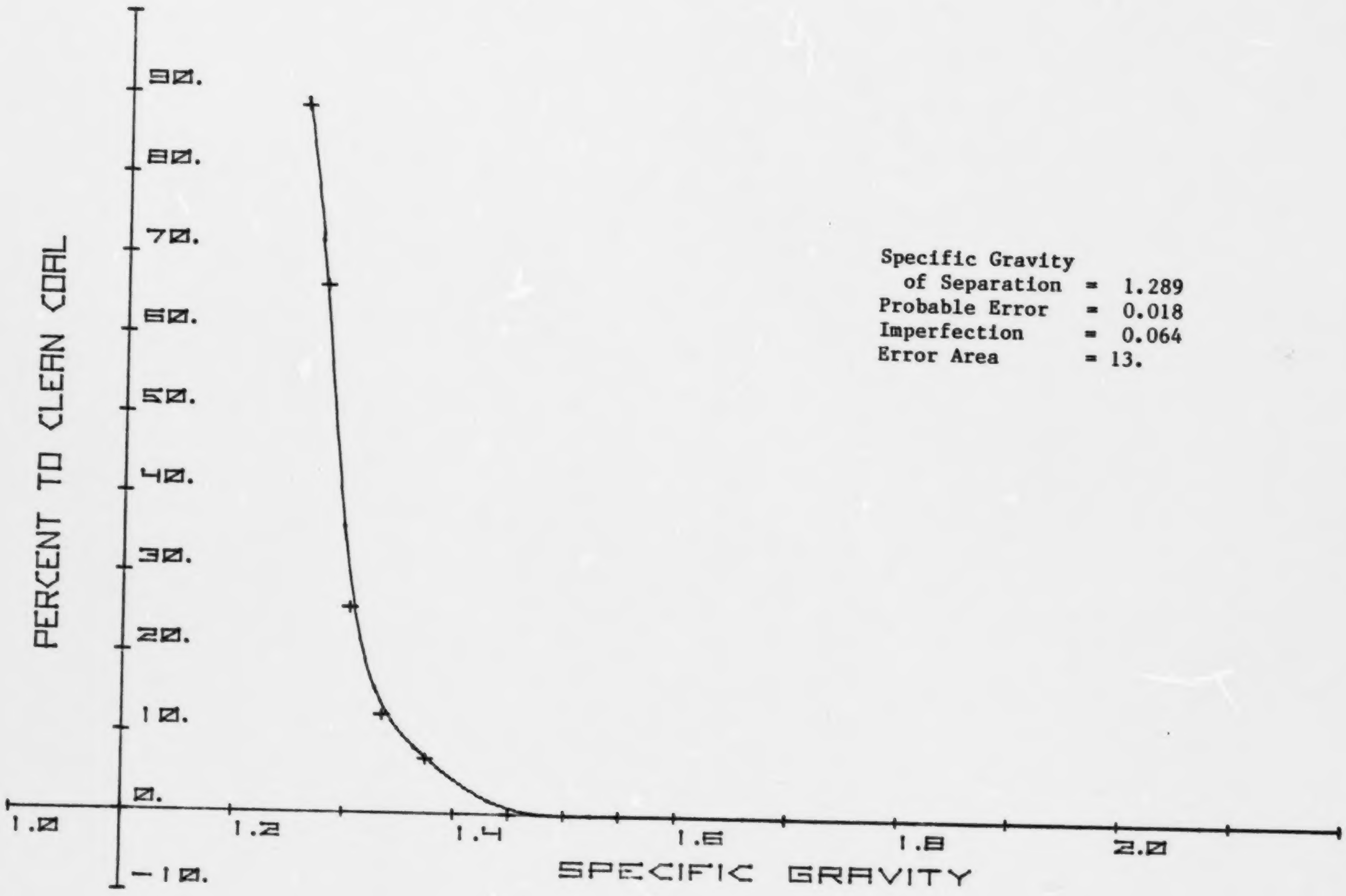
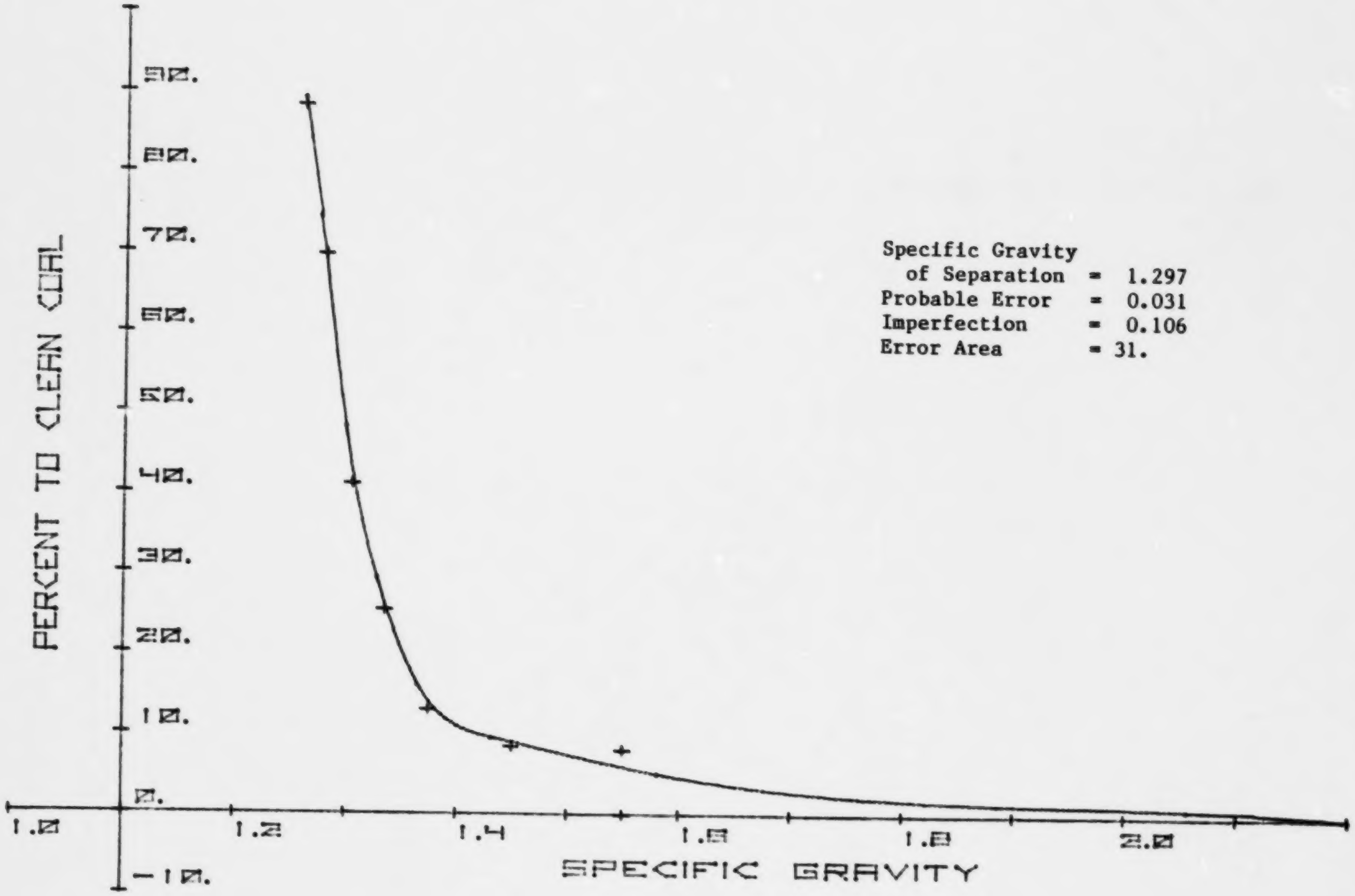
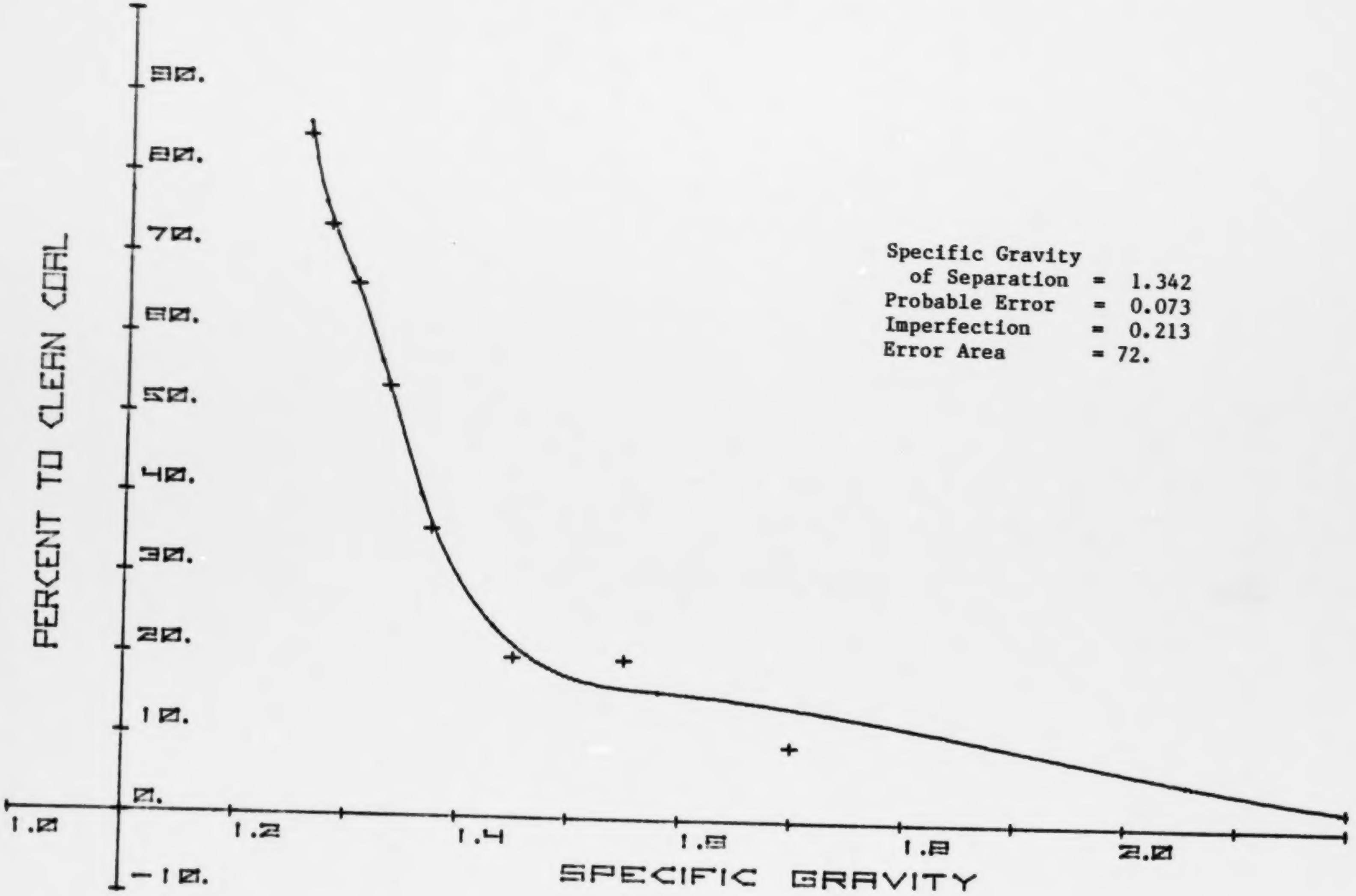


Figure A-61. Distribution Curve, Test No. 86 (9 x 14)



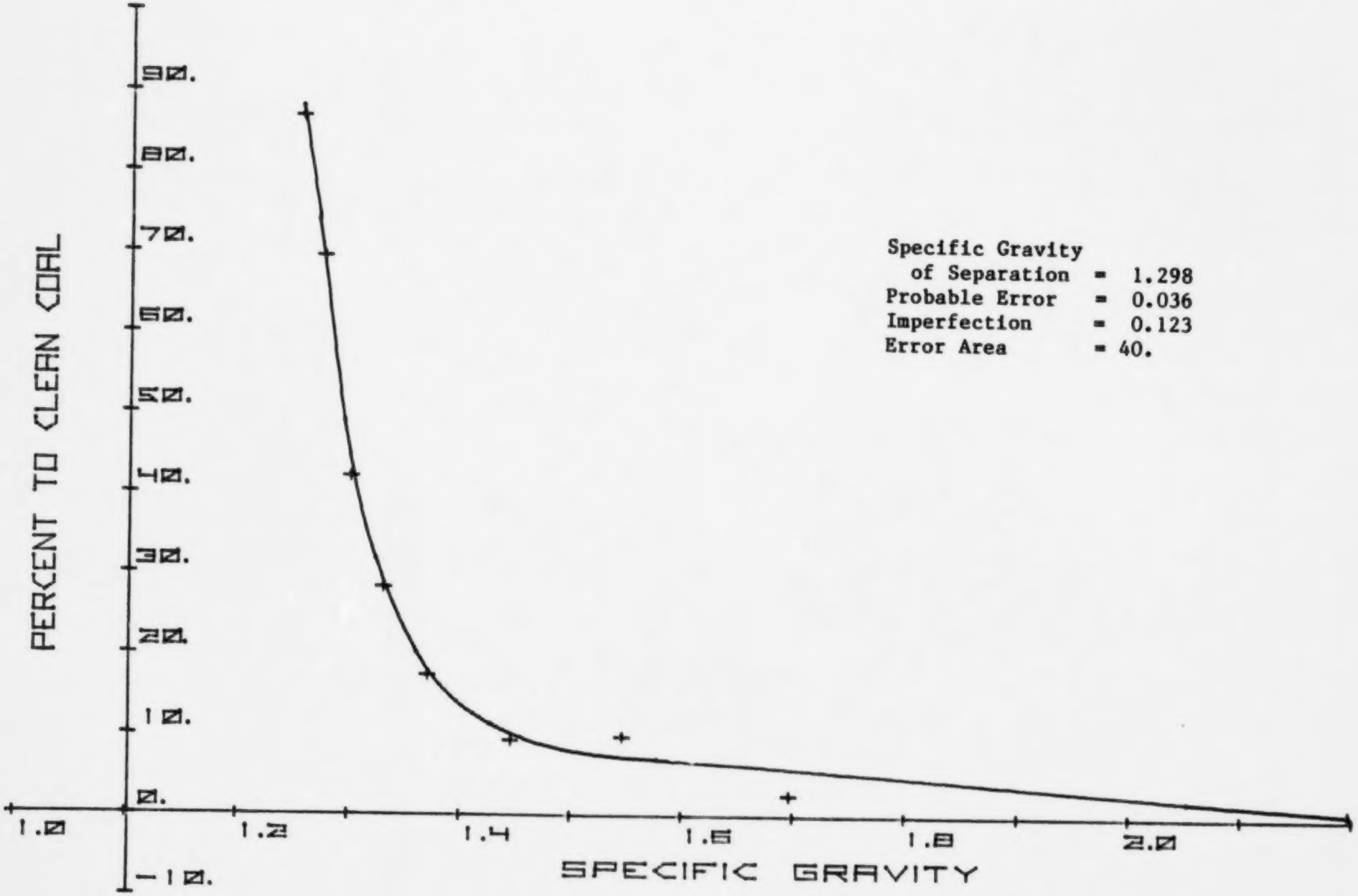
Specific Gravity
  of Separation = 1.297
Probable Error = 0.031
Imperfection = 0.106
Error Area = 31.

Figure A-62. Distribution Curve, Test No. 86 (14 x 28)



Specific Gravity
  of Separation = 1.342
Probable Error = 0.073
Imperfection = 0.213
Error Area = 72.

Figure A-63. Distribution Curve, Test No. 86 (28 x 100)



Specific Gravity
  of Separation = 1.298
Probable Error = 0.036
Imperfection = 0.123
Error Area = 40.

Figure A-64. Distribution Curve, Test No. 86 (9 x 100)

TABLE A-17. TEST 87 DATA SUMMARY

	9x14	14x28	28x100	9x100
SCREEN ANALYSIS, PERCENT:				
FEED	34.5	29.4	31.2	95.1
CLEAN COAL	23.2	29.3	40.8	93.2
REFUSE	41.5	29.4	25.3	96.2
ASH, PERCENT:				
FEED	27.8	22.1	22.1	24.2
CLEAN COAL	2.8	2.9	3.7	3.3
REFUSE	36.5	34.1	40.7	36.9
TOTAL SULFUR, PERCENT:				
FEED	3.17	2.80	3.90	3.30
CLEAN COAL	0.86	0.78	1.13	0.95
REFUSE	3.98	4.07	6.70	4.72
WEIGHT RECOVERY (YIELD)	25.9	38.4	50.3	37.8
THEORETICAL WEIGHT RECOVERY	31.1	48.7	60.5	47.8
WEIGHT RECOVERY EFFICIENCY	83.3	78.9	83.2	79.1
ASH ERROR	0.3	0.6	1.1	0.9
FLOAT IN REFUSE...% OF PRODUCT	9.4	16.3	20.0	14.1
SINK IN CLEAN COAL	17.1	10.7	10.4	13.1
TOTAL MISPLACED MATERIAL..% OF FEED	11.4	14.1	15.2	13.7
NEAR GRAVITY MATERIAL	57.2	66.2	66.1	62.9
SPECIFIC GRAVITY OF SEPARATION	1.284	1.289	1.326	1.291
PROBABLE ERROR	0.015	0.020	0.057	0.023
IMPERFECTION	0.052	0.070	0.176	0.080
ERROR AREA	10.	14.	57.	27.
DISTRIBUTION, PERCENT TO CLEANED COAL (SPECIFIC GRAVITY FRACTION):				
FLOAT 1.27	86.0	84.8	88.6	86.7
1.27-1.29	58.9	65.2	77.8	66.9
1.29-1.32	12.6	27.6	63.6	31.8
1.32-1.35	6.3	12.6	44.2	17.8
1.35-1.40	3.3	7.0	30.6	10.4
1.40-1.50	0.0	0.0	16.1	4.5
1.50-1.60	0.0	0.0	14.4	5.0
1.60-1.80	0.0	0.0	8.9	1.5
SINK 1.80	0.0	0.0	1.9	0.7

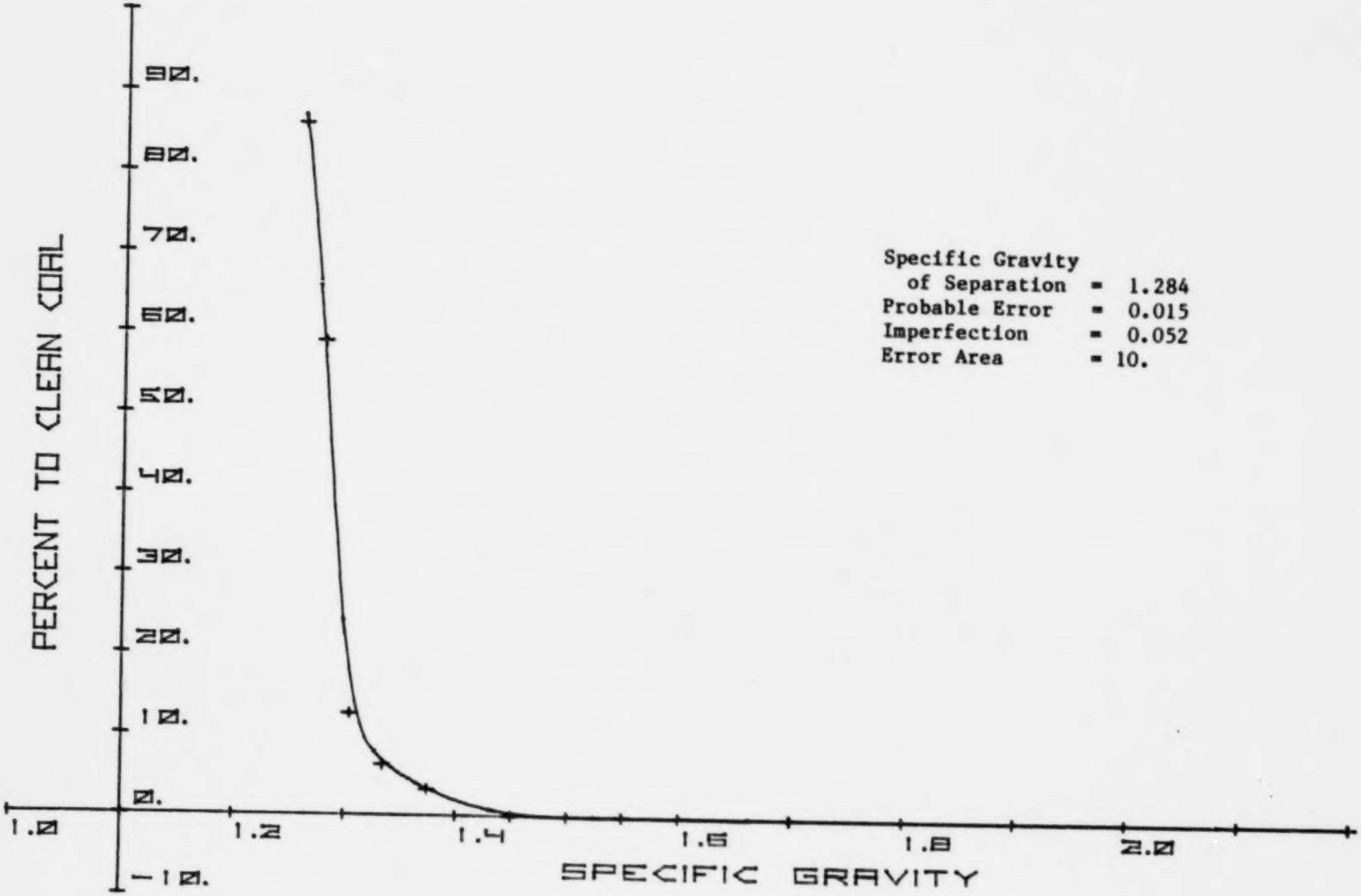
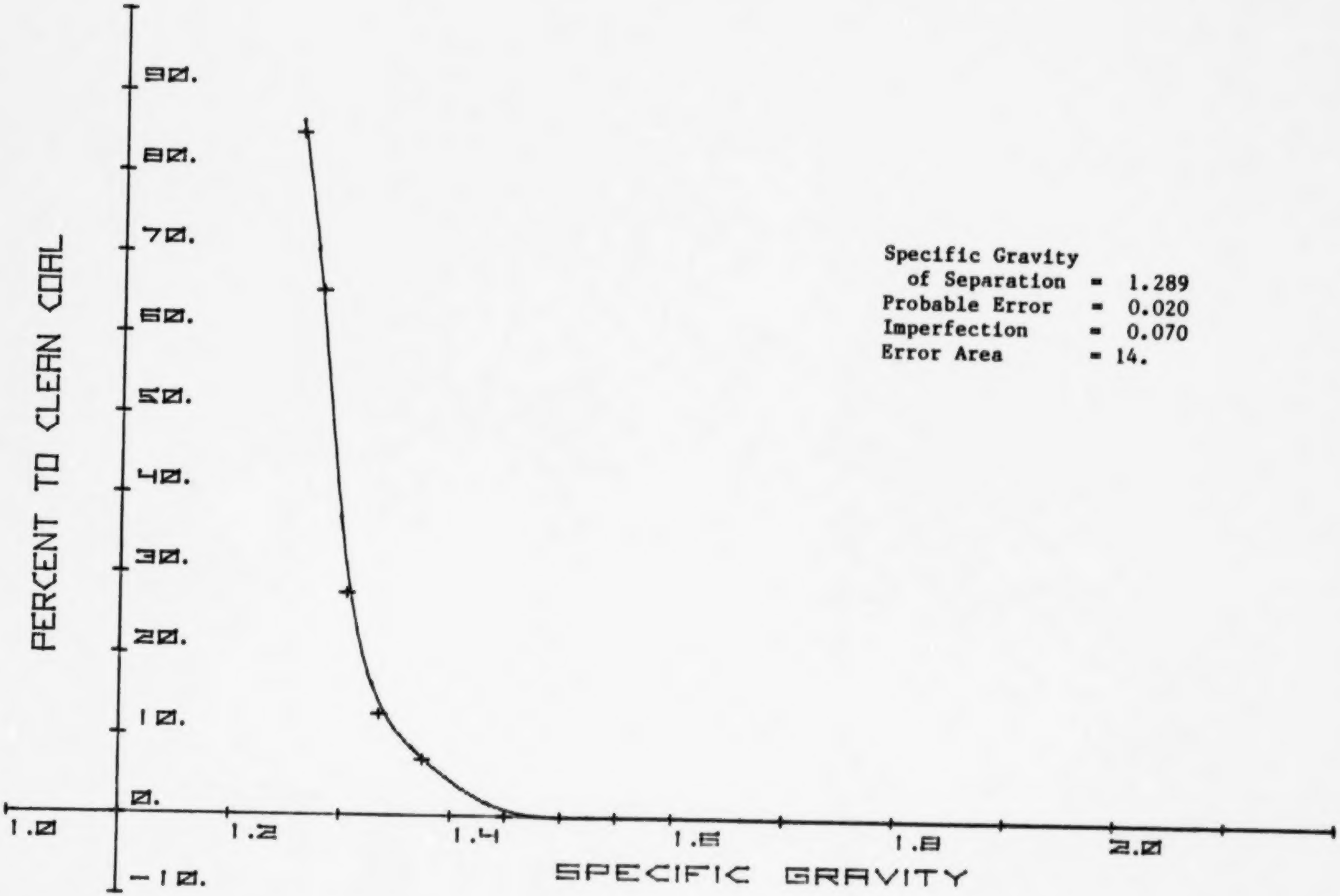


Figure A-65. Distribution Curve, Test No. 87 (9 x 14)



Specific Gravity
  of Separation = 1.289
Probable Error = 0.020
Imperfection = 0.070
Error Area = 14.

Figure A-66. Distribution Curve, Test No. 87 (14 x 28)

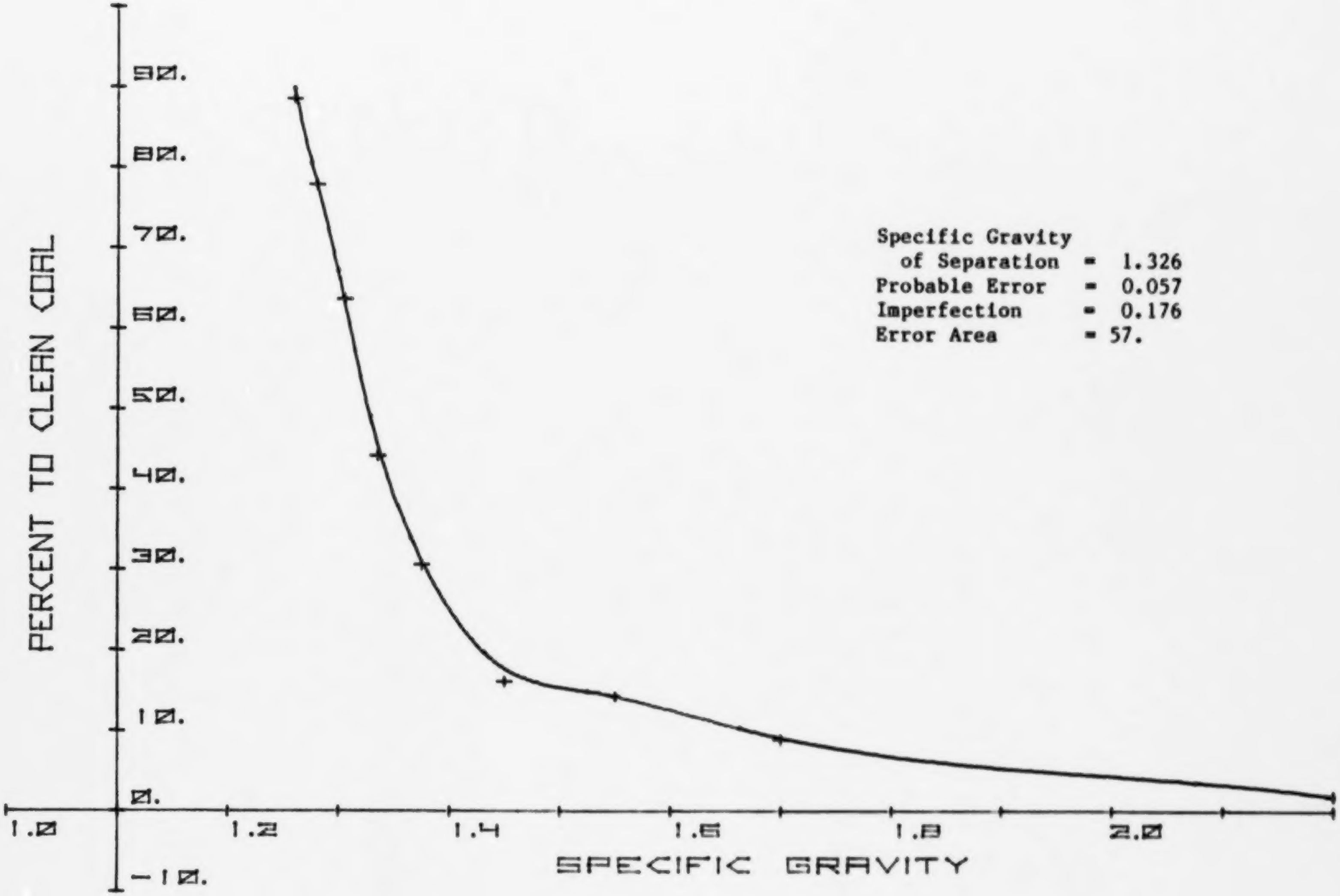
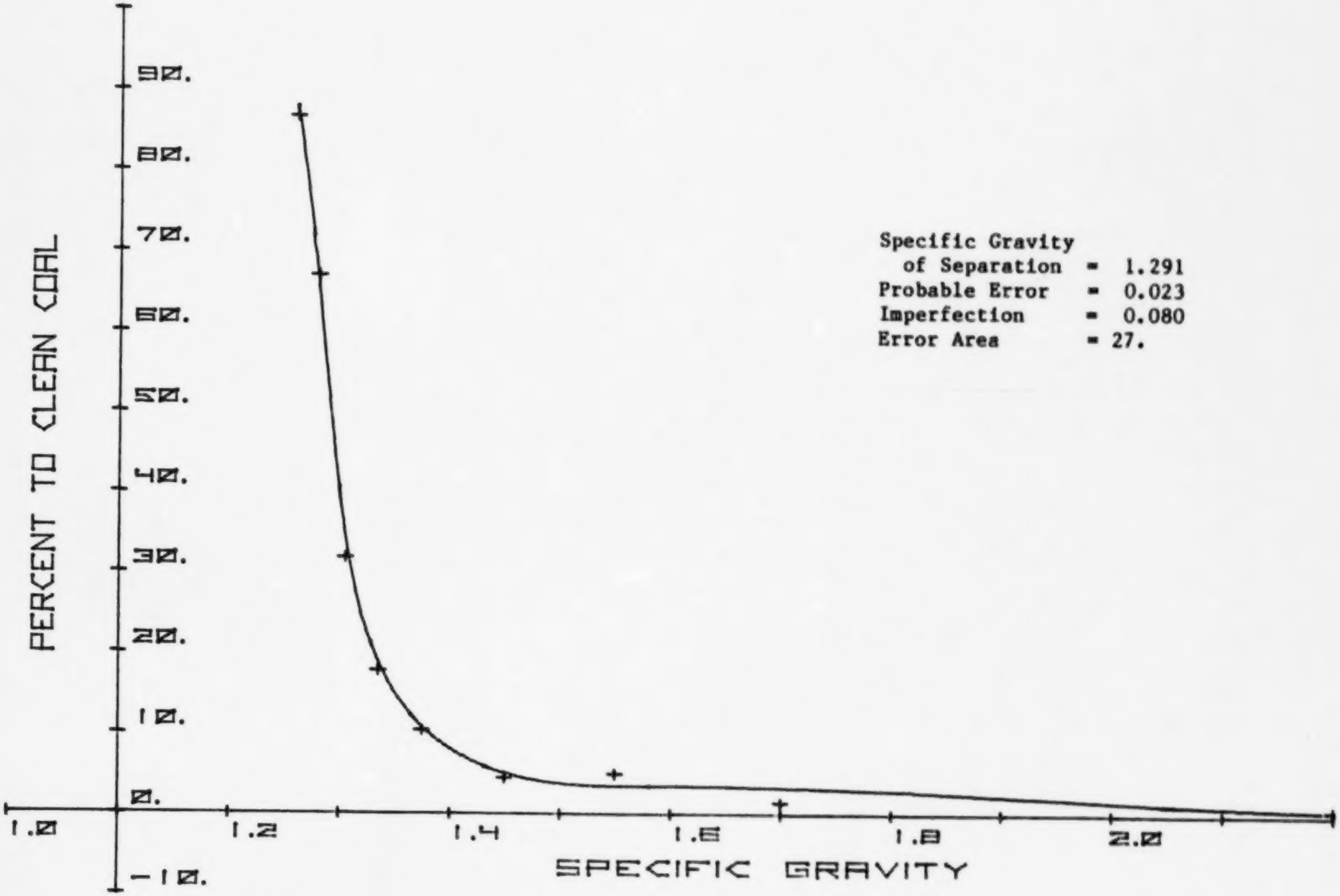


Figure A-67. Distribution Curve, Test No. 87 (28 x 100)

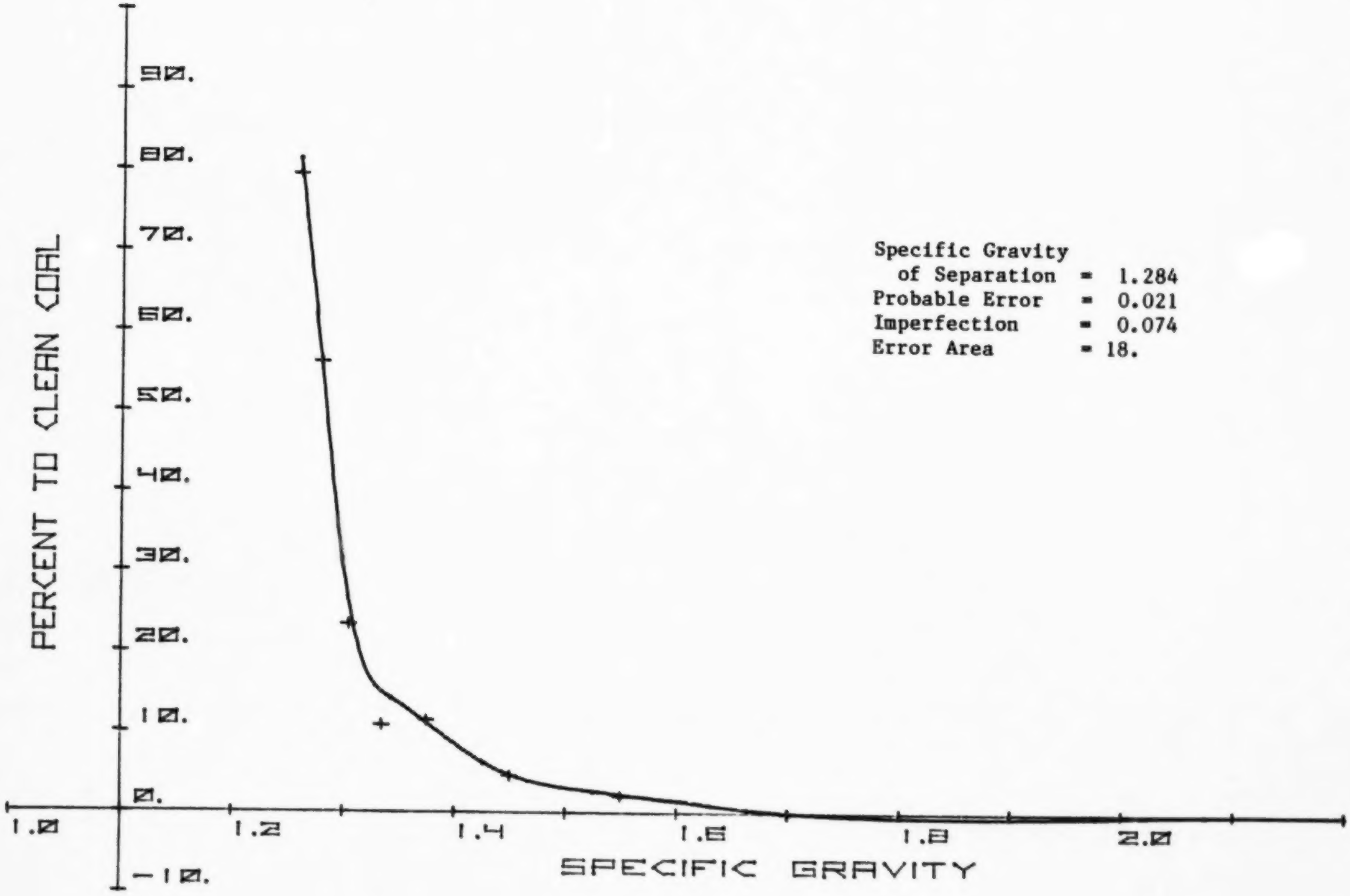


Specific Gravity
  of Separation = 1.291
Probable Error = 0.023
Imperfection = 0.080
Error Area = 27.

Figure A-68. Distribution Curve, Test No. 87 (9 x 100)

TABLE A-18. TEST 88 DATA SUMMARY

	9x14	14x28	28x100	9x100
SCREEN ANALYSIS, PERCENT:				
FEED............................	33.0	29.3	32.3	94.6
CLEAN COAL......................	24.5	28.2	40.4	93.2
REFUSE..........................	38.3	29.9	27.3	95.5
ASH, PERCENT:				
FEED............................	24.1	22.6	24.1	23.6
CLEAN COAL......................	3.9	4.3	5.8	4.8
REFUSE..........................	32.1	33.2	40.8	34.9
TOTAL SULFUR, PERCENT:				
FEED............................	2.74	3.42	4.57	3.58
CLEAN COAL......................	0.95	0.99	1.07	1.01
REFUSE..........................	3.45	4.83	7.78	5.12
WEIGHT RECOVERY (YIELD)............	28.3	36.8	47.8	37.6
THEORETICAL WEIGHT RECOVERY........	41.5	54.1	66.9	57.0
WEIGHT RECOVERY EFFICIENCY.........	68.4	68.1	71.5	66.0
ASH ERROR..........................	1.0	1.5	2.6	2.0
FLOAT IN REFUSE...% OF PRODUCT.....	11.8	18.2	25.8	17.7
SINK IN CLEAN COAL.................	31.3	21.2	13.6	23.5
TOTAL MISPLACED MATERIAL..% OF FEED	17.4	19.3	20.0	19.8
NEAR GRAVITY MATERIAL..............	60.7	64.7	63.8	62.7
SPECIFIC GRAVITY OF SEPARATION.....	1.284	1.298	1.350	1.296
PROBABLE ERROR.....................	0.021	0.032	0.104	0.043
IMPERFECTION.......................	0.074	0.107	0.296	0.146
ERROR AREA.........................	18.	34.	74.	44.
DISTRIBUTION, PERCENT TO CLEANED COAL (SPECIFIC GRAVITY FRACTION):				
FLOAT 1.27	79.5	79.1	82.1	80.5
1.27-1.29	56.1	63.9	73.0	63.8
1.29-1.32	23.4	44.3	71.3	41.9
1.32-1.35	10.9	19.6	47.3	22.9
1.35-1.40	11.5	14.8	42.8	22.4
1.40-1.50	4.6	14.2	29.5	14.8
1.50-1.60	2.1	4.6	16.9	6.8
1.60-1.80	0.0	2.6	10.3	5.1
SINK 1.80	0.0	1.0	4.0	1.8



Specific Gravity
of Separation = 1.284
Probable Error = 0.021
Imperfection = 0.074
Error Area = 18.

Figure A-69. Distribution Curve, Test No. 88 (9 x 14)

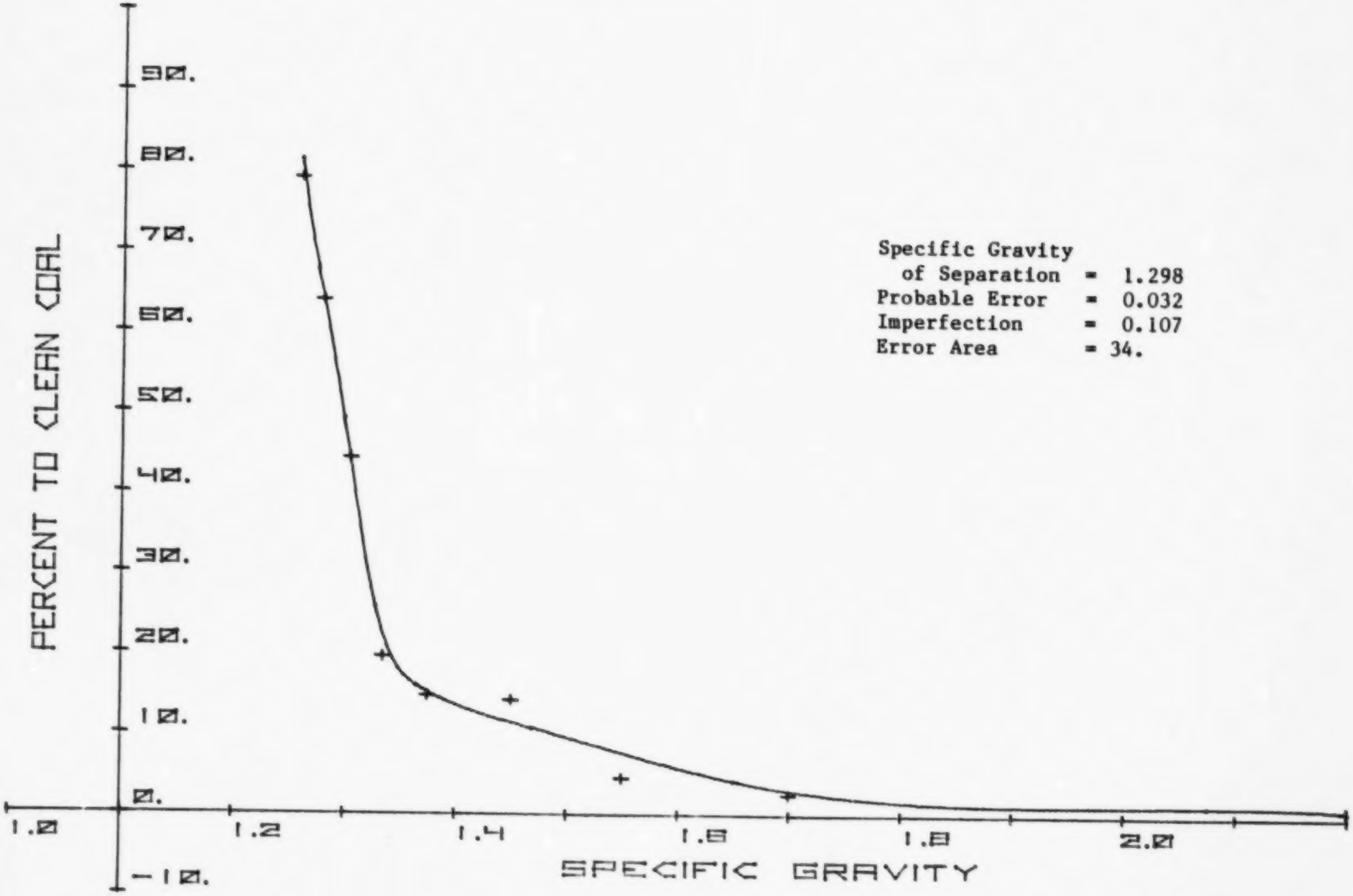


Figure A-70. Distribution Curve, Test No. 88 (14 x 28)

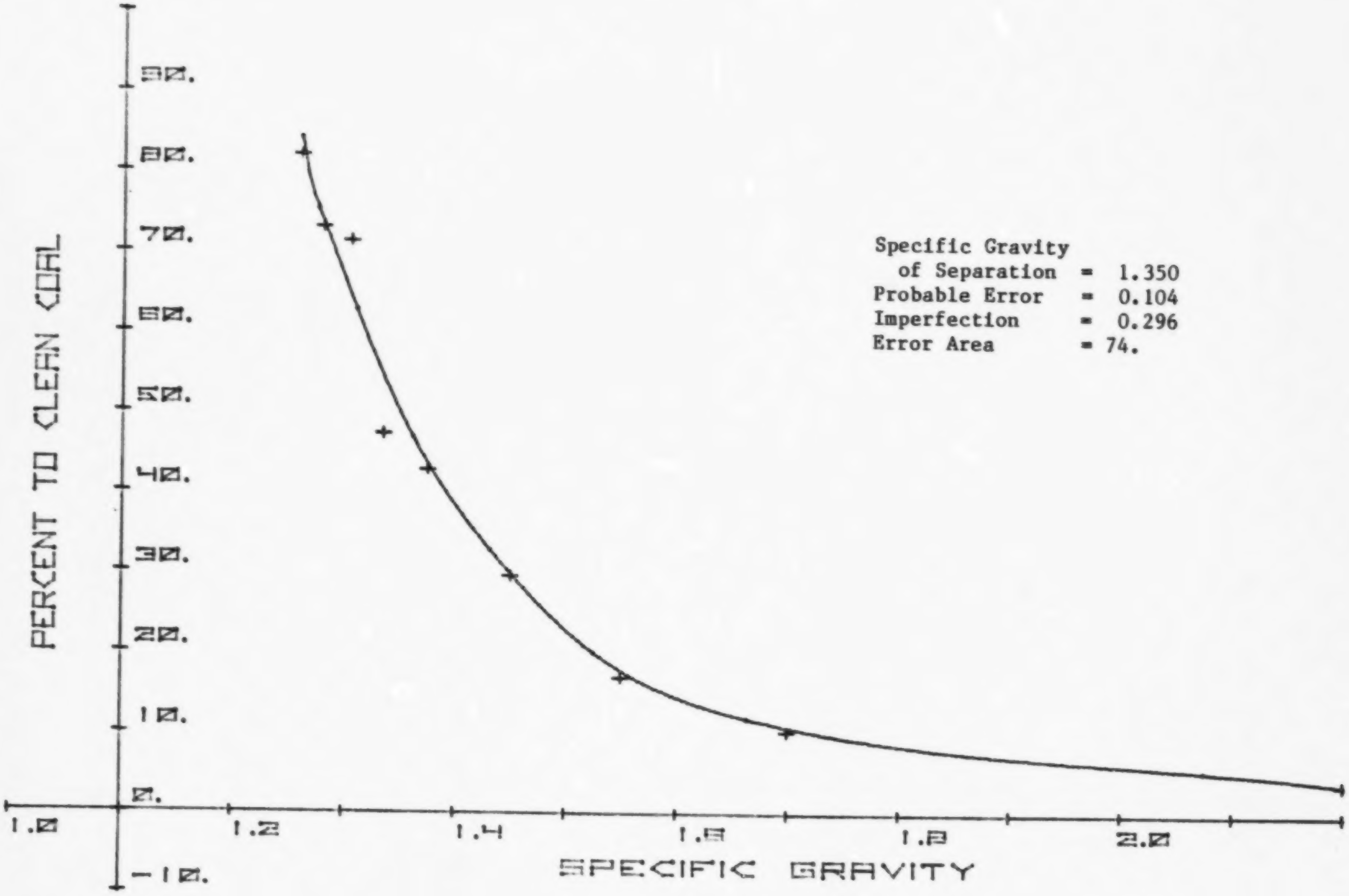
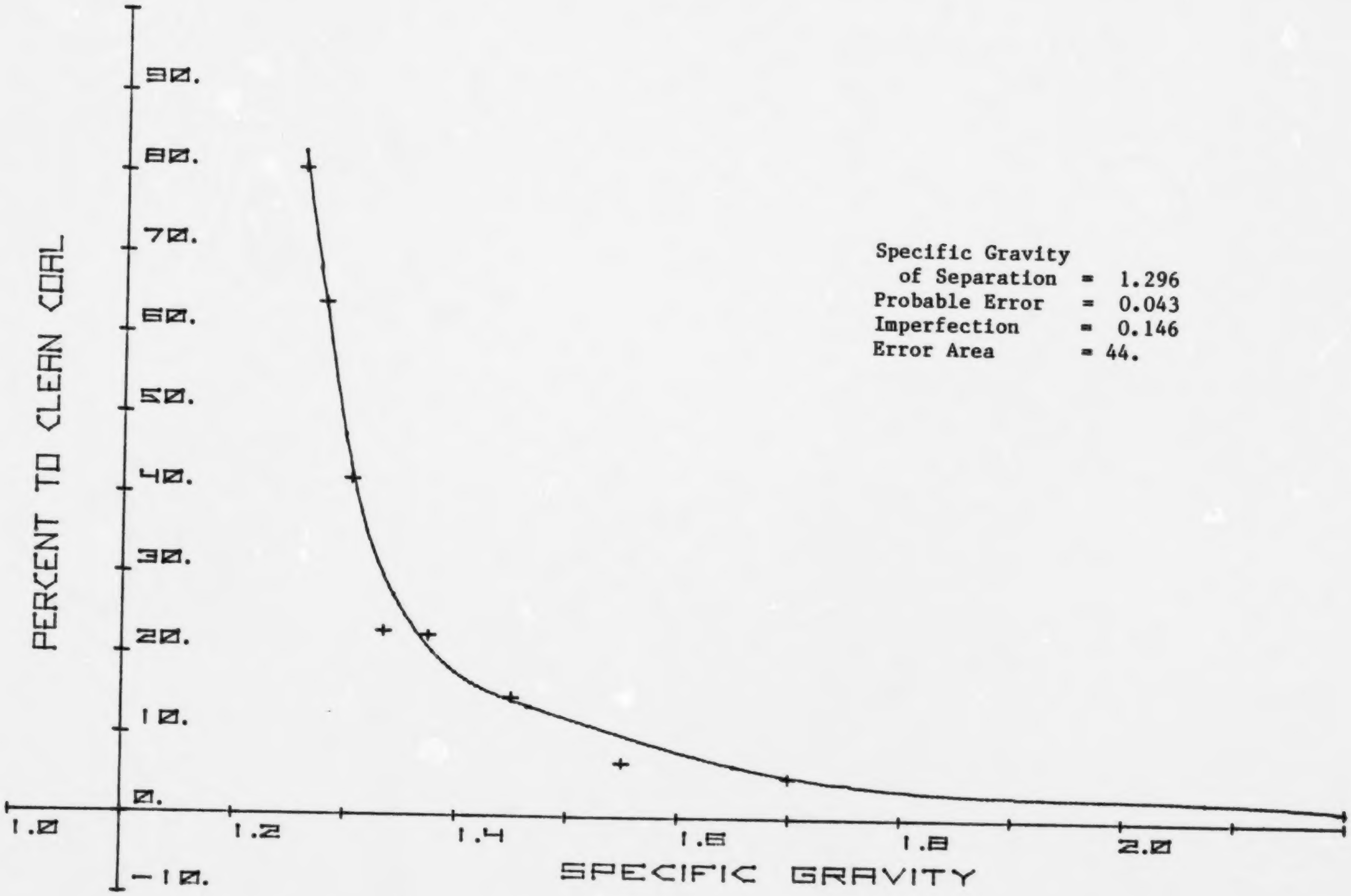


Figure A-71. Distribution Curve, Test No. 88 (28 x 100)

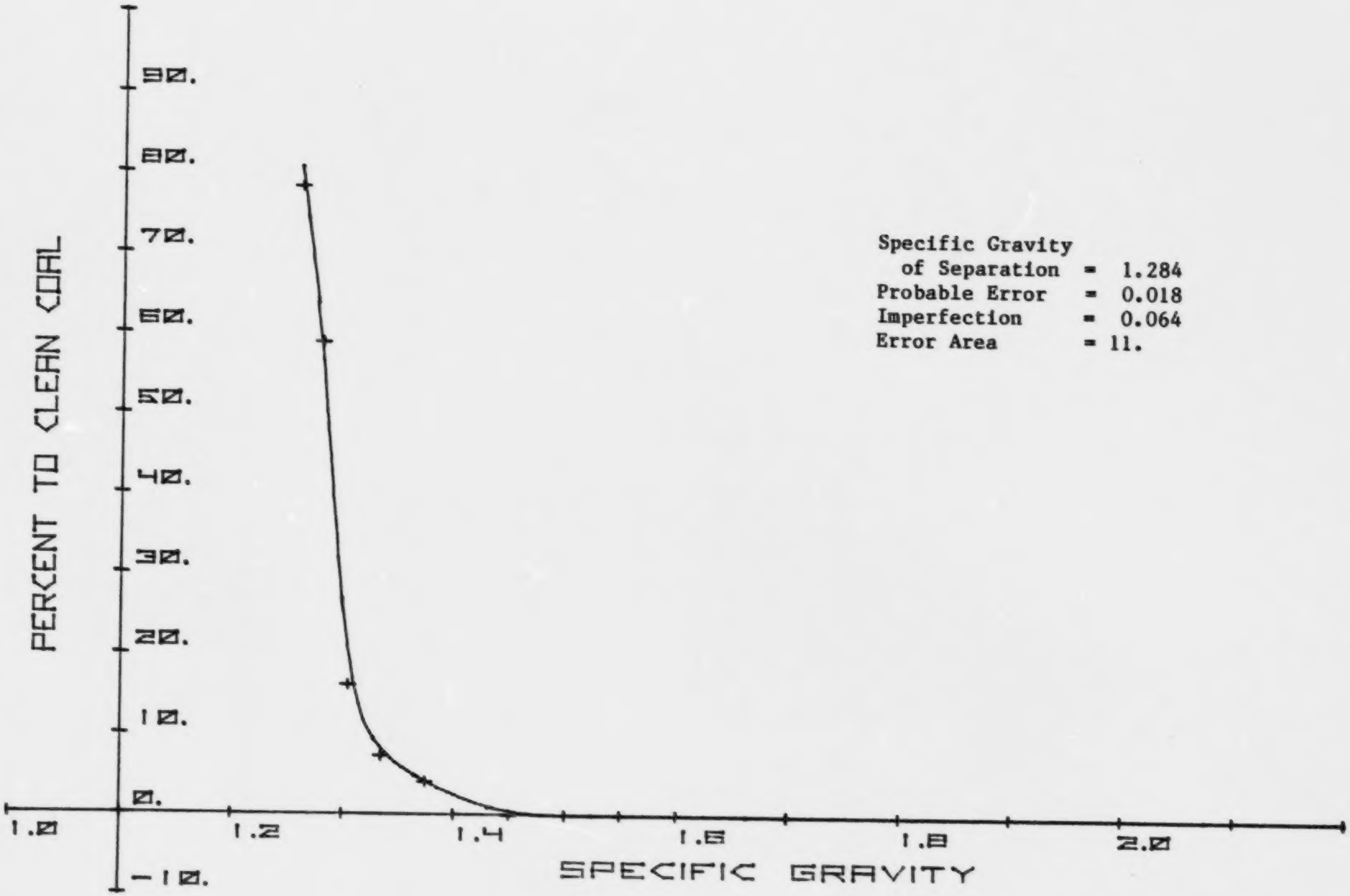


Specific Gravity
  of Separation = 1.296
Probable Error = 0.043
Imperfection = 0.146
Error Area = 44.

Figure A-72. Distribution Curve, Test No. 88 (9 x 100)

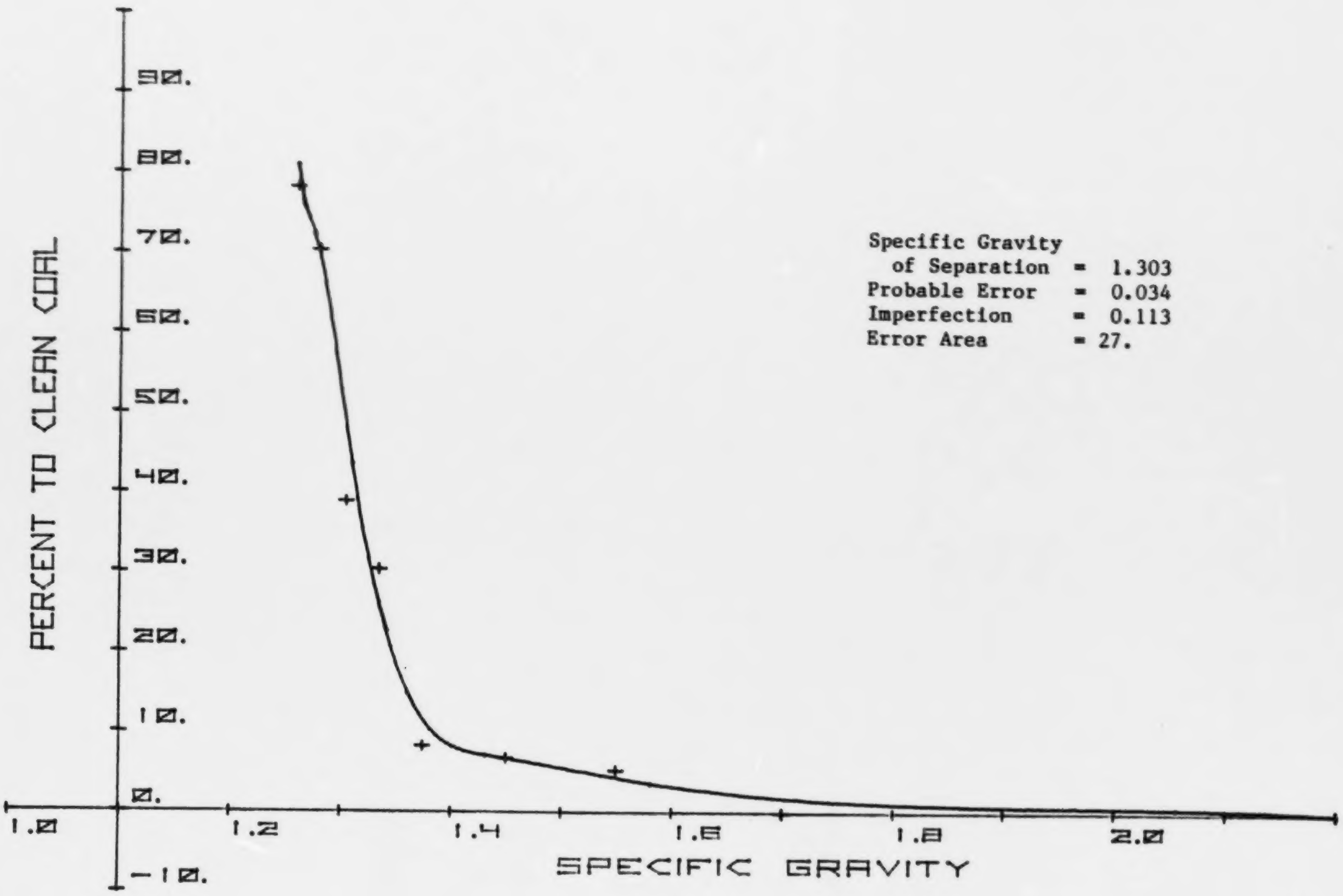
TABLE A-19. TEST 89 DATA SUMMARY

	9x14	14x28	28x100	9x100
SCREEN ANALYSIS, PERCENT:				
FEED	29.0	29.5	35.5	93.9
CLEAN COAL	17.2	27.4	46.9	91.5
REFUSE	37.3	31.0	27.4	95.7
ASH, PERCENT:				
FEED	24.0	21.1	20.0	21.5
CLEAN COAL	3.8	4.0	5.3	4.6
REFUSE	30.5	31.7	37.7	33.0
TOTAL SULFUR, PERCENT:				
FEED	2.37	2.52	3.12	2.70
CLEAN COAL	0.87	0.85	0.98	0.92
REFUSE	2.86	3.57	5.71	3.91
WEIGHT RECOVERY (YIELD)	24.6	38.4	54.7	40.3
THEORETICAL WEIGHT RECOVERY	38.0	52.6	71.0	58.6
WEIGHT RECOVERY EFFICIENCY	64.7	73.0	77.0	68.7
ASH ERROR	1.1	1.3	2.1	1.8
FLOAT IN REFUSE...% OF PRODUCT	10.9	18.6	26.8	16.7
SINK IN CLEAN COAL	21.1	17.4	11.1	19.3
TOTAL MISPLACED MATERIAL..% OF FEED	13.4	18.1	18.2	17.8
NEAR GRAVITY MATERIAL	60.3	67.7	69.4	65.9
SPECIFIC GRAVITY OF SEPARATION	1.284	1.303	1.362	1.302
PROBABLE ERROR	0.018	0.034	0.108	0.038
IMPERFECTION	0.064	0.113	0.300	0.125
ERROR AREA	11.	27.	74.	36.
DISTRIBUTION, PERCENT TO CLEANED COAL (SPECIFIC GRAVITY FRACTION):				
FLOAT 1.27	78.1	78.2	90.0	83.9
1.27-1.29	58.8	70.3	74.7	68.2
1.29-1.32	16.1	38.9	62.9	36.4
1.32-1.35	7.3	30.4	59.4	34.0
1.35-1.40	4.1	8.3	42.9	16.0
1.40-1.50	0.0	6.8	31.5	14.2
1.50-1.60	0.0	5.3	18.4	8.5
1.60-1.80	0.0	0.0	9.7	1.9
SINK 1.80	0.0	0.0	3.3	1.4



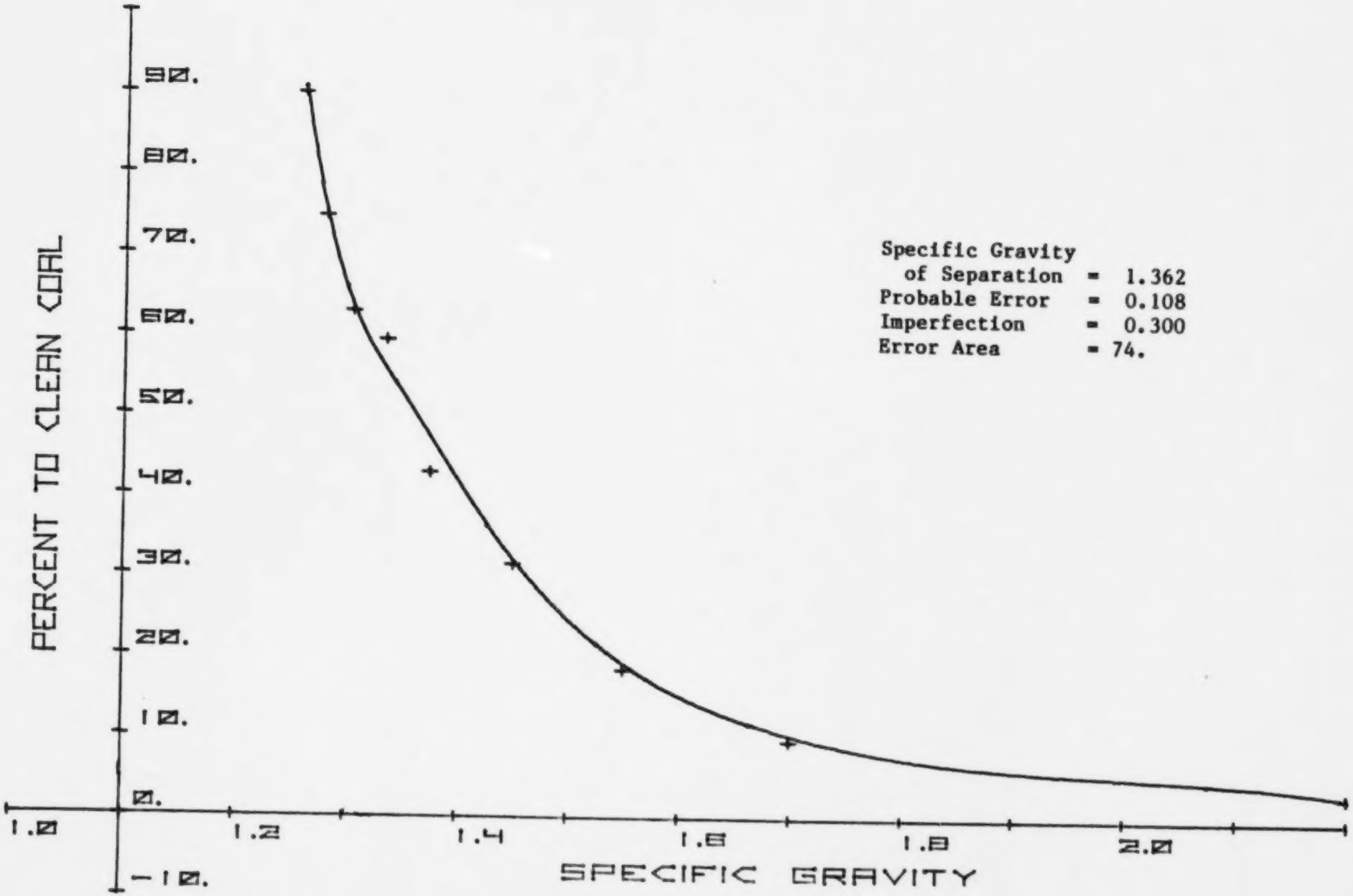
Specific Gravity
  of Separation = 1.284
Probable Error = 0.018
Imperfection = 0.064
Error Area = 11.

Figure A-73. Distribution Curve, Test No. 89 (9 x 14)



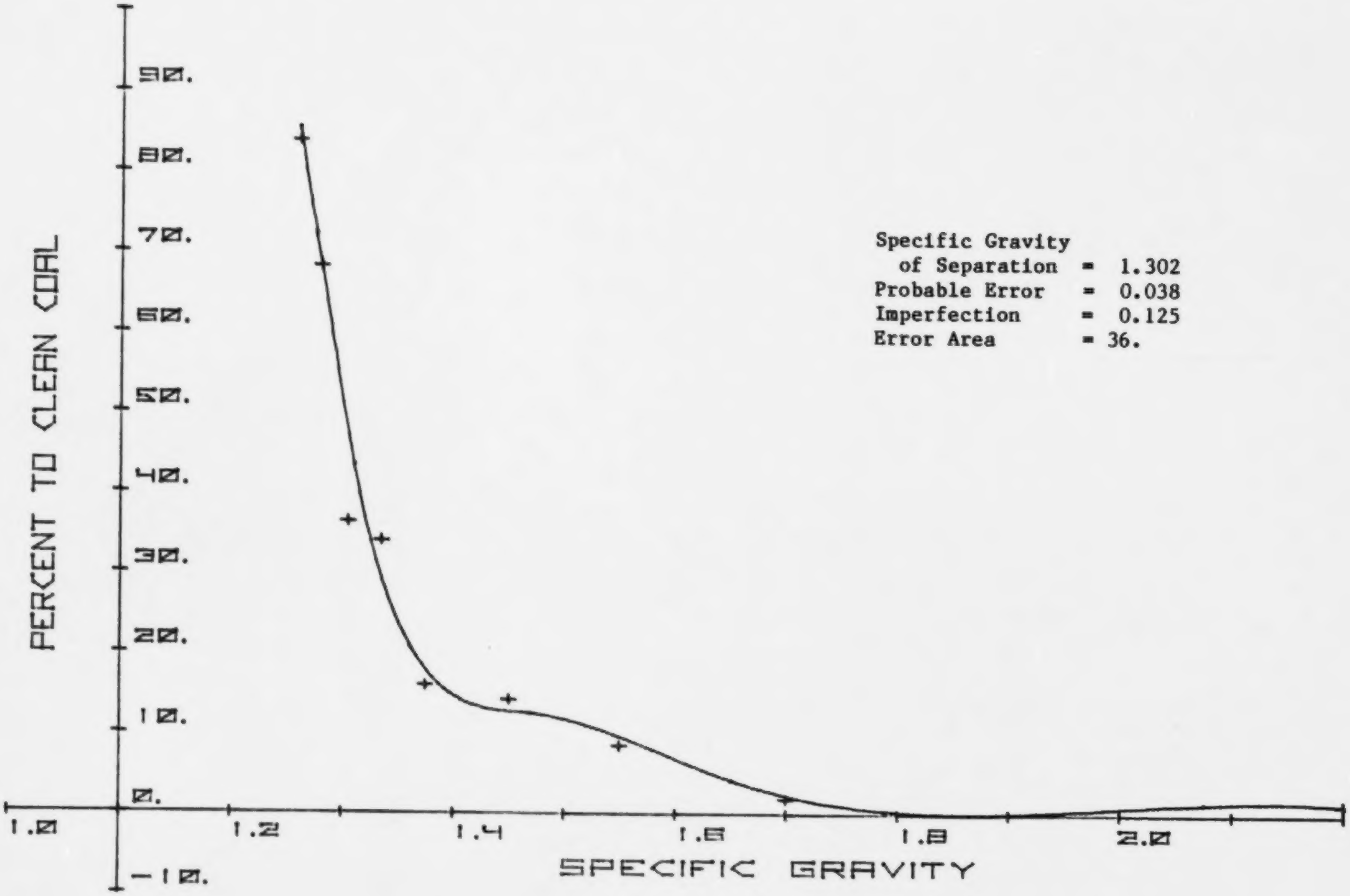
Specific Gravity
of Separation = 1.303
Probable Error = 0.034
Imperfection = 0.113
Error Area = 27.

Figure A-74. Distribution Curve, Test No. 89 (14 x 28)



Specific Gravity
of Separation = 1.362
Probable Error = 0.108
Imperfection = 0.300
Error Area = 74.

Figure A-75. Distribution Curve, Test No. 89 (28 x 100)



Specific Gravity
  of Separation = 1.302
Probable Error = 0.038
Imperfection = 0.125
Error Area = 36.

Figure A-76. Distribution Curve, Test No. 89 (9 x 100)

TABLE A-20. TEST 90 DATA SUMMARY

	9x14	14x28	28x100	9x100
SCREEN ANALYSIS, PERCENT:				
FEED	27.8	28.7	37.2	93.7
CLEAN COAL	19.5	27.3	45.7	92.5
REFUSE	33.7	29.8	31.1	94.6
ASH, PERCENT:				
FEED	23.9	20.9	18.3	20.7
CLEAN COAL	3.6	3.8	4.7	4.2
REFUSE	32.5	32.2	32.7	32.5
TOTAL SULFUR, PERCENT:				
FEED	2.30	2.62	2.69	2.55
CLEAN COAL	1.02	0.99	0.94	0.97
REFUSE	2.84	3.70	4.56	3.68
WEIGHT RECOVERY (YIELD)	29.6	39.9	51.6	41.5
THEORETICAL WEIGHT RECOVERY	37.4	47.4	69.9	55.0
WEIGHT RECOVERY EFFICIENCY	79.0	84.2	73.8	75.5
ASH ERROR	0.6	0.9	2.0	1.4
FLOAT IN REFUSE...% OF PRODUCT	11.5	13.5	27.2	15.5
SINK IN CLEAN COAL	11.7	17.6	13.6	19.3
TOTAL MISPLACED MATERIAL..% OF FEED	11.5	15.1	20.2	17.1
NEAR GRAVITY MATERIAL	61.5	67.7	70.6	66.6
SPECIFIC GRAVITY OF SEPARATION	1.292	1.294	1.329	1.300
PROBABLE ERROR	0.021	0.023	0.099	0.038
IMPERFECTION	0.073	0.079	0.300	0.126
ERROR AREA	25.	25.	64.	35.
DISTRIBUTION, PERCENT TO CLEANED COAL (SPECIFIC GRAVITY FRACTION):				
FLOAT 1.27	77.5	86.4	85.8	84.0
1.27-1.29	82.1	68.1	70.3	72.5
1.29-1.32	28.8	36.8	61.5	43.8
1.32-1.35	3.2	14.1	39.7	15.6
1.35-1.40	5.1	12.2	38.1	20.5
1.40-1.50	6.6	15.0	26.3	17.9
1.50-1.60	0.0	4.6	17.4	7.5
1.60-1.80	0.0	0.0	7.0	1.5
SINK 1.80	0.0	0.0	2.9	1.3

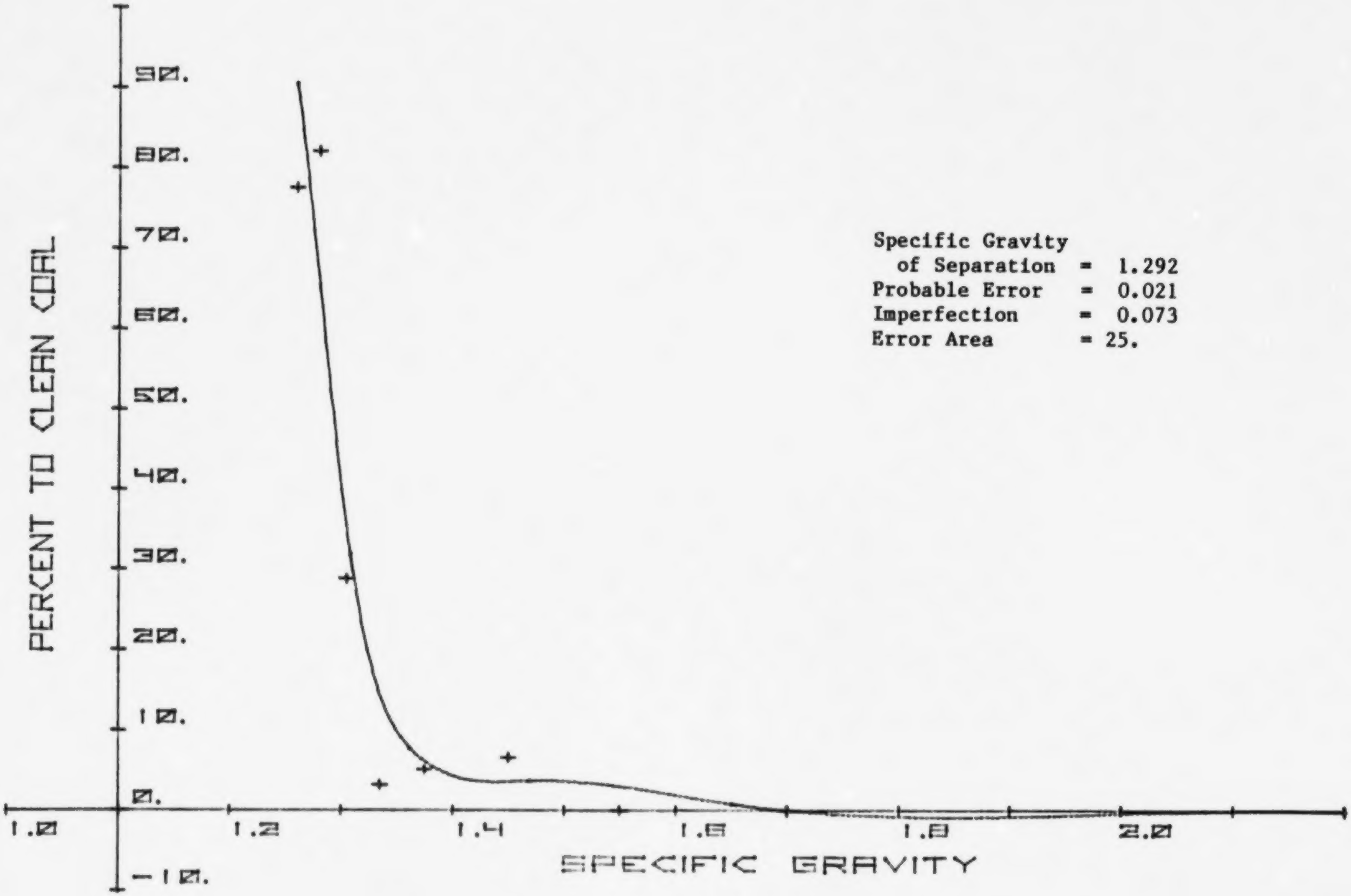
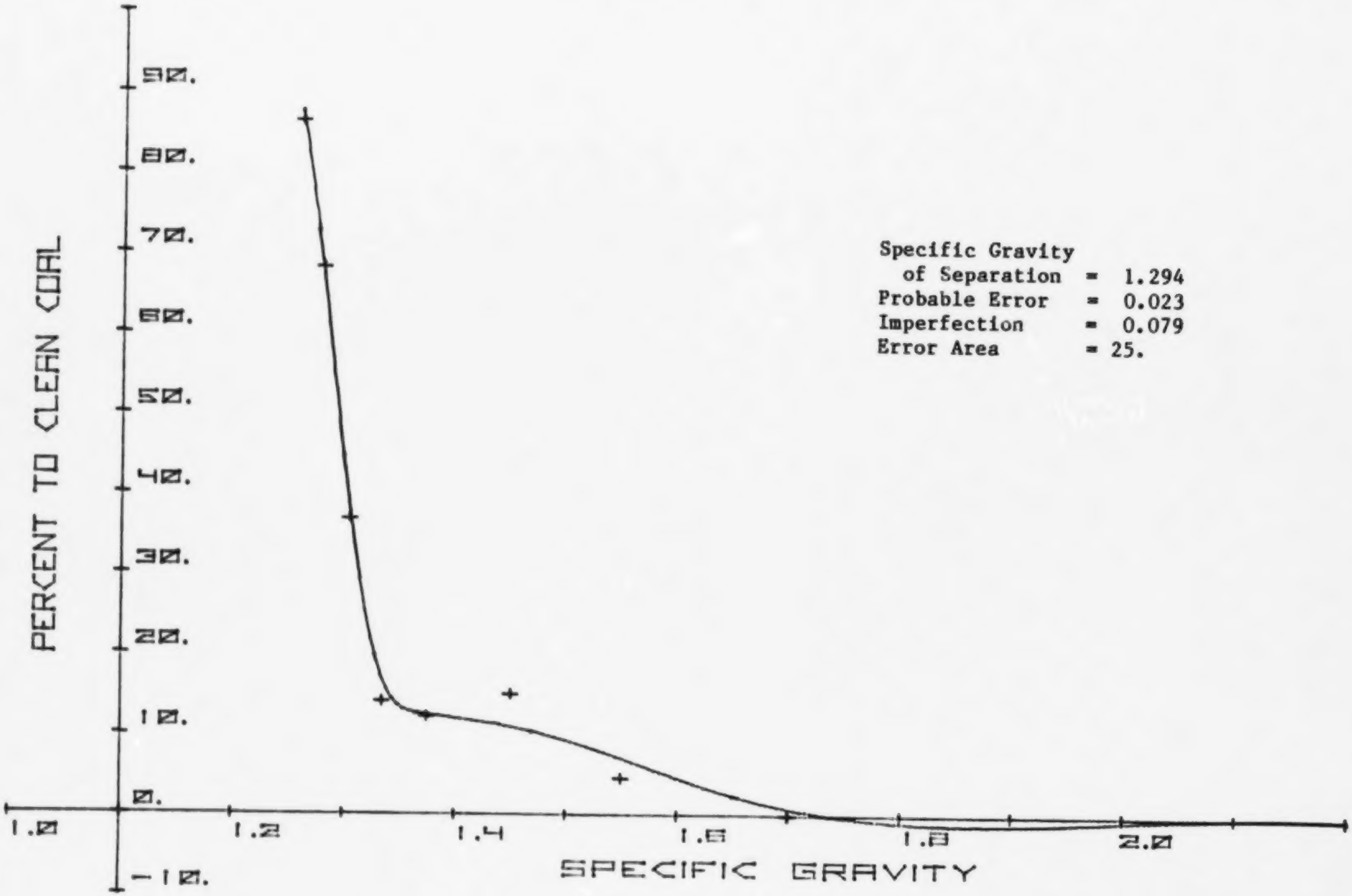
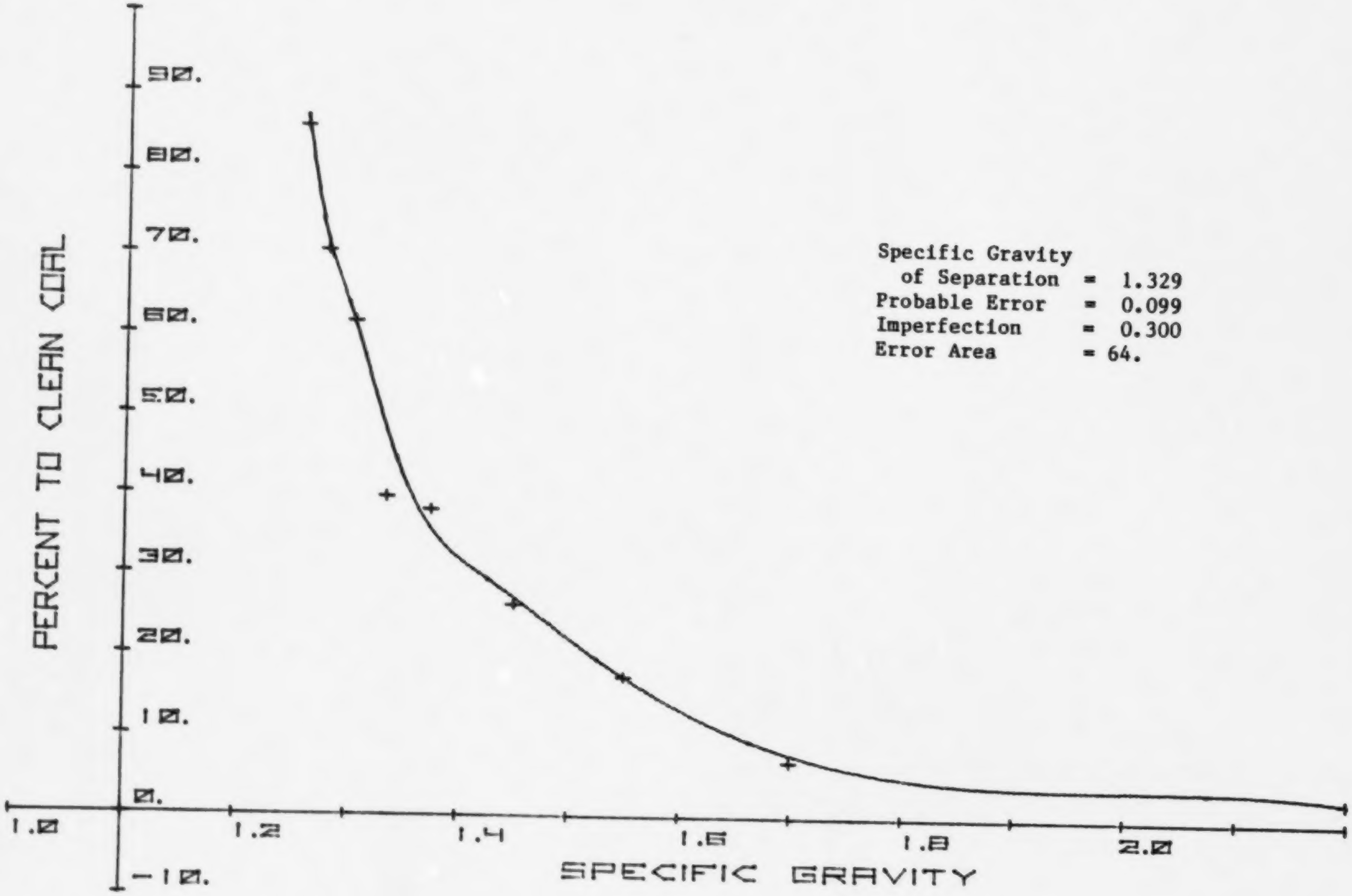


Figure A-77. Distribution Curve, Test No. 90 (9 x 14)



Specific Gravity
  of Separation = 1.294
Probable Error = 0.023
Imperfection = 0.079
Error Area = 25.

Figure A-78. Distribution Curve, Test No. 90 (14 x 28)



Specific Gravity
  of Separation = 1.329
Probable Error = 0.099
Imperfection = 0.300
Error Area = 64.

Figure A-79. Distribution Curve, Test No. 90 (28 x 100)

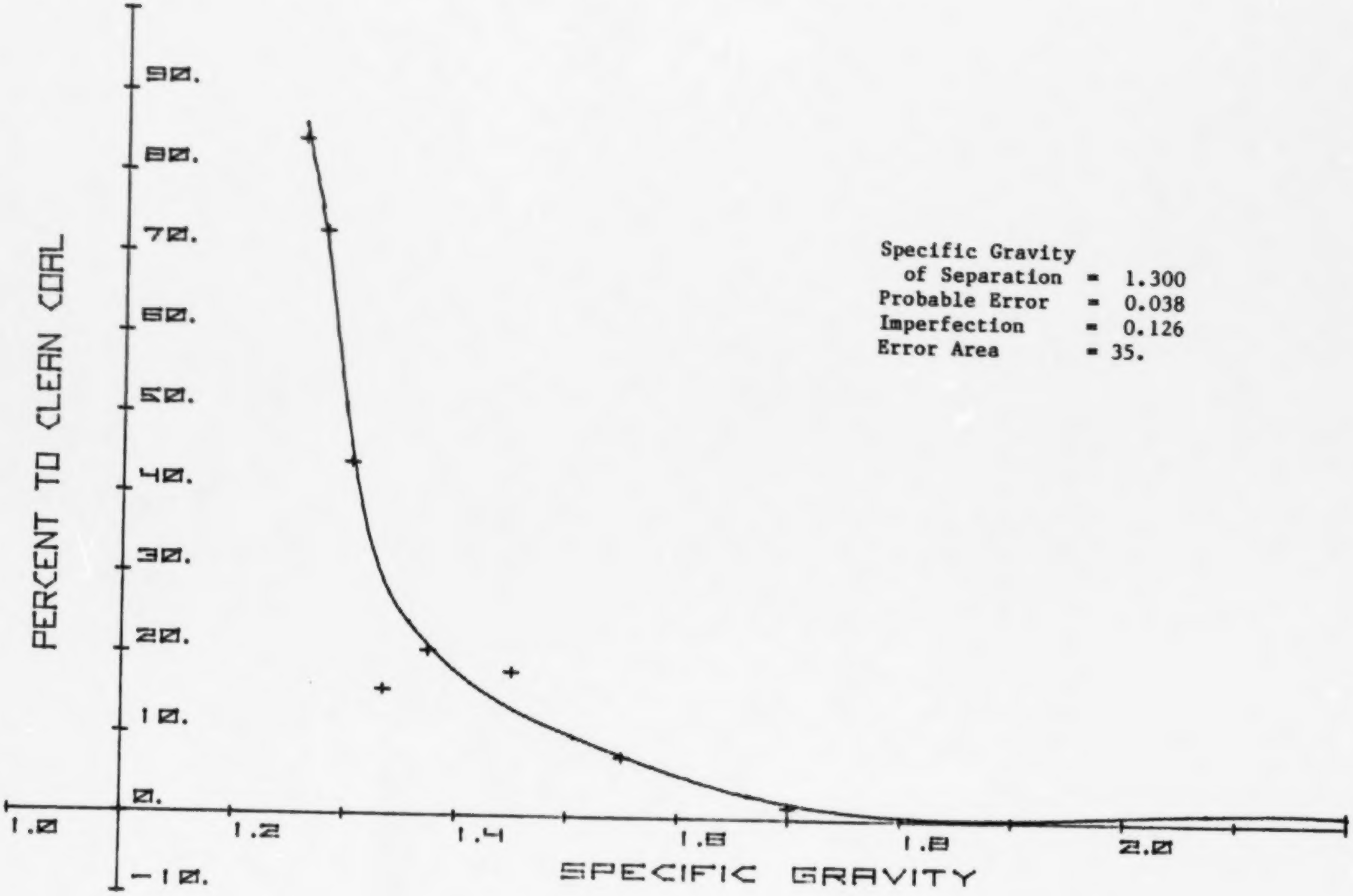
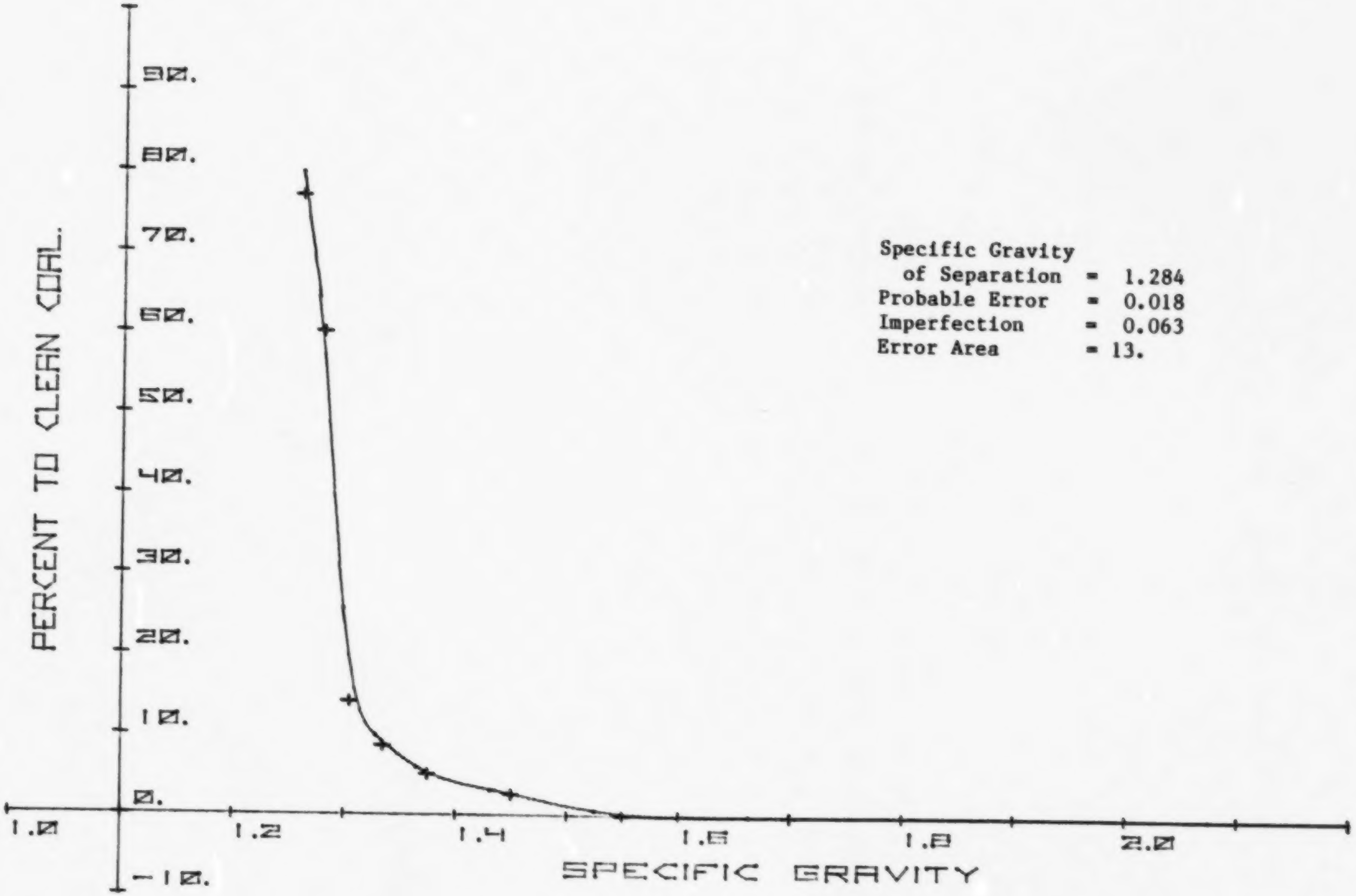


Figure A-80. Distribution Curve, Test No. 90 (9 x 100)

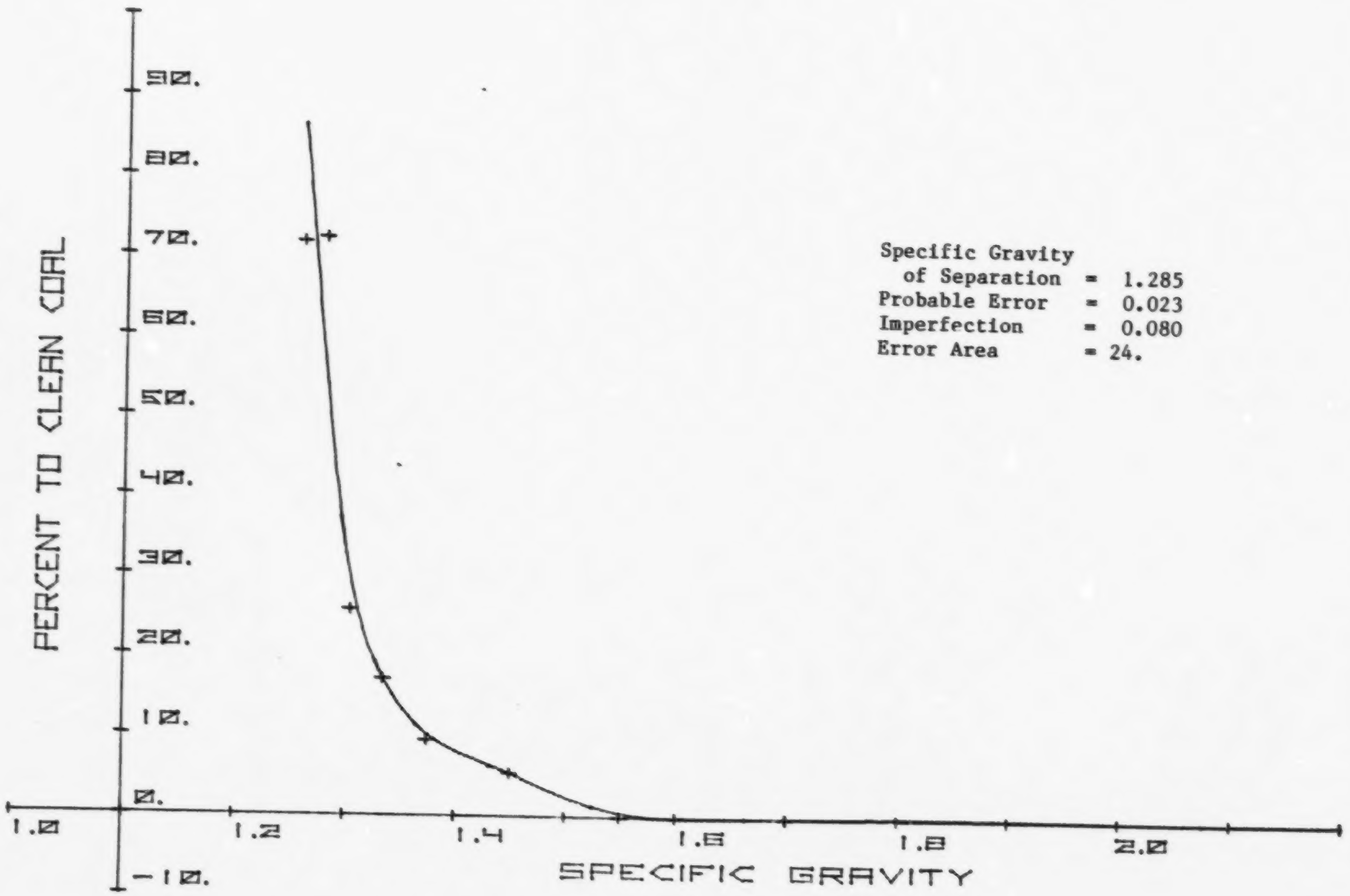
TABLE A-21. TEST 91 DATA SUMMARY

	9x14	14x28	28x100	9x100
SCREEN ANALYSIS, PERCENT:				
FEED	26.6	28.4	37.9	92.9
CLEAN COAL	16.6	24.8	48.8	90.2
REFUSE	32.9	30.7	30.9	94.6
ASH, PERCENT:				
FEED	20.4	18.9	18.4	19.1
CLEAN COAL	3.5	3.1	4.2	3.8
REFUSE	25.9	27.1	32.6	28.5
TOTAL SULFUR, PERCENT:				
FEED	2.63	2.78	3.19	2.90
CLEAN COAL	1.13	0.93	0.96	0.99
REFUSE	3.11	3.73	5.45	4.08
WEIGHT RECOVERY (YIELD)	24.4	34.0	50.3	37.9
THEORETICAL WEIGHT RECOVERY	40.8	50.3	68.0	57.9
WEIGHT RECOVERY EFFICIENCY	59.8	67.7	74.0	65.5
ASH ERROR	1.2	0.9	1.8	1.6
FLOAT IN REFUSE...% OF PRODUCT	10.6	17.7	29.3	17.3
SINK IN CLEAN COAL	26.2	15.8	14.1	19.2
TOTAL MISPLACED MATERIAL..% OF FEED	14.4	17.1	21.6	18.0
NEAR GRAVITY MATERIAL	63.9	69.1	72.0	68.4
SPECIFIC GRAVITY OF SEPARATION	1.284	1.285	1.319	1.288
PROBABLE ERROR	0.018	0.023	0.074	0.030
IMPERFECTION	0.063	0.080	0.233	0.105
ERROR AREA	13.	24.	64.	40.
DISTRIBUTION, PERCENT TO CLEANED COAL (SPECIFIC GRAVITY FRACTION):				
FLOAT 1.27	77.0	71.7	78.3	76.1
1.27-1.29	60.1	72.2	84.9	73.7
1.29-1.32	14.1	25.7	52.2	29.8
1.32-1.35	8.6	17.1	46.1	24.1
1.35-1.40	5.1	9.5	32.9	16.1
1.40-1.50	2.6	5.4	21.4	10.1
1.50-1.60	0.0	0.0	13.1	5.3
1.60-1.80	0.0	0.0	9.9	3.6
SINK 1.80	0.0	0.0	2.7	1.1



Specific Gravity
of Separation = 1.284
Probable Error = 0.018
Imperfection = 0.063
Error Area = 13.

Figure A-81. Distribution Curve, Test No. 91 (9 x 14)



Specific Gravity
  of Separation = 1.285
Probable Error = 0.023
Imperfection = 0.080
Error Area = 24.

Figure A-82. Distribution Curve, Test No. 91 (14 x 28)

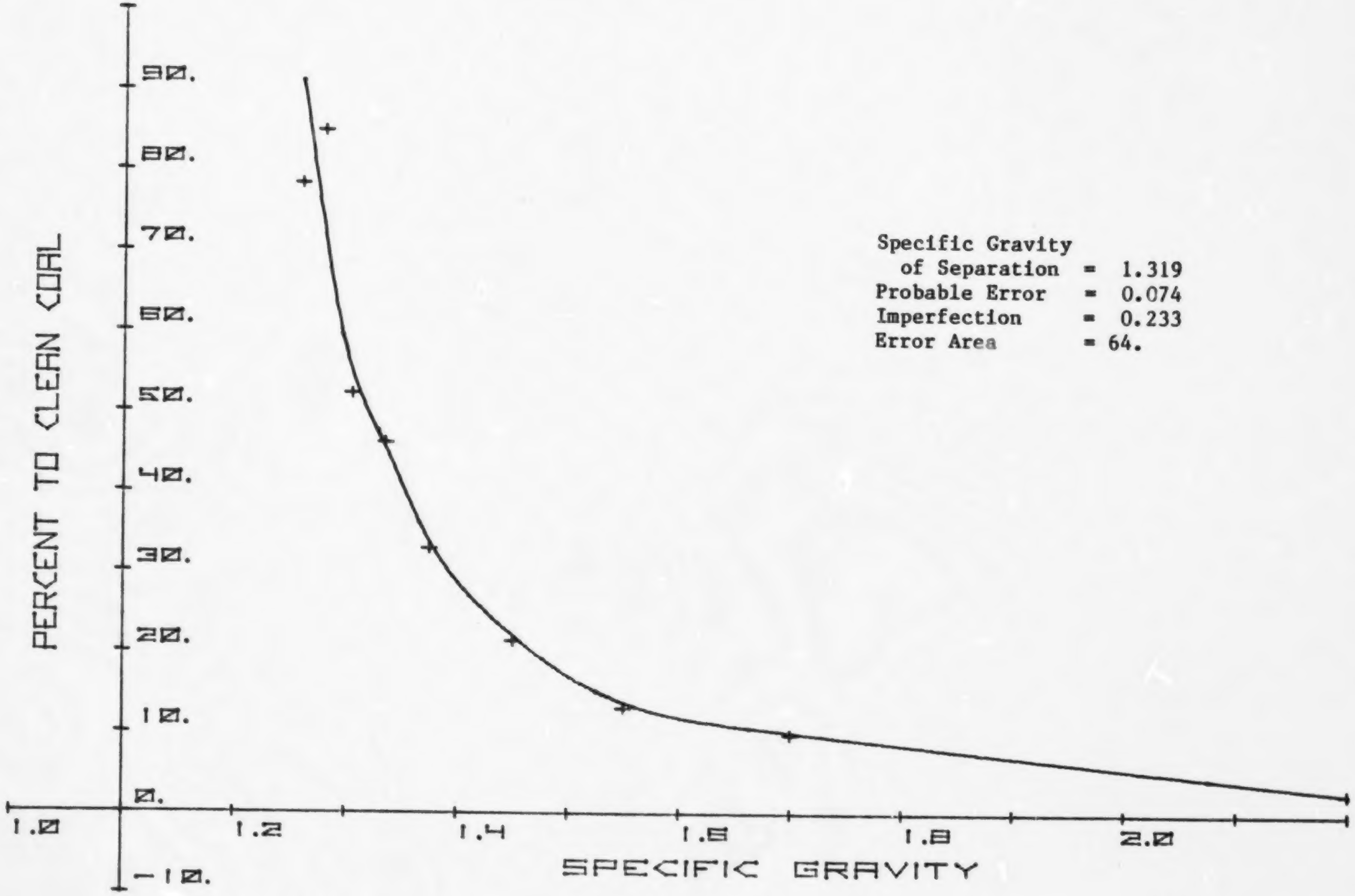
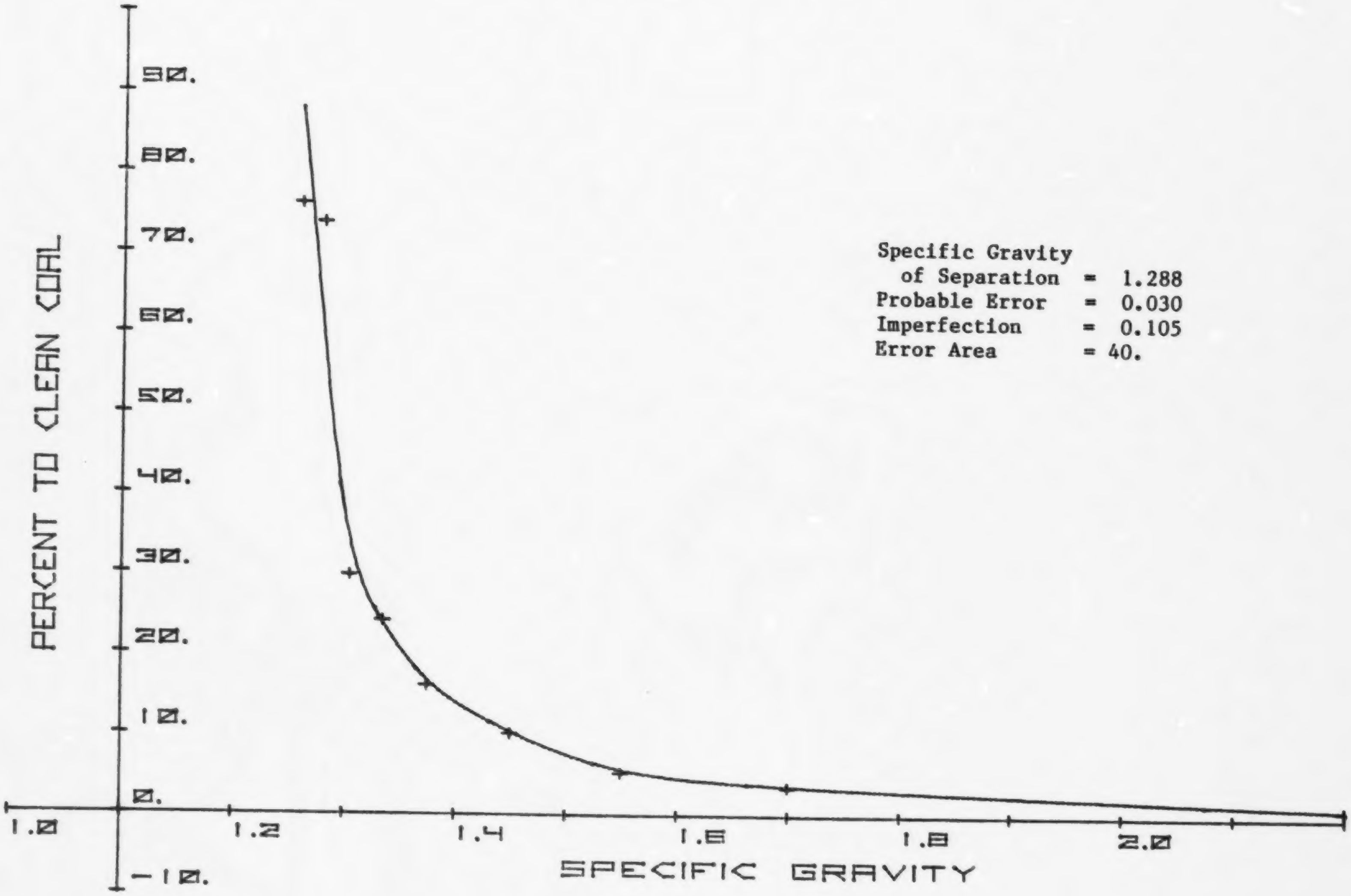


Figure A-83. Distribution Curve, Test No. 91 (28 x 100)

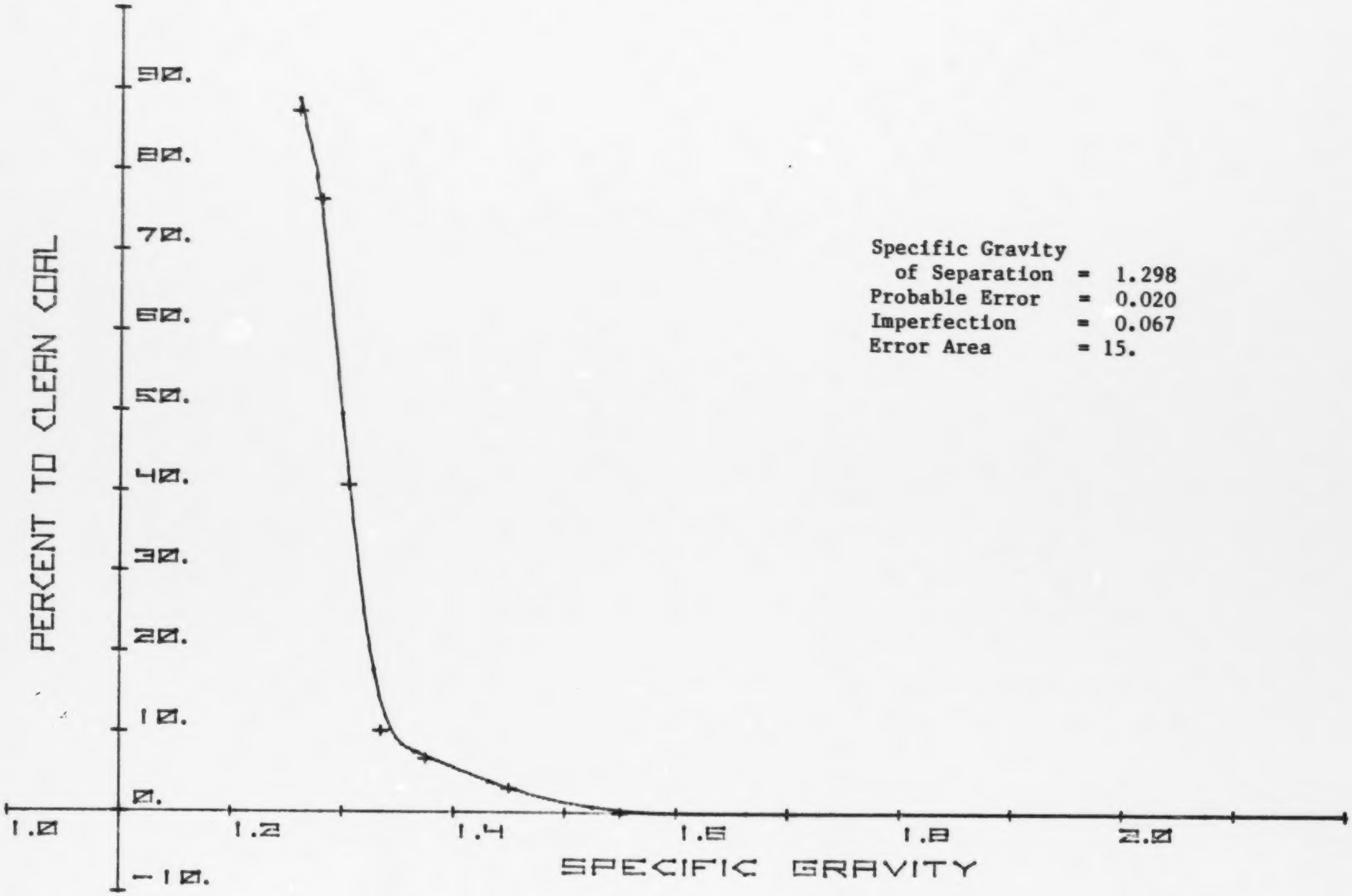


Specific Gravity of Separation = 1.288
Probable Error = 0.030
Imperfection = 0.105
Error Area = 40.

Figure A-84. Distribution Curve, Test No. 91 (9 x 100)

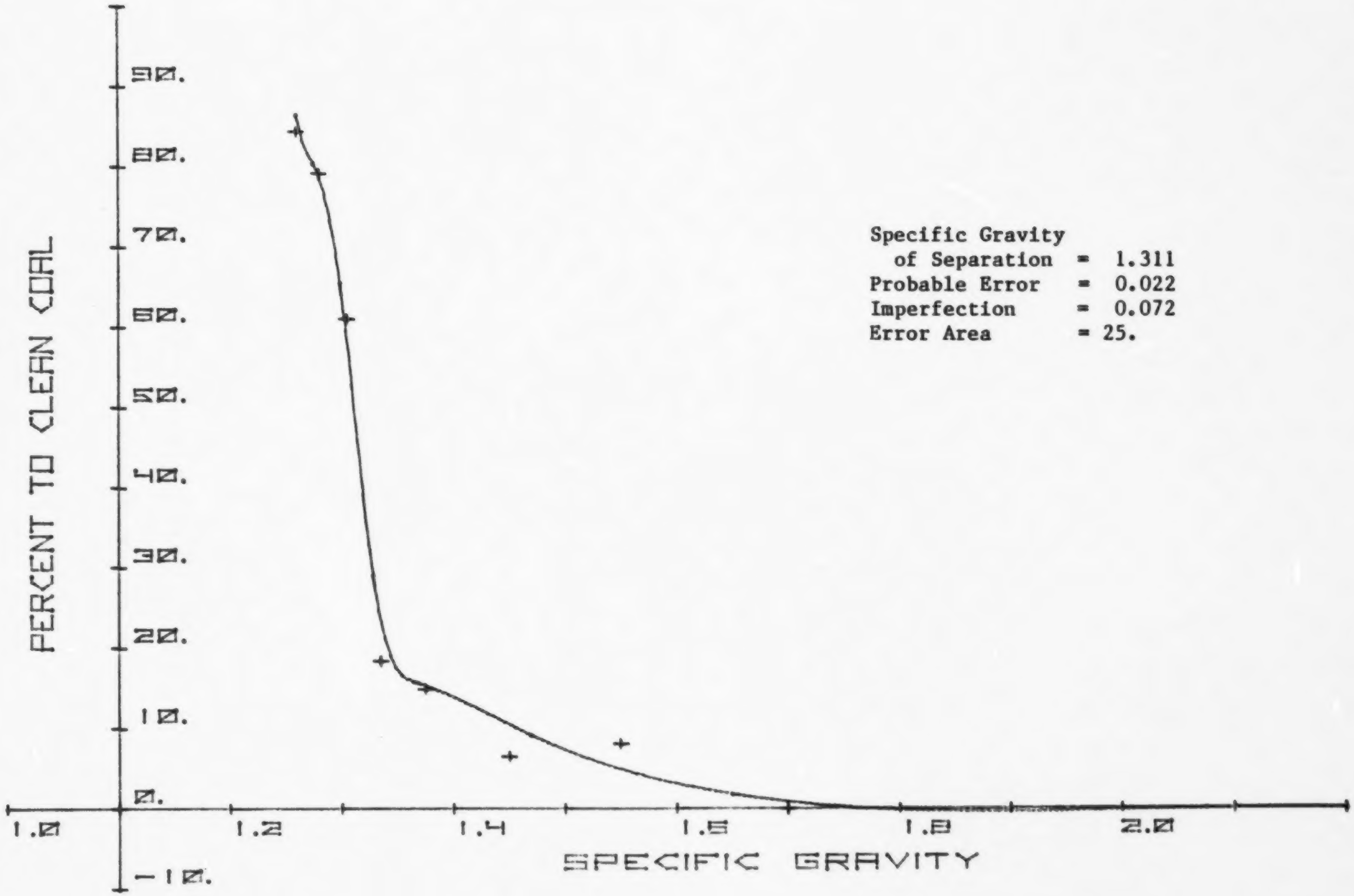
TABLE A-22. TEST 92 DATA SUMMARY

	9x14	14x28	28x100	9x100
SCREEN ANALYSIS, PERCENT:				
FEED	27.9	28.1	37.9	93.9
CLEAN COAL	20.5	28.6	44.3	93.4
REFUSE	33.2	27.7	33.3	94.2
ASH, PERCENT:				
FEED	25.0	20.3	20.5	21.8
CLEAN COAL	3.7	3.9	5.2	4.4
REFUSE	34.4	32.4	35.0	34.0
TOTAL SULFUR, PERCENT:				
FEED	3.08	2.75	3.75	3.25
CLEAN COAL	1.01	0.77	0.99	0.93
REFUSE	3.99	4.21	6.39	4.90
WEIGHT RECOVERY (YIELD)	30.7	42.5	48.7	41.5
THEORETICAL WEIGHT RECOVERY	35.2	55.8	68.5	58.3
WEIGHT RECOVERY EFFICIENCY	87.3	76.0	71.1	71.2
ASH ERROR	0.3	1.0	2.3	1.5
FLOAT IN REFUSE...% OF PRODUCT	10.5	17.2	25.8	16.6
SINK IN CLEAN COAL	14.9	12.4	14.1	17.5
TOTAL MISPLACED MATERIAL..% OF FEED	11.9	15.2	20.1	17.0
NEAR GRAVITY MATERIAL	62.6	68.6	67.8	66.4
SPECIFIC GRAVITY OF SEPARATION	1.298	1.311	1.339	1.311
PROBABLE ERROR	0.020	0.022	0.086	0.029
IMPERFECTION	0.067	0.072	0.253	0.093
ERROR AREA	15.	25.	65.	36.
DISTRIBUTION, PERCENT TO CLEANED COAL (SPECIFIC GRAVITY FRACTION):				
FLOAT 1.27	87.2	84.4	83.0	84.3
1.27-1.29	76.2	79.1	74.8	76.6
1.29-1.32	40.7	61.0	71.0	58.4
1.32-1.35	10.1	18.4	45.0	24.2
1.35-1.40	6.7	14.9	36.7	18.9
1.40-1.50	3.0	6.5	20.3	9.9
1.50-1.60	0.0	8.1	17.5	10.3
1.60-1.80	0.0	0.0	8.2	2.6
SINK 1.80	0.0	0.0	2.7	1.1



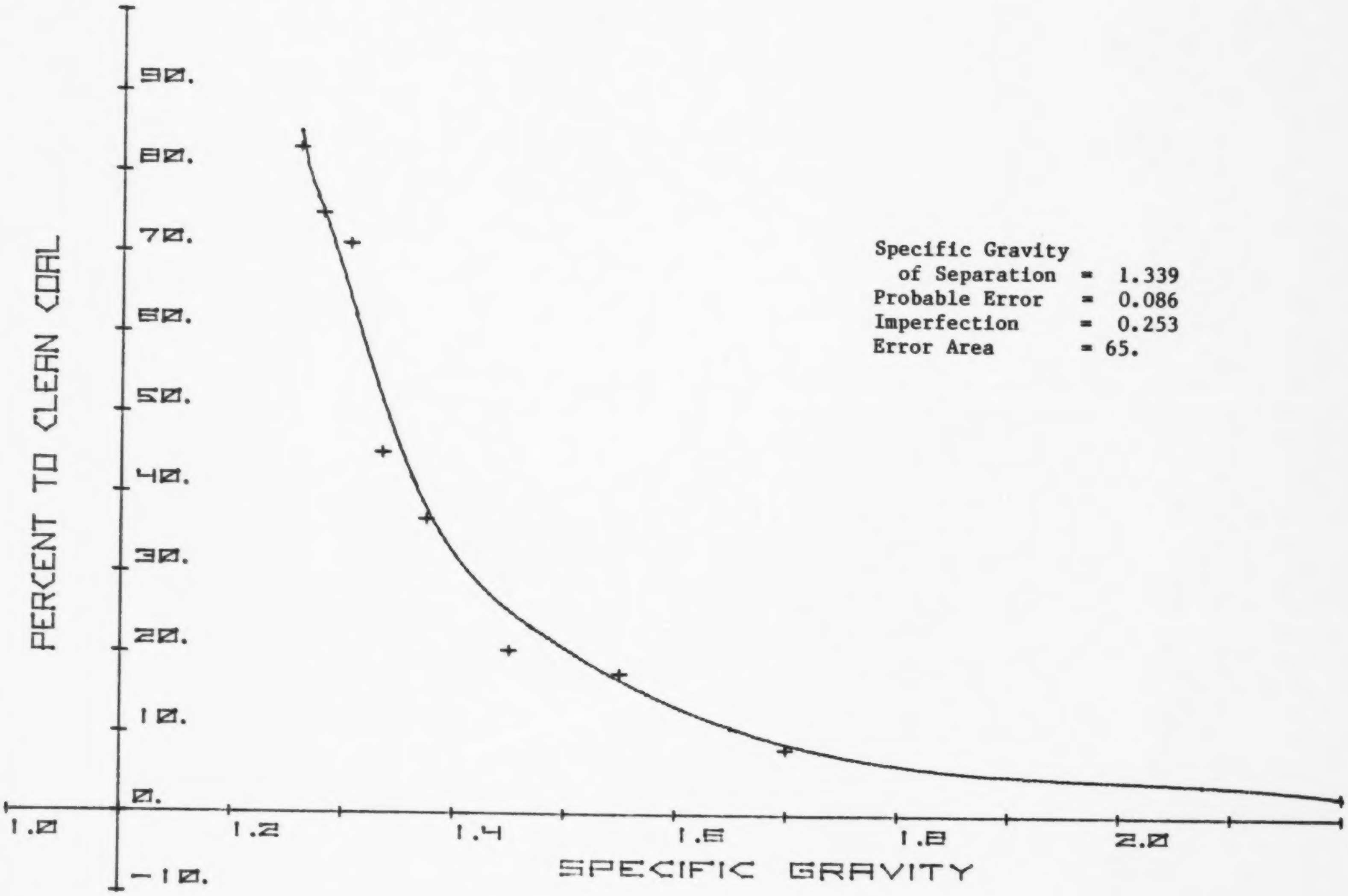
Specific Gravity
  of Separation = 1.298
Probable Error = 0.020
Imperfection = 0.067
Error Area = 15.

Figure A-85. Distribution Curve, Test No. 92 (9 x 14)



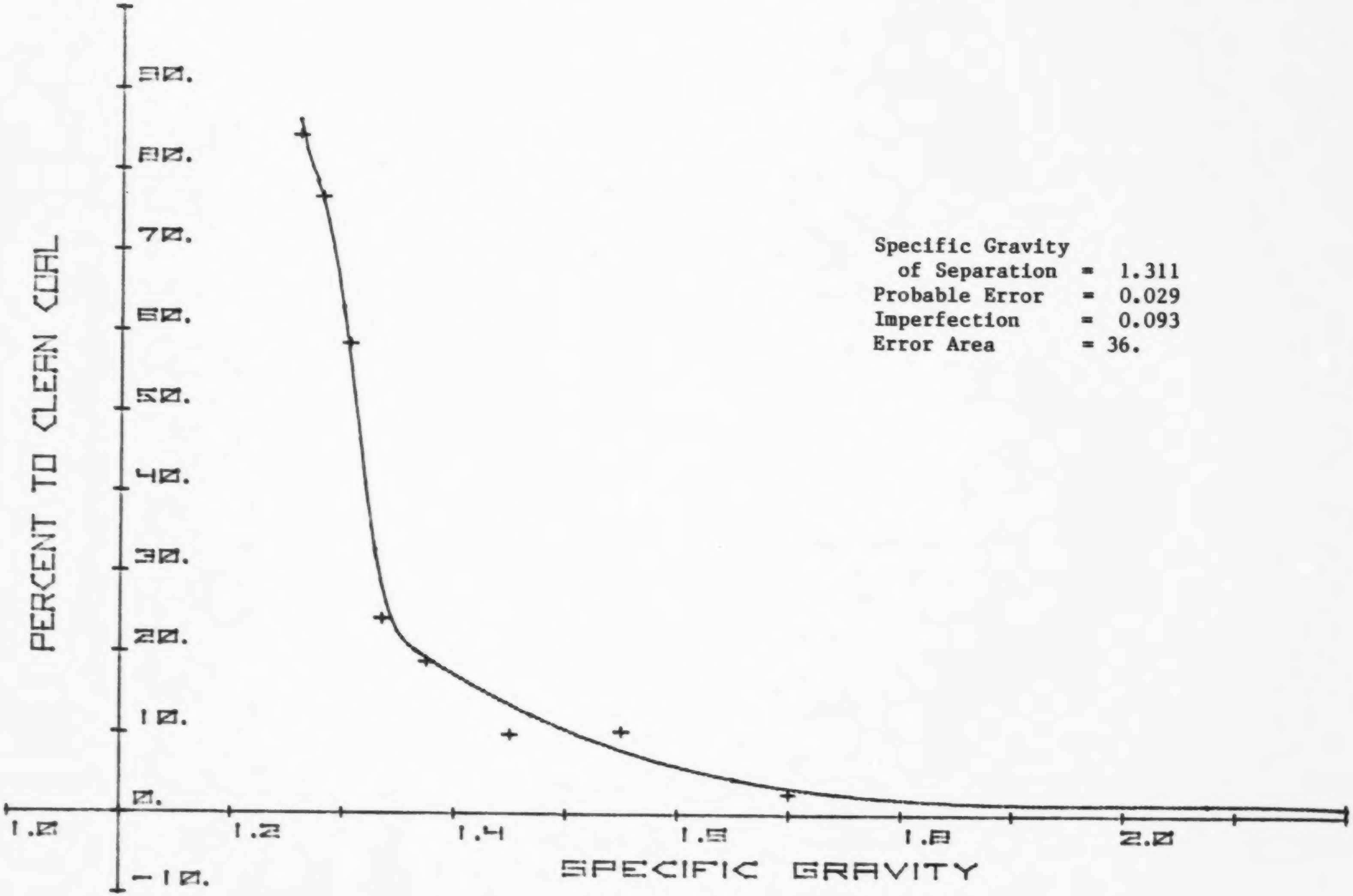
Specific Gravity
  of Separation = 1.311
Probable Error = 0.022
Imperfection = 0.072
Error Area = 25.

Figure A-86. Distribution Curve, Test No. 92 (14 x 28)



Specific Gravity
  of Separation = 1.339
Probable Error = 0.086
Imperfection = 0.253
Error Area = 65.

Figure A-87. Distribution Curve, Test No. 92 (28 x 100)

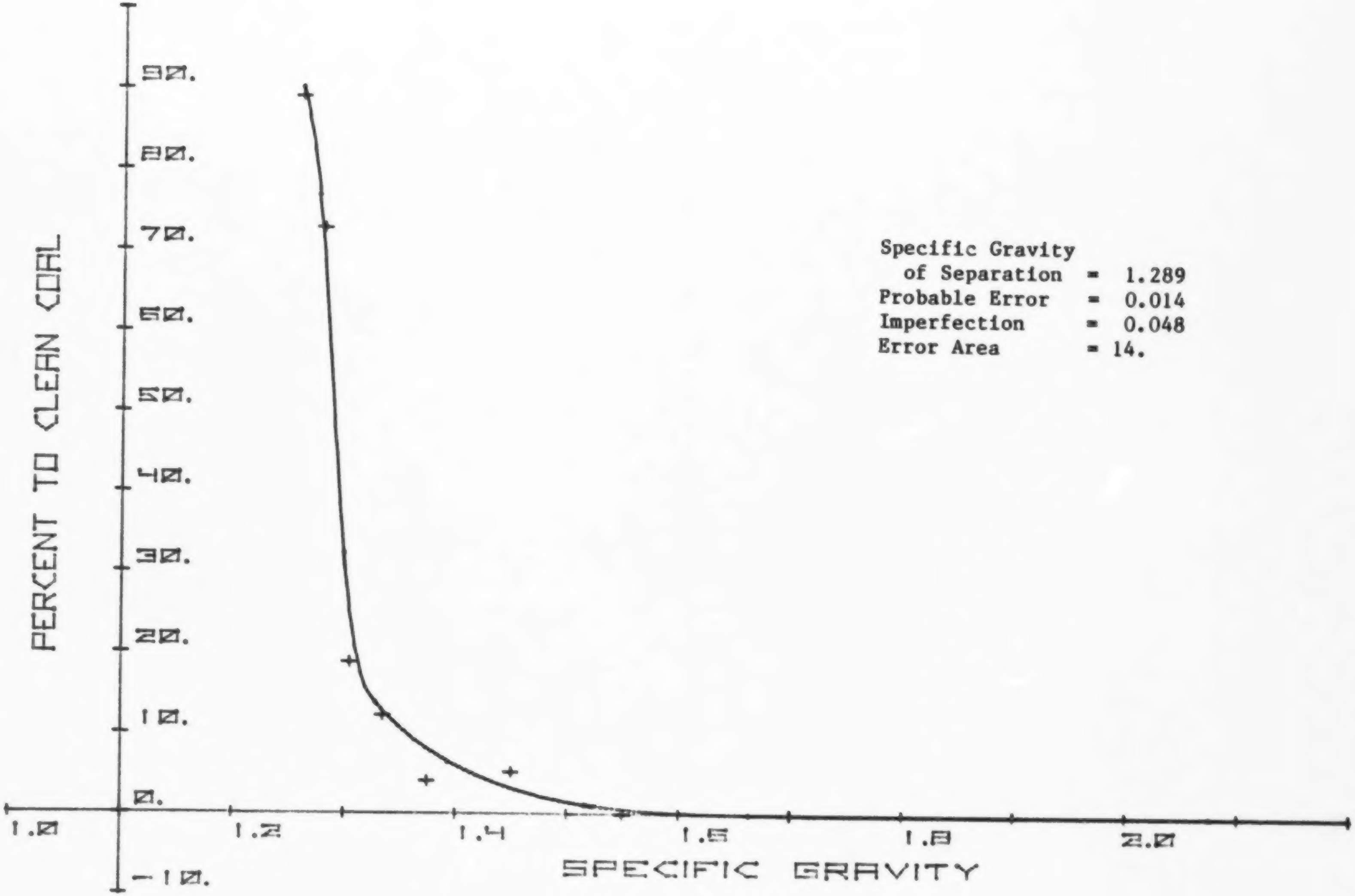


Specific Gravity
  of Separation = 1.311
Probable Error = 0.029
Imperfection = 0.093
Error Area = 36.

Figure A-88. Distribution Curve, Test No. 92 (9 x 100)

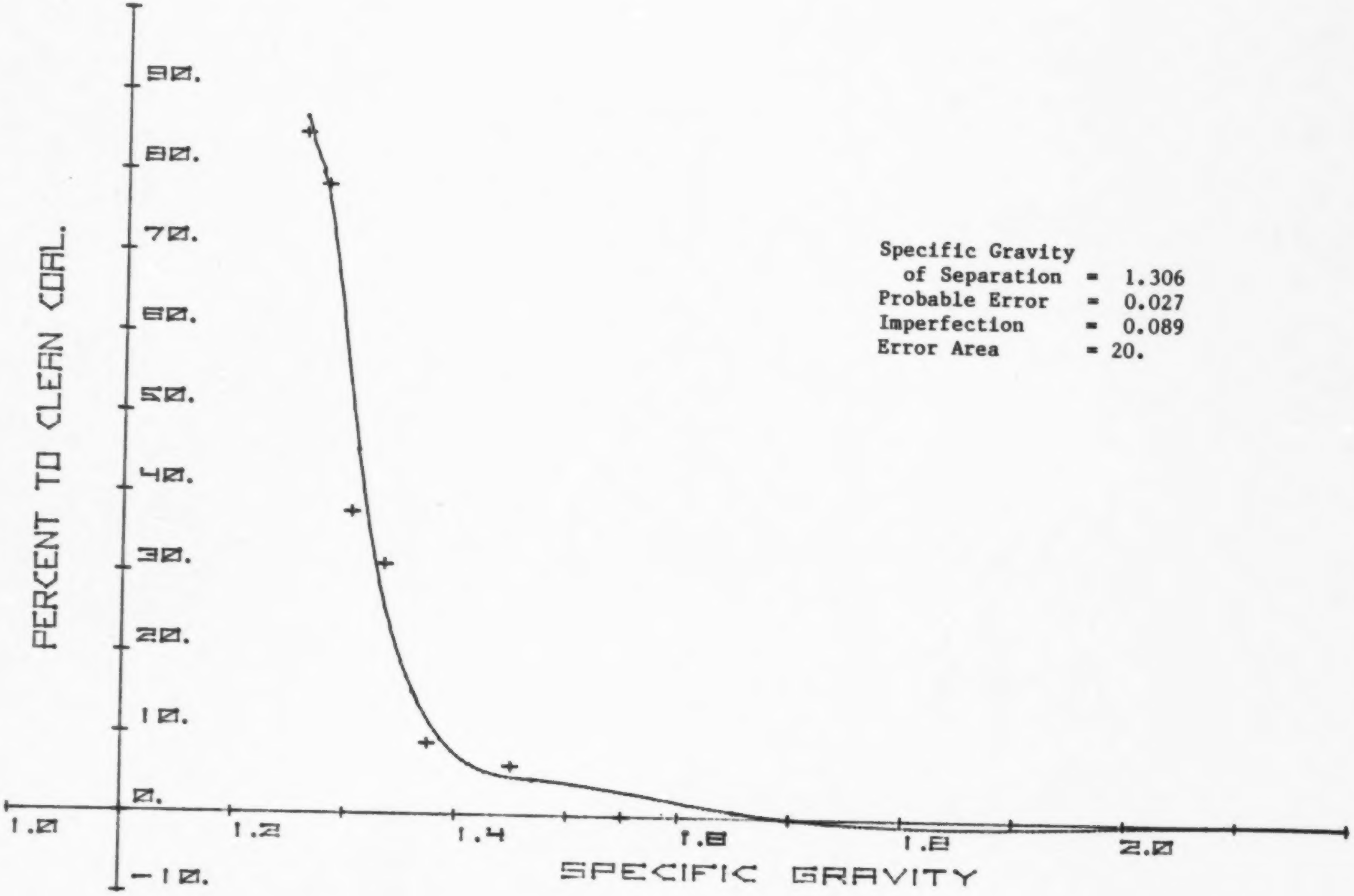
TABLE A-23. TEST 93 DATA SUMMARY

	9x14	14x28	28x100	9x100
SCREEN ANALYSIS, PERCENT:				
FEED	27.0	28.5	38.0	93.5
CLEAN COAL	18.1	26.2	47.2	91.5
REFUSE	33.9	30.3	30.8	95.0
ASH, PERCENT:				
FEED	23.4	21.1	19.4	21.0
CLEAN COAL	3.2	3.2	4.4	3.8
REFUSE	31.7	33.1	37.3	34.0
TOTAL SULFUR, PERCENT:				
FEED	2.83	3.25	3.66	3.30
CLEAN COAL	1.01	1.01	1.01	1.01
REFUSE	3.58	4.77	6.83	5.01
WEIGHT RECOVERY (YIELD)	29.3	40.3	54.5	42.9
THEORETICAL WEIGHT RECOVERY	35.4	49.7	68.3	54.2
WEIGHT RECOVERY EFFICIENCY	83.0	81.1	79.7	79.1
ASH ERROR	0.5	0.8	1.4	1.1
FLOAT IN REFUSE...% OF PRODUCT	8.2	15.8	27.8	17.3
SINK IN CLEAN COAL	15.7	14.6	8.7	18.3
TOTAL MISPLACED MATERIAL..% OF FEED	10.4	15.3	17.4	17.7
NEAR GRAVITY MATERIAL	59.7	67.4	71.2	66.9
SPECIFIC GRAVITY OF SEPARATION	1.289	1.306	1.343	1.310
PROBABLE ERROR	0.014	0.027	0.070	0.034
IMPERFECTION	0.048	0.089	0.204	0.110
ERROR AREA	14.	20.	61.	34.
DISTRIBUTION, PERCENT TO CLEANED COAL (SPECIFIC GRAVITY FRACTION):				
FLOAT 1.27	89.0	84.7	86.9	86.6
1.27-1.29	72.7	78.2	83.9	78.8
1.29-1.32	18.8	37.6	60.0	38.3
1.32-1.35	12.2	31.1	67.9	42.2
1.35-1.40	4.1	8.9	34.2	15.8
1.40-1.50	5.2	6.1	26.5	12.8
1.50-1.60	0.0	0.0	12.3	4.5
1.60-1.80	0.0	0.0	7.8	2.7
SINK 1.80	0.0	0.0	2.8	1.1



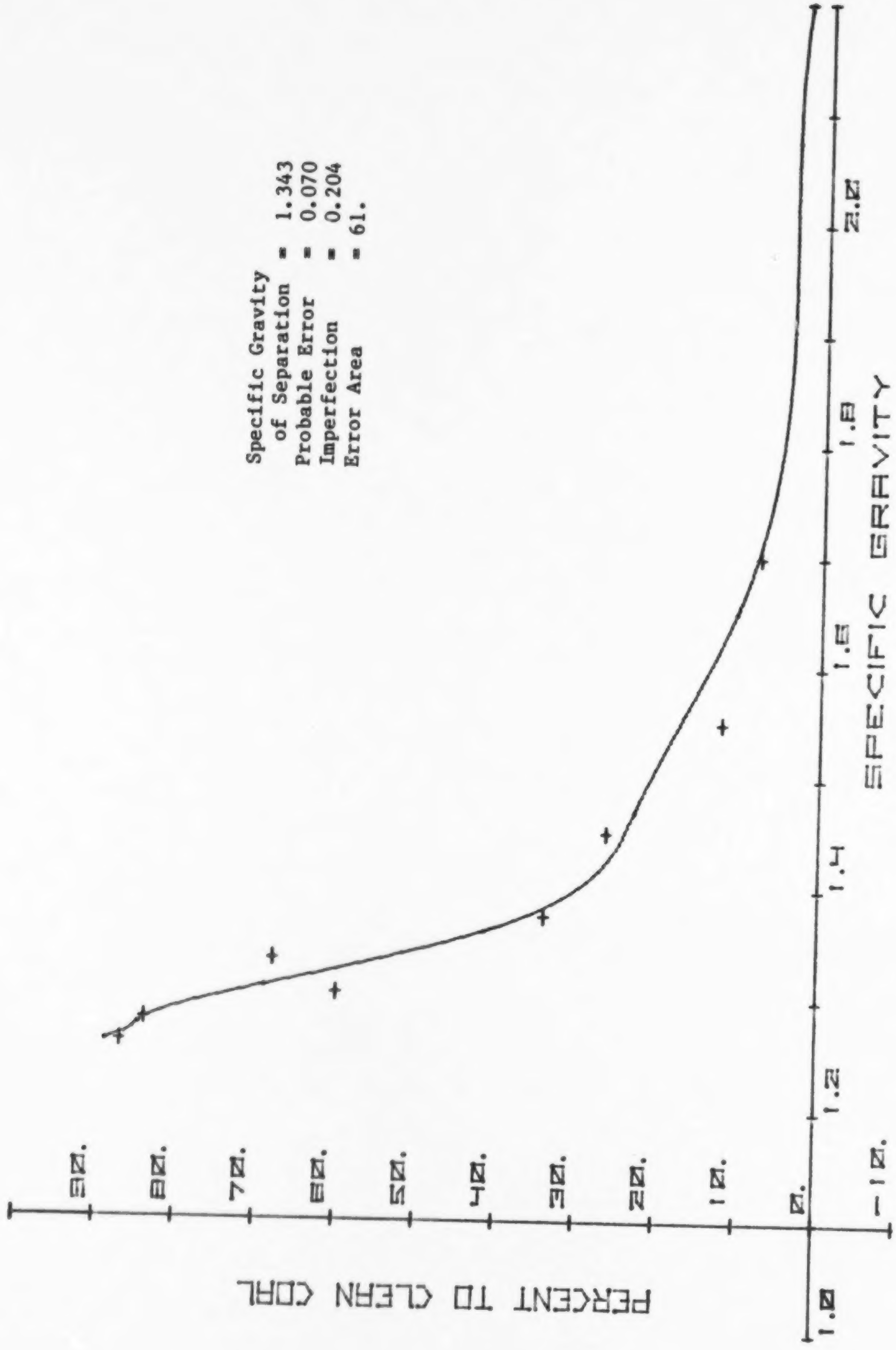
Specific Gravity
of Separation = 1.289
Probable Error = 0.014
Imperfection = 0.048
Error Area = 14.

Figure A-89. Distribution Curve, Test No. 93 (9 x 14)



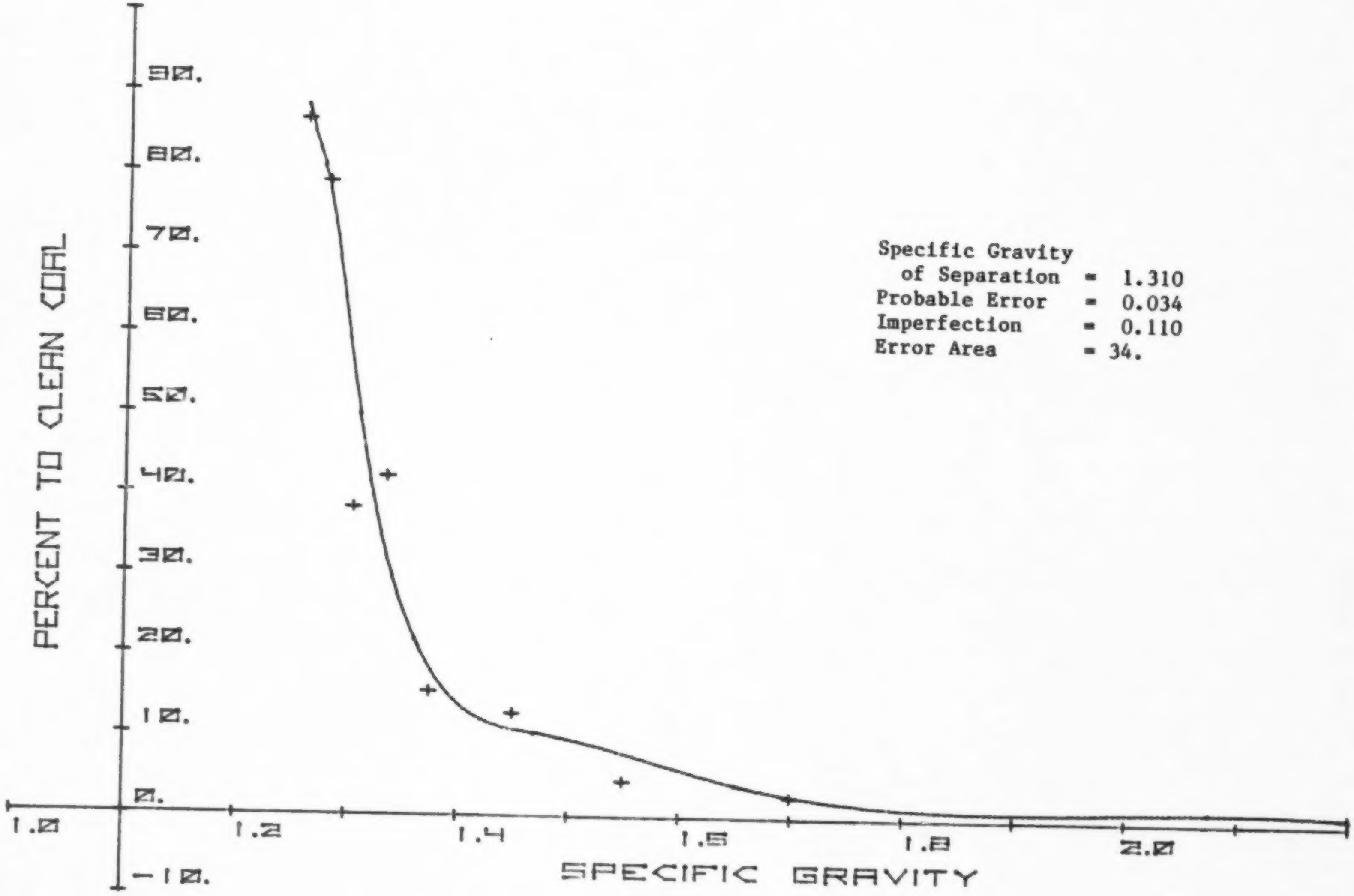
Specific Gravity
of Separation = 1.306
Probable Error = 0.027
Imperfection = 0.089
Error Area = 20.

Figure A-90. Distribution Curve, Test No. 93 (14 x 28)



Specific Gravity
of Separation = 1.343
Probable Error = 0.070
Imperfection = 0.204
Error Area = 61.

Figure A-91. Distribution Curve, Test No. 93 (28 x 100)

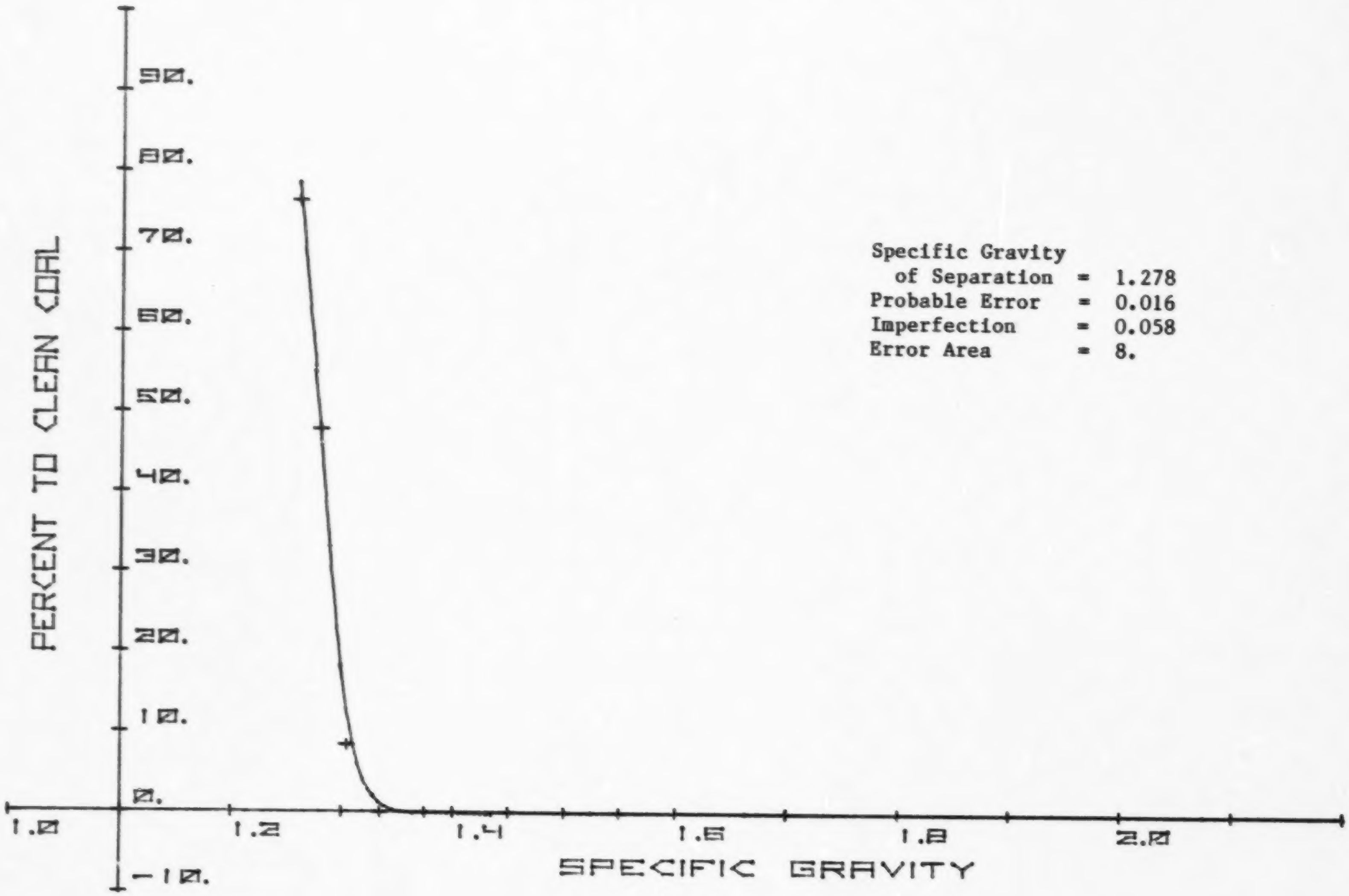


Specific Gravity of Separation = 1.310
Probable Error = 0.034
Imperfection = 0.110
Error Area = 34.

Figure A-92. Distribution Curve, Test No. 93 (9 x 100)

TABLE A-24. TEST 94 DATA SUMMARY

	9x14	14x28	28x100	9x100
SCREEN ANALYSIS, PERCENT:				
FEED	30.3	28.1	35.7	94.1
CLEAN COAL	21.5	27.9	43.4	92.9
REFUSE	37.1	28.1	29.8	95.0
ASH, PERCENT:				
FEED	22.6	21.1	20.8	21.5
CLEAN COAL	3.8	3.2	5.5	4.4
REFUSE	31.0	34.8	38.2	34.4
TOTAL SULFUR, PERCENT:				
FEED	2.69	3.08	3.68	3.18
CLEAN COAL	1.14	1.17	1.20	1.18
REFUSE	3.39	4.55	6.48	4.70
WEIGHT RECOVERY (YIELD)	31.0	43.5	53.0	43.1
THEORETICAL WEIGHT RECOVERY	36.5	52.2	71.7	58.3
WEIGHT RECOVERY EFFICIENCY	85.0	83.3	73.9	73.9
ASH ERROR	0.4	0.6	2.1	1.4
FLOAT IN REFUSE...% OF PRODUCT	15.3	22.0	32.9	23.3
SINK IN CLEAN COAL	16.6	17.4	8.0	15.9
TOTAL MISPLACED MATERIAL..% OF FEED	15.7	20.0	19.7	20.1
NEAR GRAVITY MATERIAL	63.6	68.4	70.7	67.2
SPECIFIC GRAVITY OF SEPARATION	1.278	1.281	1.354	1.295
PROBABLE ERROR	0.016	0.038	0.095	0.046
IMPERFECTION	0.058	0.134	0.267	0.156
ERROR AREA	8.	20.	64.	37.
DISTRIBUTION, PERCENT TO CLEANED COAL (SPECIFIC GRAVITY FRACTION):				
FLOAT 1.27	76.3	79.1	79.6	78.6
1.27-1.29	47.8	49.3	76.2	57.0
1.29-1.32	8.4	33.1	64.3	32.9
1.32-1.35	0.0	25.9	72.8	44.6
1.35-1.40	0.0	15.0	39.4	17.7
1.40-1.50	0.0	0.0	27.8	10.0
1.50-1.60	0.0	0.0	17.1	5.9
1.60-1.80	0.0	0.0	7.0	2.4
SINK 1.80	0.0	0.0	3.3	1.3



Specific Gravity
of Separation = 1.278
Probable Error = 0.016
Imperfection = 0.058
Error Area = 8.

Figure A-93. Distribution Curve, Test No. 94 (9 x 14)

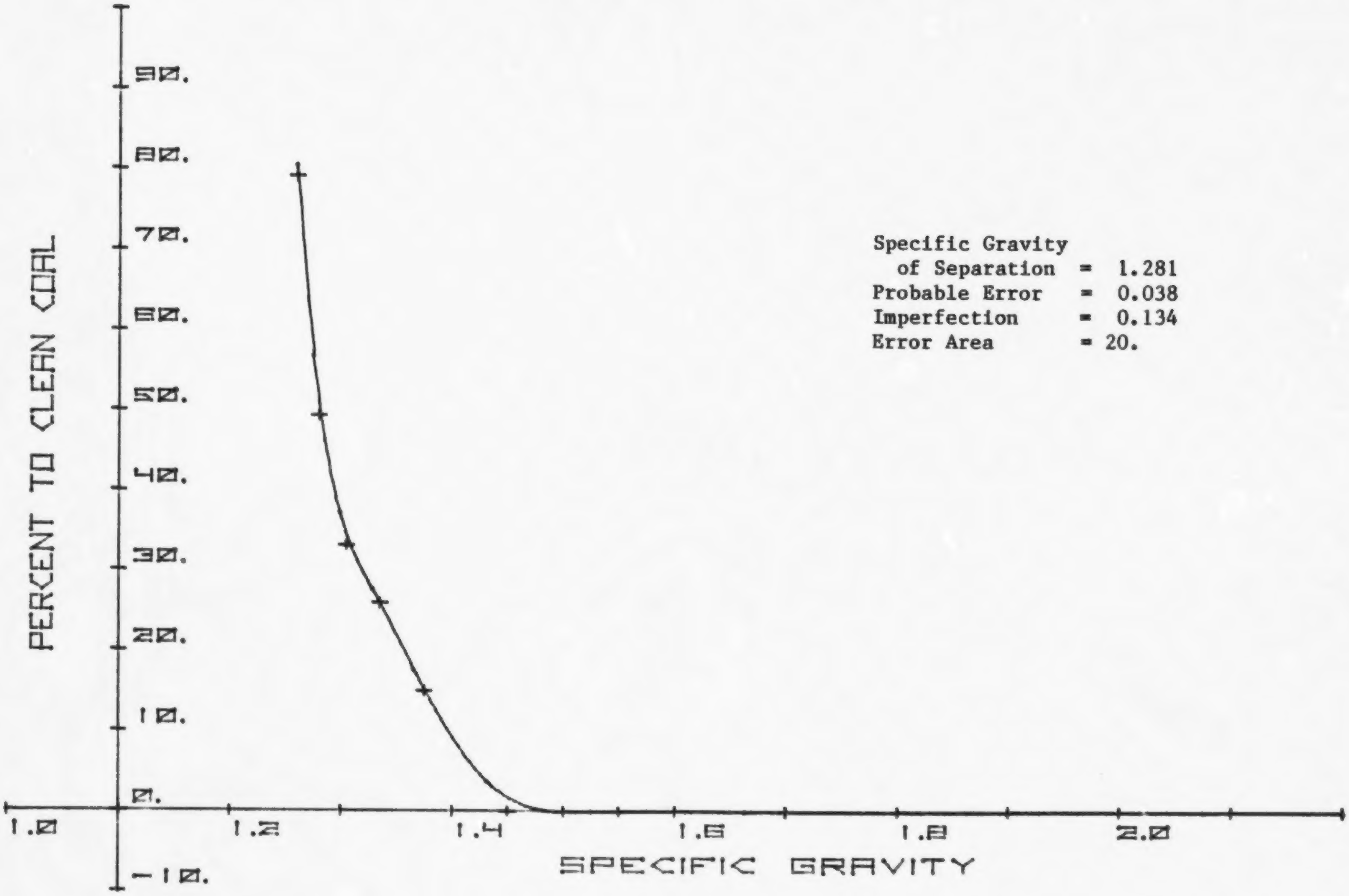


Figure A-94. Distribution Curve, Test No. 94 (14 x 28)

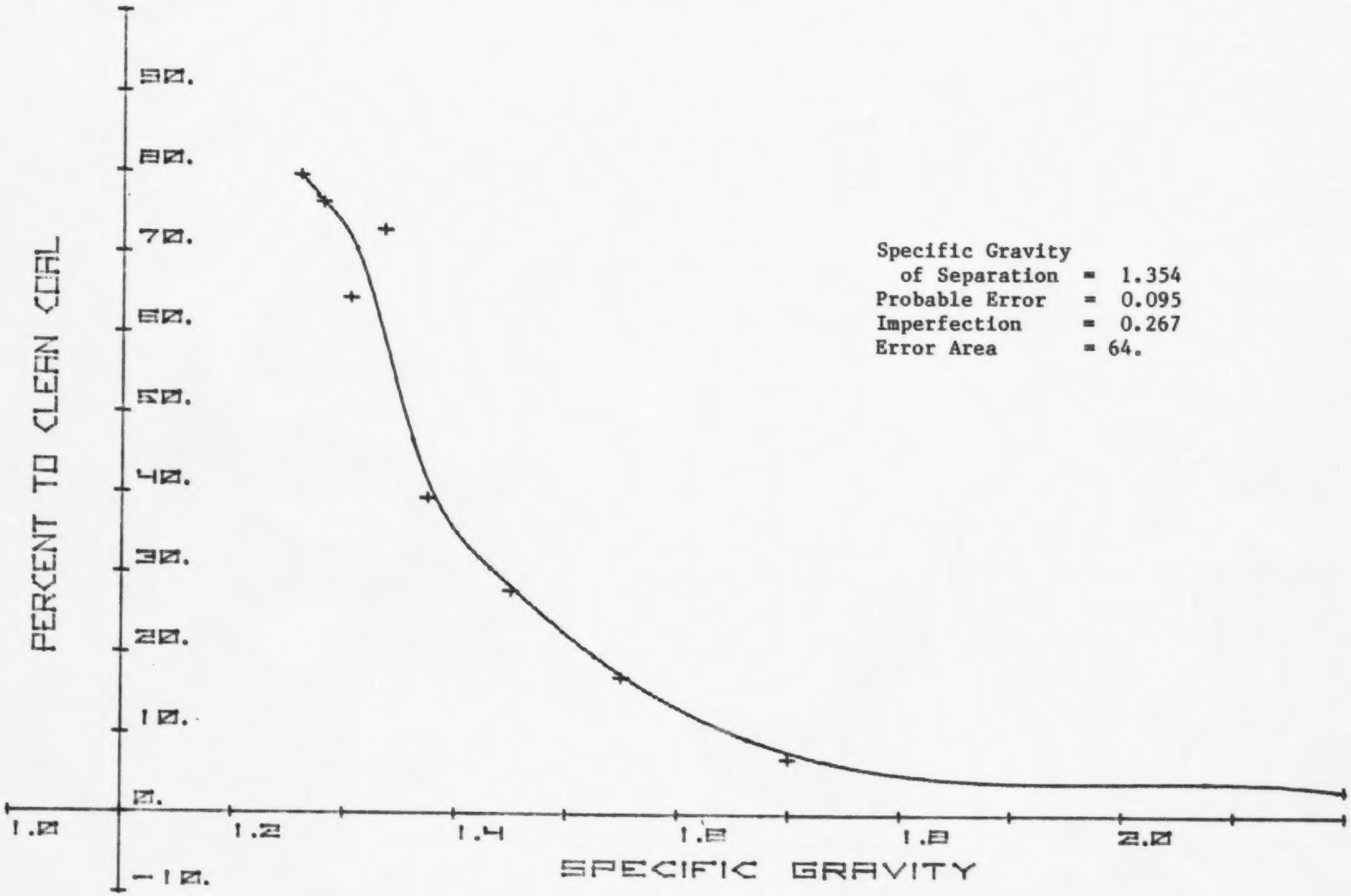
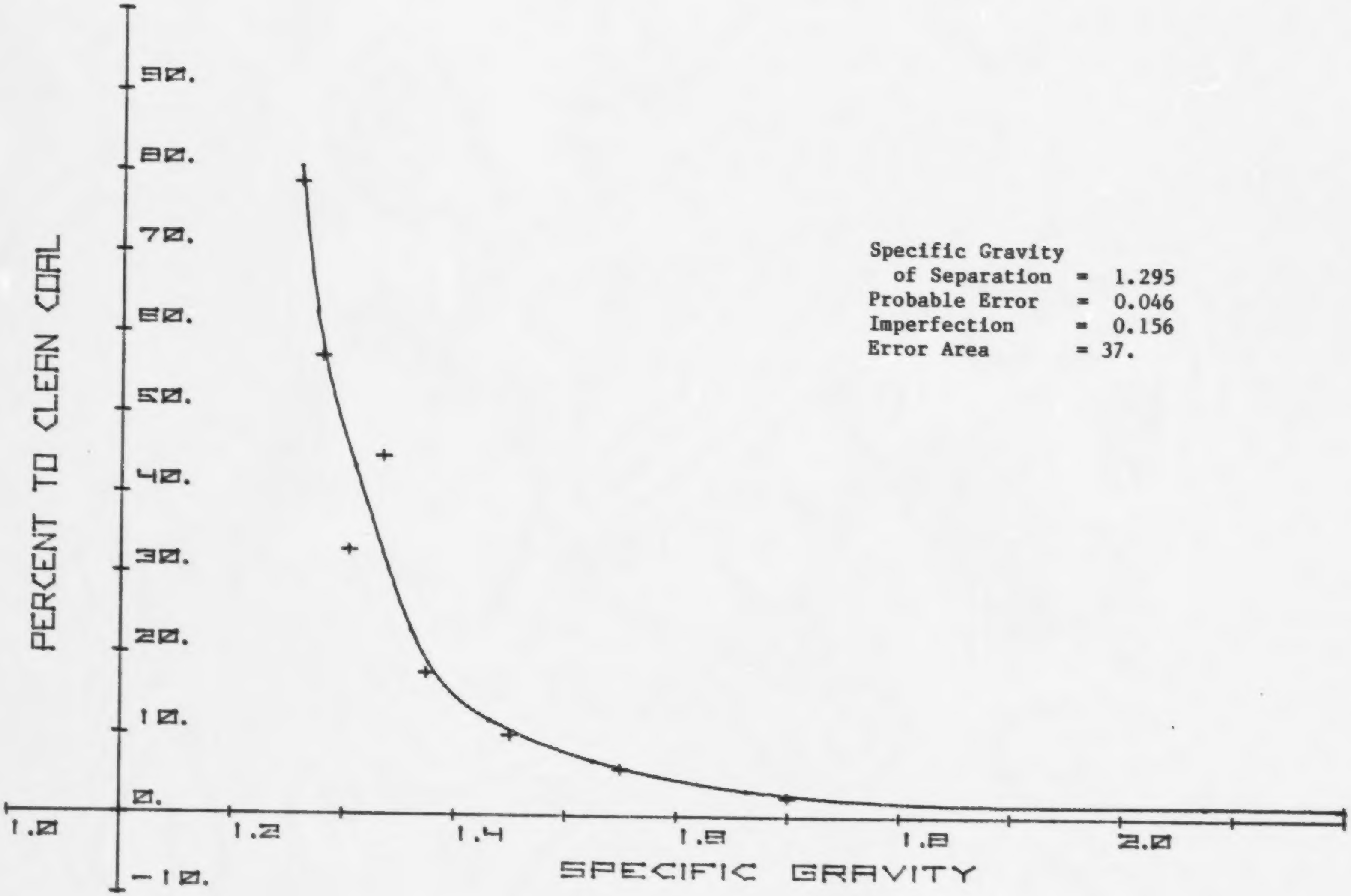


Figure A-95. Distribution Curve, Test No. 94 (28 x 100)



Specific Gravity
of Separation = 1.295
Probable Error = 0.046
Imperfection = 0.156
Error Area = 37.

Figure A-96. Distribution Curve, Test No. 94 (9 x 100)

TABLE A-25. TEST 95 DATA SUMMARY

	9x14	14x28	28x100	9x100
SCREEN ANALYSIS, PERCENT:				
FEED	29.3	27.9	36.4	93.7
CLEAN COAL	19.8	26.5	45.6	91.9
REFUSE	37.8	29.1	28.3	95.3
ASH, PERCENT:				
FEED	25.7	21.0	19.8	22.0
CLEAN COAL	3.8	3.7	5.7	4.7
REFUSE	35.8	35.0	40.0	36.8
TOTAL SULFUR, PERCENT:				
FEED	2.85	3.01	3.41	3.11
CLEAN COAL	0.78	0.82	1.03	0.92
REFUSE	3.80	4.78	6.81	4.99
WEIGHT RECOVERY (YIELD)	31.7	44.7	58.8	46.1
THEORETICAL WEIGHT RECOVERY	40.3	55.2	75.0	60.8
WEIGHT RECOVERY EFFICIENCY	78.5	80.9	78.5	75.9
ASH ERROR	0.8	0.8	2.3	1.6
FLOAT IN REFUSE...% OF PRODUCT	9.7	21.3	32.9	21.9
SINK IN CLEAN COAL	16.4	13.6	7.0	15.4
TOTAL MISPLACED MATERIAL..% OF FEED	11.8	17.8	17.7	18.9
NEAR GRAVITY MATERIAL	58.9	68.0	23.4	66.5
SPECIFIC GRAVITY OF SEPARATION	1.291	1.308	1.386	1.318
PROBABLE ERROR	0.021	0.041	0.117	0.059
IMPERFECTION	0.071	0.134	0.303	0.184
ERROR AREA	25.	31.	82.	48.
DISTRIBUTION, PERCENT TO CLEANED COAL (SPECIFIC GRAVITY FRACTION):				
FLOAT 1.27	81.9	81.7	87.9	84.7
1.27-1.29	85.5	87.5	85.7	86.2
1.29-1.32	30.2	48.9	71.6	49.2
1.32-1.35	15.1	36.8	76.5	44.5
1.35-1.40	10.2	19.0	51.6	27.7
1.40-1.50	6.9	12.8	37.7	19.5
1.50-1.60	3.5	6.9	24.5	11.2
1.60-1.80	0.0	0.0	12.3	4.3
SINK 1.80	0.0	0.0	5.2	1.9

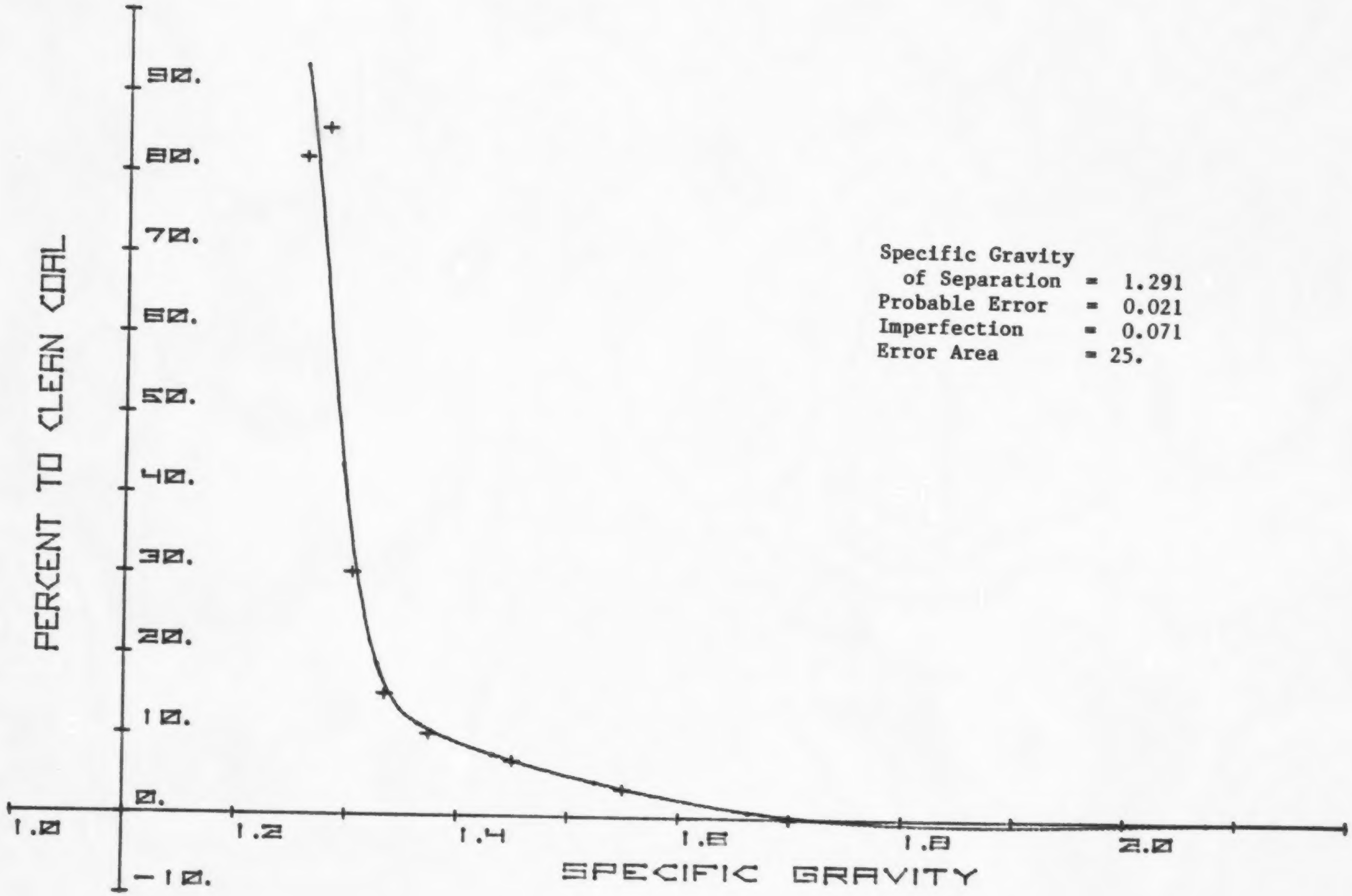
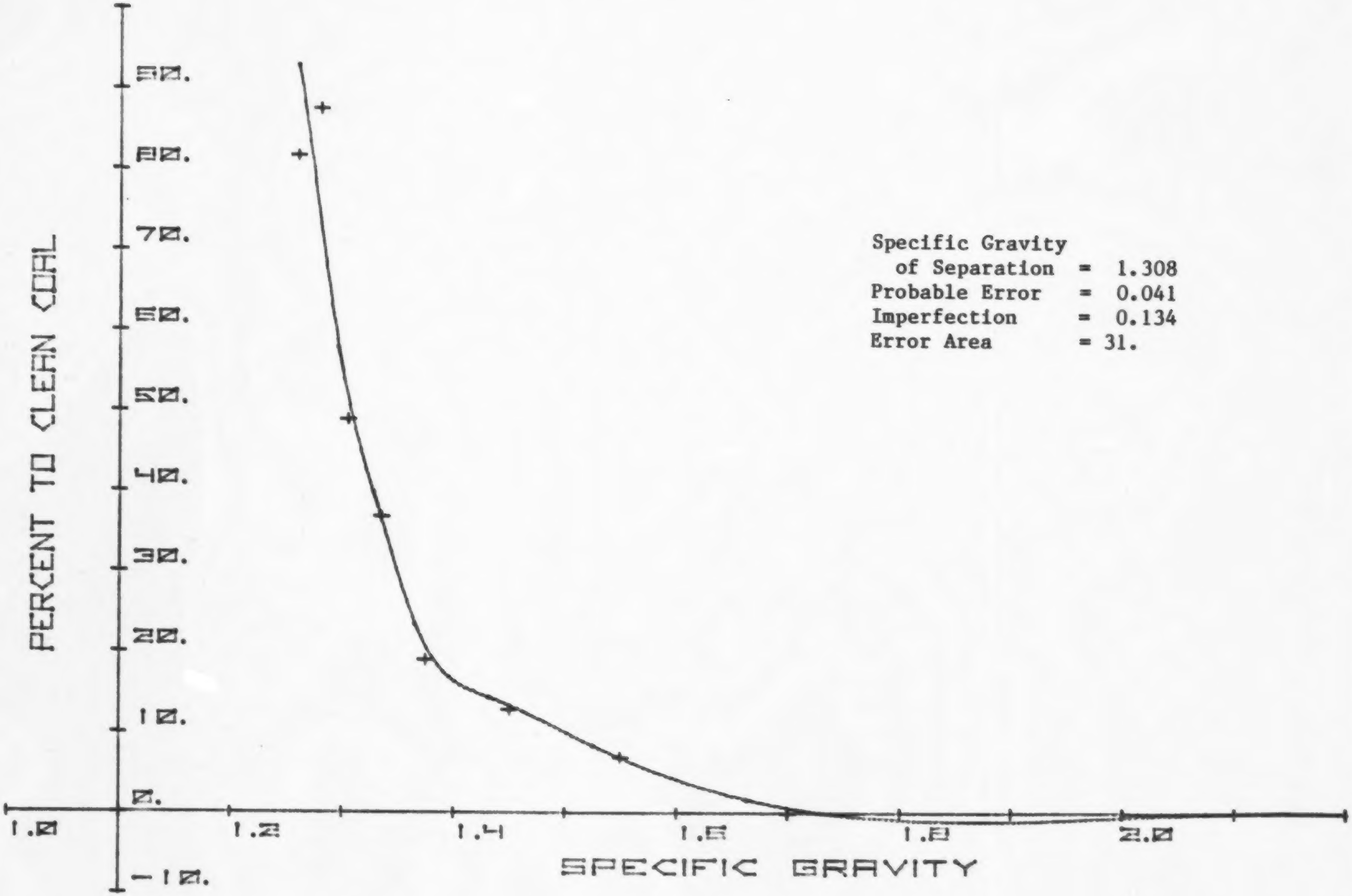
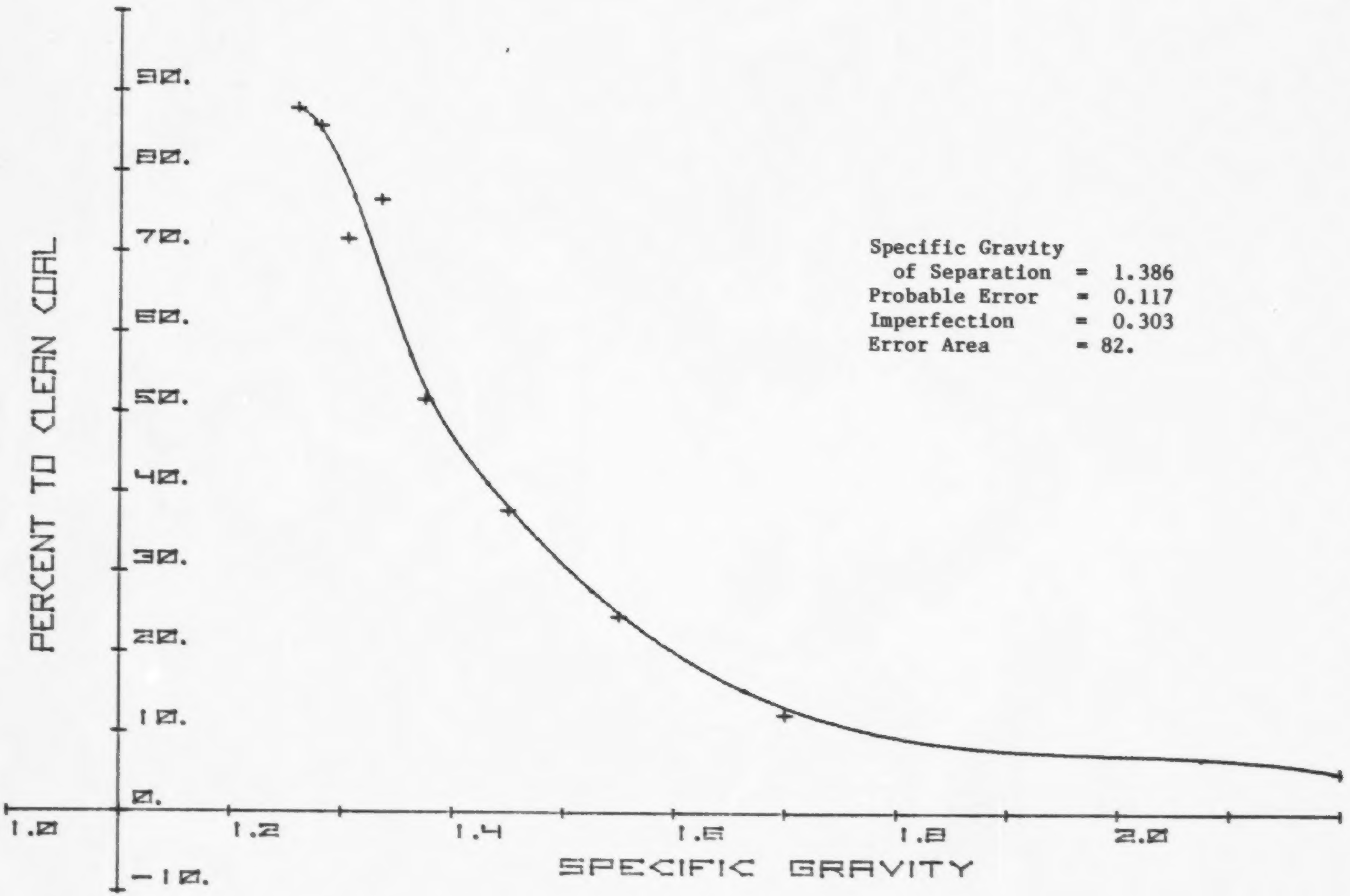


Figure A-97. Distribution Curve, Test No. 95 (9 x 14)



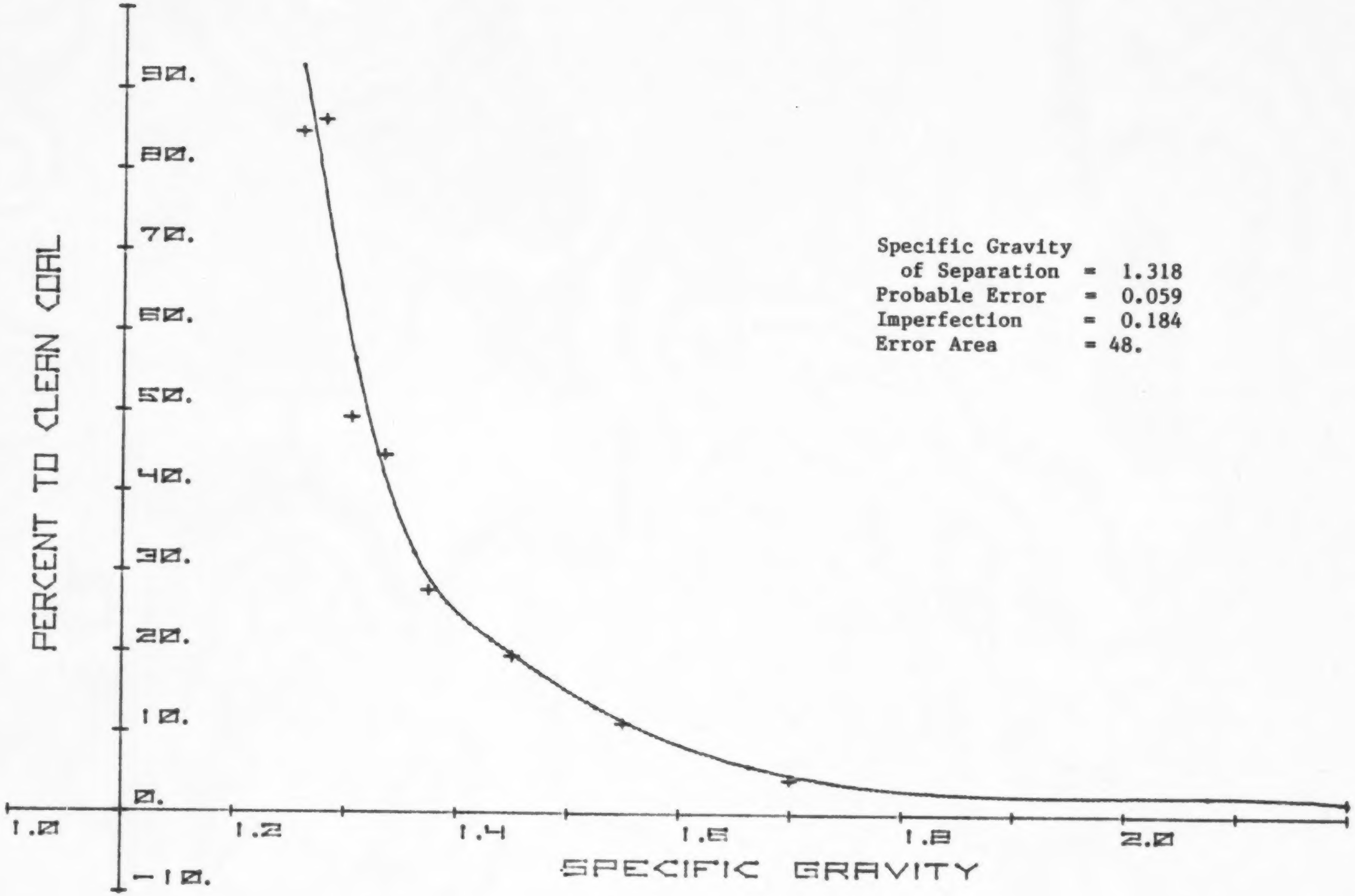
Specific Gravity
  of Separation = 1.308
Probable Error = 0.041
Imperfection = 0.134
Error Area = 31.

Figure A-98. Distribution Curve, Test No. 95 (14 x 28)



Specific Gravity
  of Separation = 1.386
Probable Error = 0.117
Imperfection = 0.303
Error Area = 82.

Figure A-99. Distribution Curve, Test No. 95 (28 x 100)

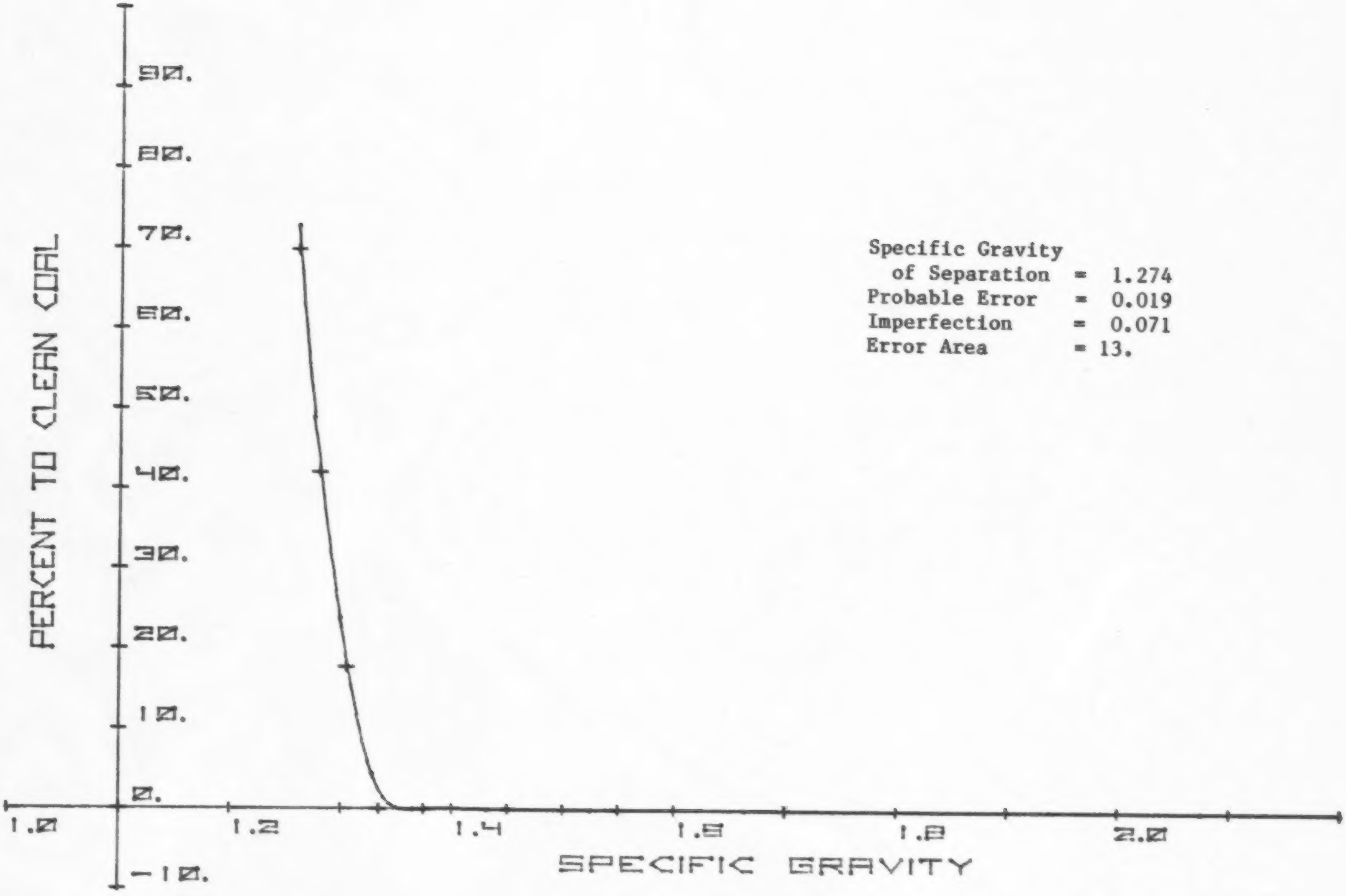


Specific Gravity
  of Separation = 1.318
Probable Error = 0.059
Imperfection = 0.184
Error Area = 48.

Figure A-100. Distribution Curve, Test No. 95 (9 x 100)

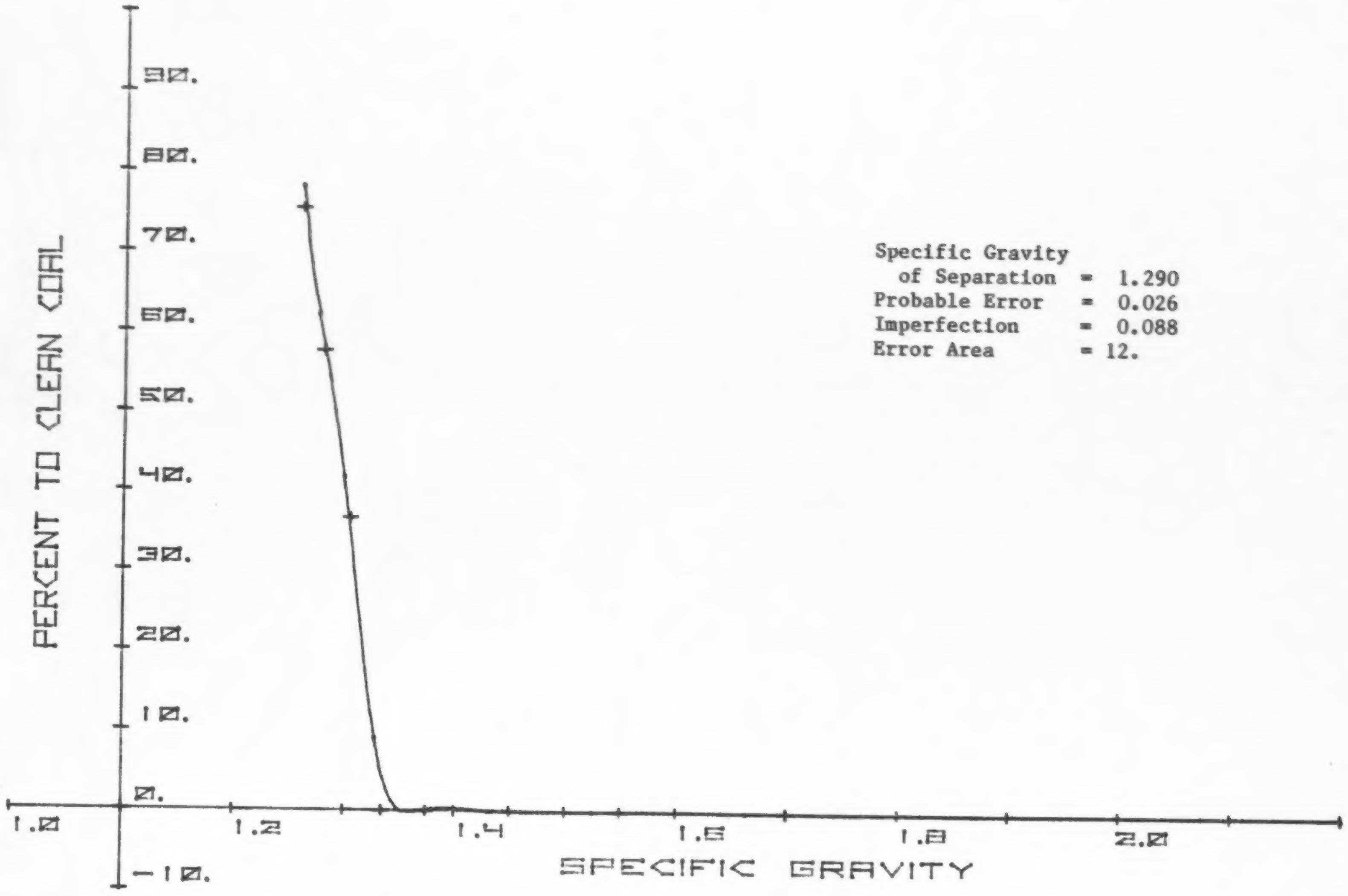
TABLE A-26. TEST 96 DATA SUMMARY

	9x14	14x28	28x100	9x100
SCREEN ANALYSIS, PERCENT:				
FEED	28.5	27.2	37.7	93.4
CLEAN COAL	16.7	26.0	48.6	91.2
REFUSE	36.1	28.0	30.7	94.9
ASH, PERCENT:				
FEED	27.3	21.5	20.7	22.9
CLEAN COAL	3.1	2.8	5.0	4.0
REFUSE	34.4	32.7	36.5	34.6
TOTAL SULFUR, PERCENT:				
FEED	3.50	3.25	4.27	3.74
CLEAN COAL	1.38	1.45	1.30	1.36
REFUSE	4.13	4.32	7.26	5.20
WEIGHT RECOVERY (YIELD)	22.8	37.2	50.3	38.1
THEORETICAL WEIGHT RECOVERY	31.1	46.1	71.1	56.9
WEIGHT RECOVERY EFFICIENCY	73.3	80.8	70.8	66.9
ASH ERROR	0.5	0.5	2.6	1.6
FLOAT IN REFUSE...% OF PRODUCT	10.6	21.9	29.3	20.1
SINK IN CLEAN COAL	26.4	11.5	10.2	16.2
TOTAL MISPLACED MATERIAL..% OF FEED	14.2	18.0	19.7	18.6
NEAR GRAVITY MATERIAL	55.4	67.0	68.0	64.2
SPECIFIC GRAVITY OF SEPARATION	1.274	1.290	1.319	1.292
PROBABLE ERROR	0.019	0.026	0.072	0.034
IMPERFECTION	0.071	0.088	0.226	0.117
ERROR AREA	13.	12.	67.	32.
DISTRIBUTION, PERCENT TO CLEANED COAL (SPECIFIC GRAVITY FRACTION):				
FLOAT 1.27	69.8	75.3	81.5	76.9
1.27-1.29	42.0	57.5	76.5	60.5
1.29-1.32	17.7	36.6	57.4	39.2
1.32-1.35	0.0	0.0	29.7	5.8
1.35-1.40	0.0	0.0	33.9	11.9
1.40-1.50	0.0	0.0	26.9	11.5
1.50-1.60	0.0	0.0	15.5	5.4
1.60-1.80	0.0	0.0	10.6	4.0
SINK 1.80	0.0	0.0	3.7	1.4



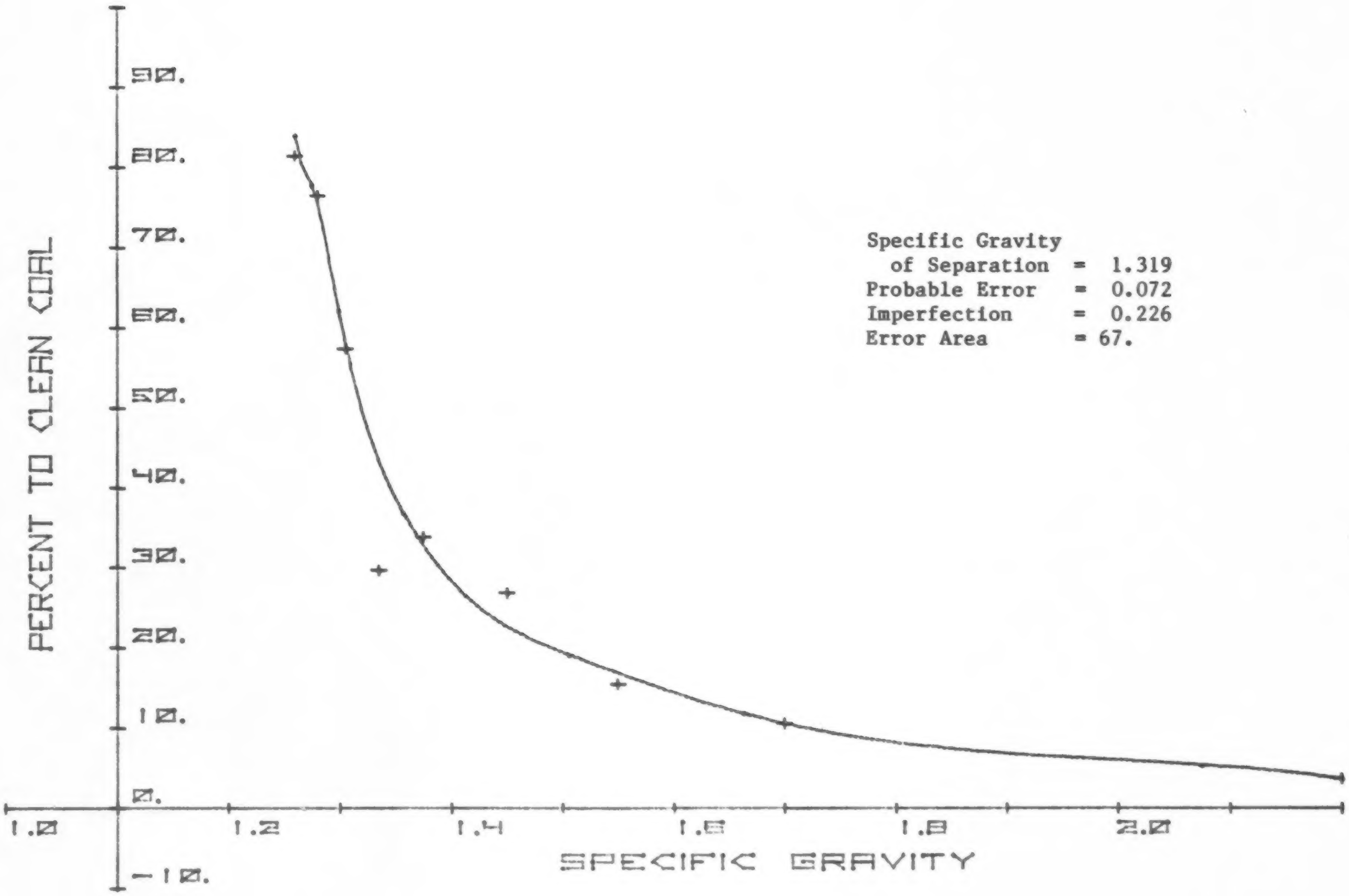
Specific Gravity
  of Separation = 1.274
Probable Error = 0.019
Imperfection = 0.071
Error Area = 13.

Figure A-101. Distribution Curve, Test No. 96 (9 x 14)



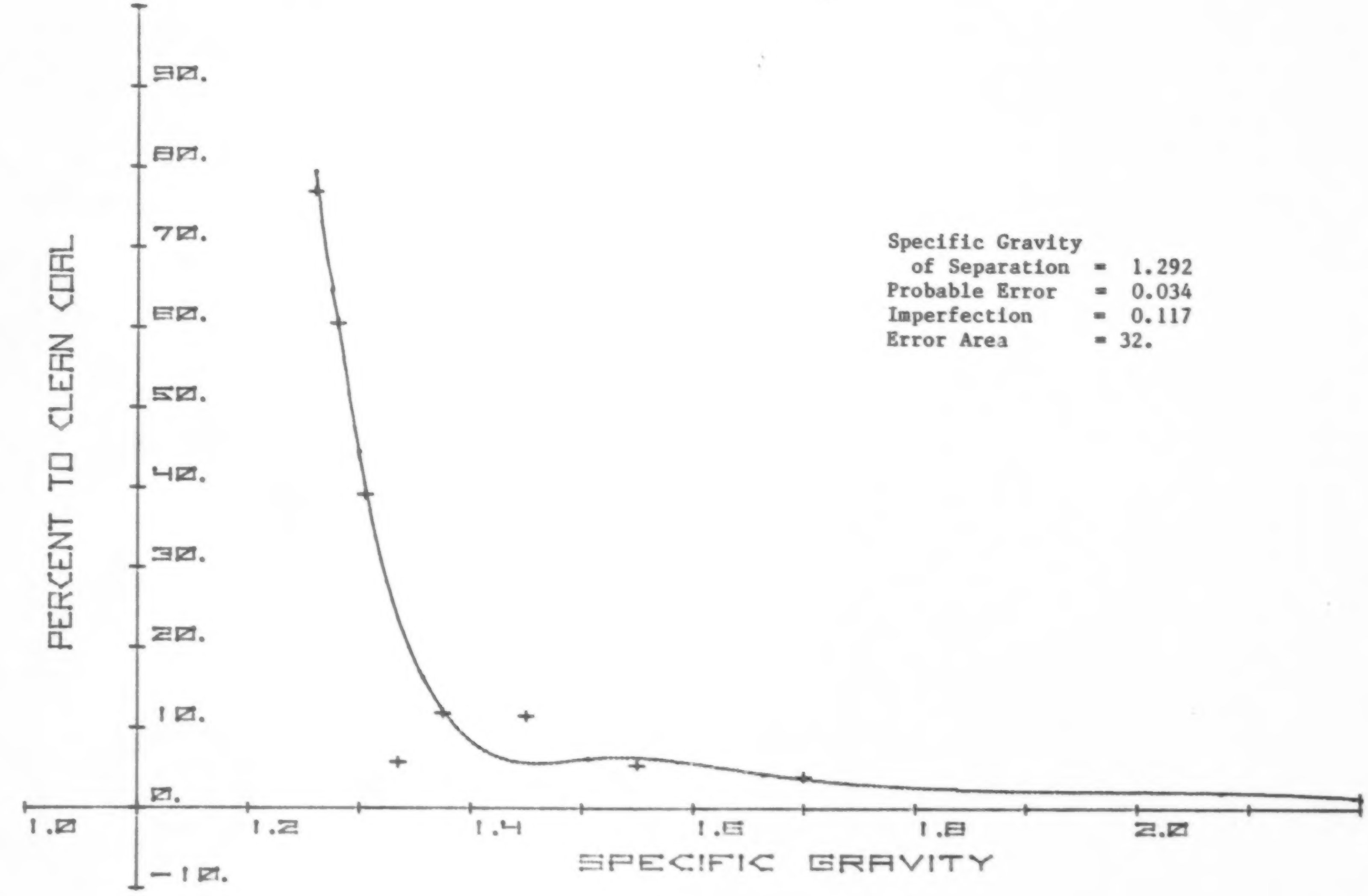
Specific Gravity
  of Separation = 1.290
Probable Error = 0.026
Imperfection = 0.088
Error Area = 12.

Figure A-102. Distribution Curve, Test No. 96 (14 x 28)



Specific Gravity
  of Separation = 1.319
Probable Error = 0.072
Imperfection = 0.226
Error Area = 67.

Figure A-103. Distribution Curve, Test No. 96 (28 x 100)



Specific Gravity
of Separation = 1.292
Probable Error = 0.034
Imperfection = 0.117
Error Area = 32.

Figure A-104. Distribution Curve, Test No. 96 (9 x 100)

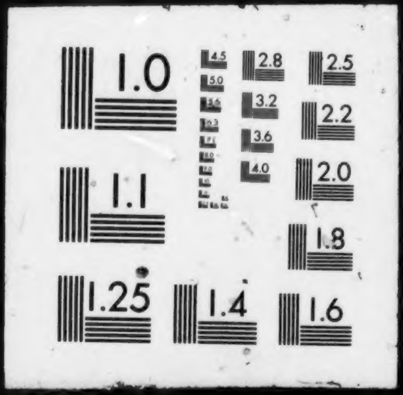
TABLE A-27. TEST 97 DATA SUMMARY

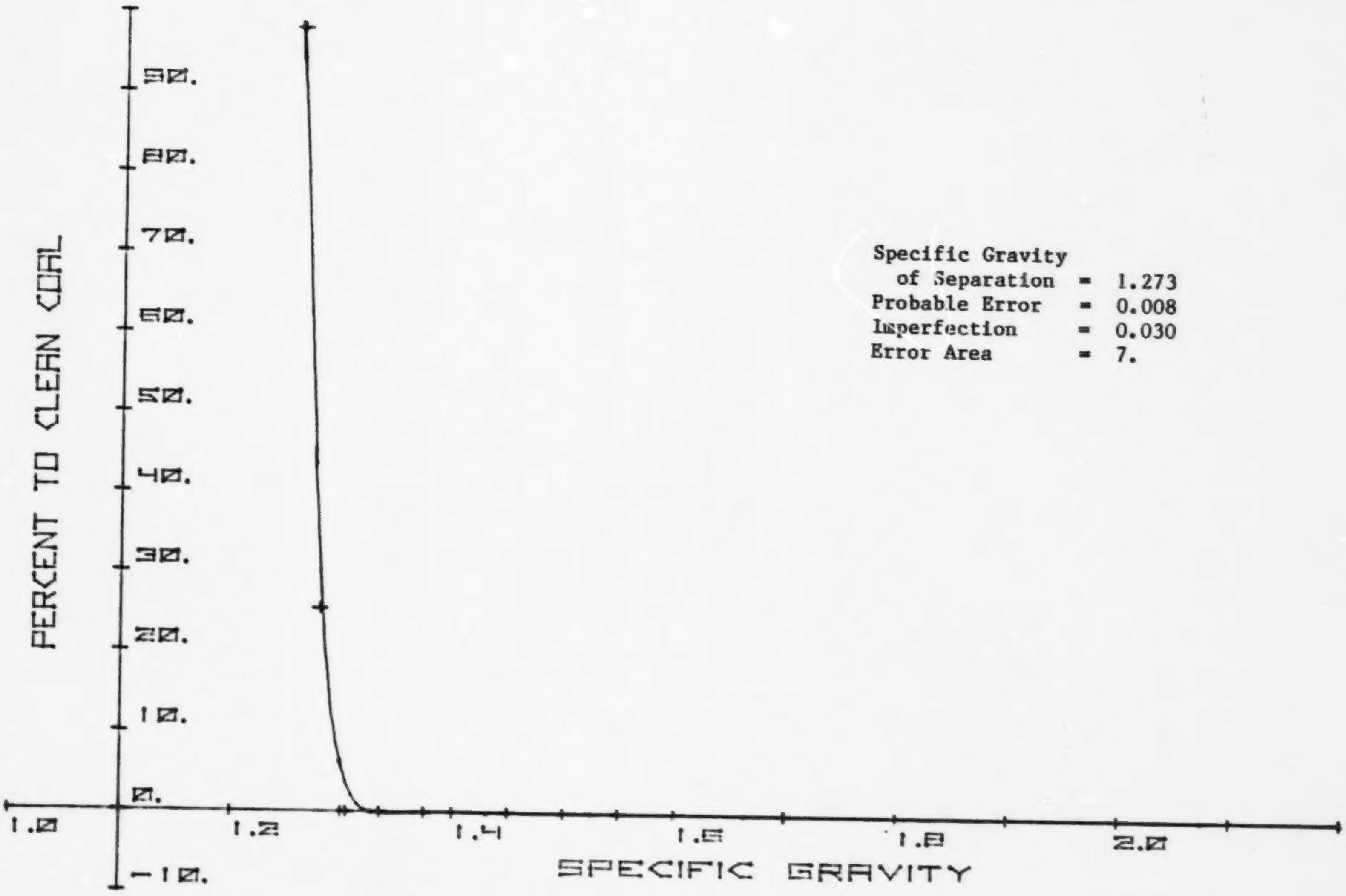
	9x14	14x28	28x100	9x100
SCREEN ANALYSIS, PERCENT:				
FEED............................	27.0	27.6	38.4	93.1
CLEAN COAL......................	17.3	24.8	48.7	90.8
REFUSE..........................	33.8	29.6	31.3	94.7
ASH, PERCENT:				
FEED............................	23.5	20.2	19.0	20.7
CLEAN COAL......................	2.9	3.0	3.0	3.2
REFUSE..........................	30.9	30.2	36.2	32.4
TOTAL SULFUR, PERCENT:				
FEED............................	2.72	3.01	3.66	3.19
CLEAN COAL......................	0.96	0.94	0.76	0.90
REFUSE..........................	3.34	4.21	6.78	4.75
WEIGHT RECOVERY (YIELD)............	26.1	36.7	51.9	39.9
THEORETICAL WEIGHT RECOVERY........	32.5	45.4	45.1	44.5
WEIGHT RECOVERY EFFICIENCY.........	80.4	80.7	100.0	89.7
ASH ERROR..........................	0.4	0.4	0.0	0.3
FLOAT IN REFUSE...% OF PRODUCT.....	3.3	9.6	21.7	9.9
SINK IN CLEAN COAL.................	14.9	14.3	8.9	21.1
TOTAL MISPLACED MATERIAL..% OF FEED	6.3	11.3	15.1	14.4
NEAR GRAVITY MATERIAL..............	61.6	67.1	70.2	66.2
SPECIFIC GRAVITY OF SEPARATION.....	1.273	1.276	1.297	1.280
PROBABLE ERROR.....................	0.008	0.010	0.042	0.014
IMPERFECTION.......................	0.030	0.036	0.140	0.050
ERROR AREA.........................	7.	6.	50.	23.
DISTRIBUTION, PERCENT TO CLEANED COAL (SPECIFIC GRAVITY FRACTION):				
FLOAT 1.27	98.0	97.1	98.4	97.9
1.27-1.29	25.4	39.0	66.8	49.1
1.29-1.32	0.0	0.0	42.9	15.1
1.32-1.35	0.0	0.0	30.0	7.1
1.35-1.40	0.0	0.0	21.6	7.7
1.40-1.50	0.0	0.0	23.3	8.7
1.50-1.60	0.0	0.0	12.7	4.7
1.60-1.80	0.0	0.0	0.0	0.0
SINK 1.80	0.0	0.0	0.0	0.0

3 OF 3

PB84

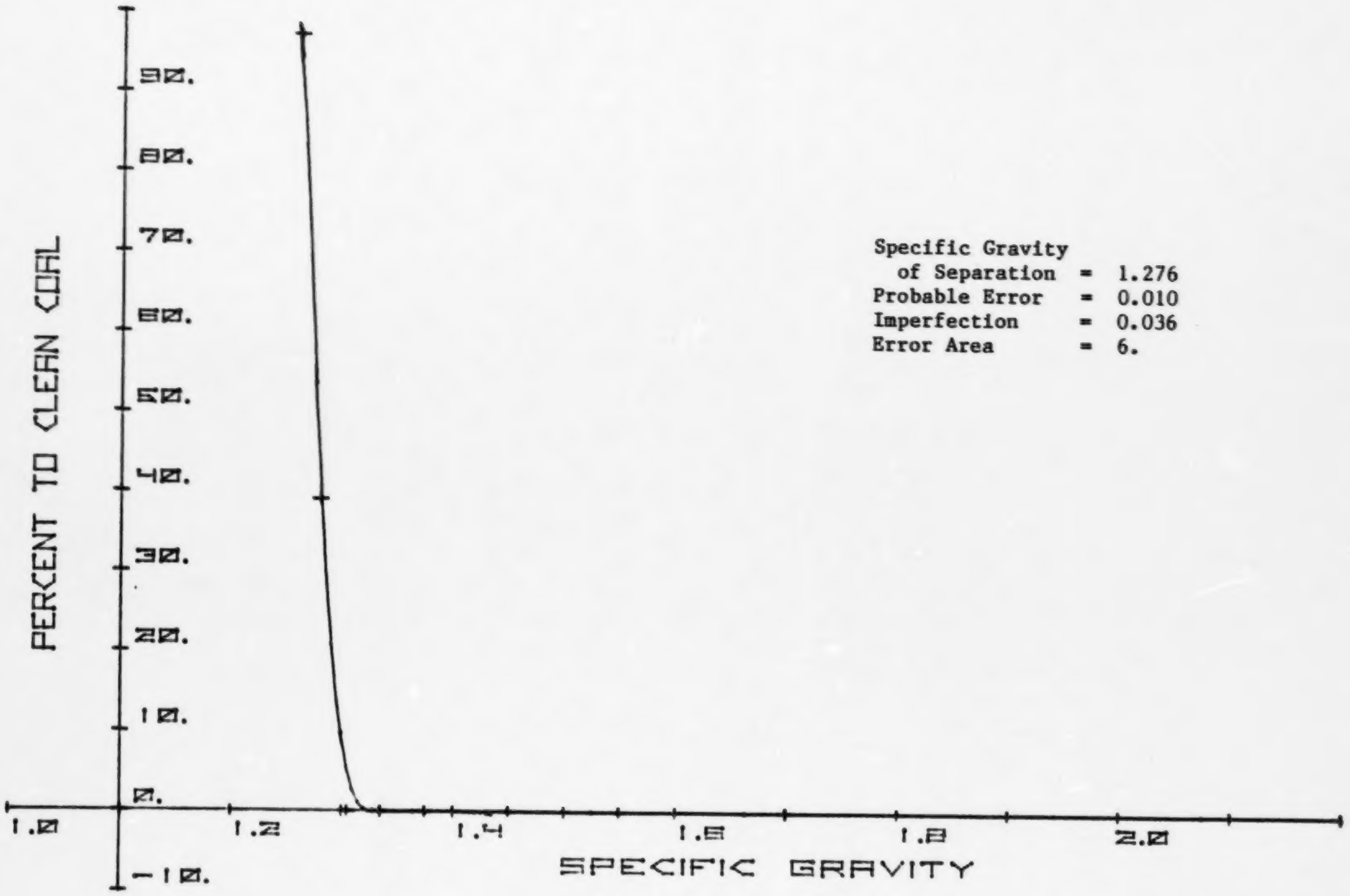
122936





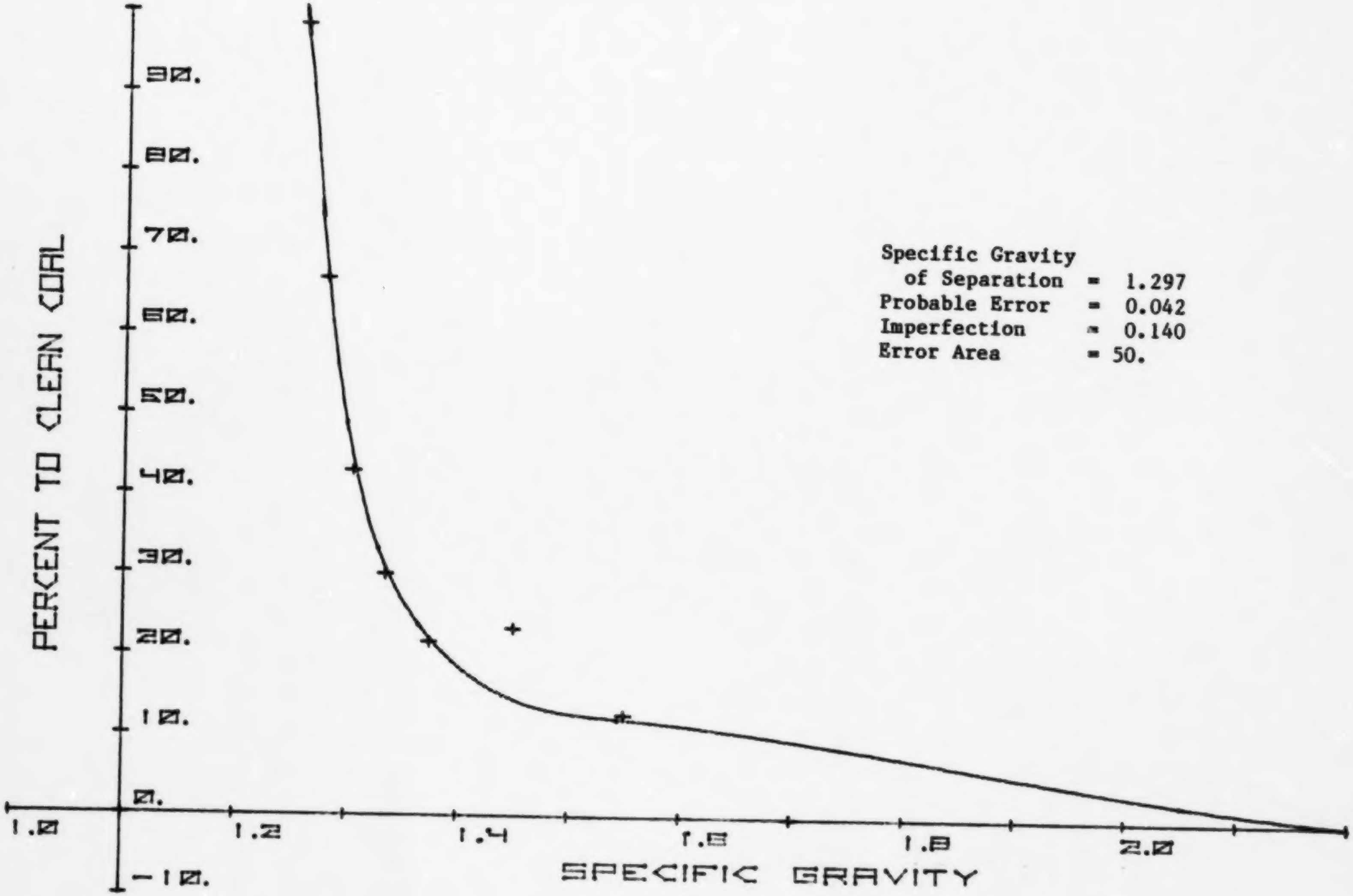
Specific Gravity
  of Separation = 1.273
Probable Error = 0.008
Imperfection = 0.030
Error Area = 7.

Figure A-105. Distribution Curve, Test No. 97 (9 x 14)



Specific Gravity
  of Separation = 1.276
Probable Error = 0.010
Imperfection = 0.036
Error Area = 6.

Figure A-106. Distribution Curve, Test No. 97 (14 x 28)



Specific Gravity
  of Separation = 1.297
Probable Error = 0.042
Imperfection = 0.140
Error Area = 50.

Figure A-107. Distribution Curve, Test No. 97 (28 x 100)

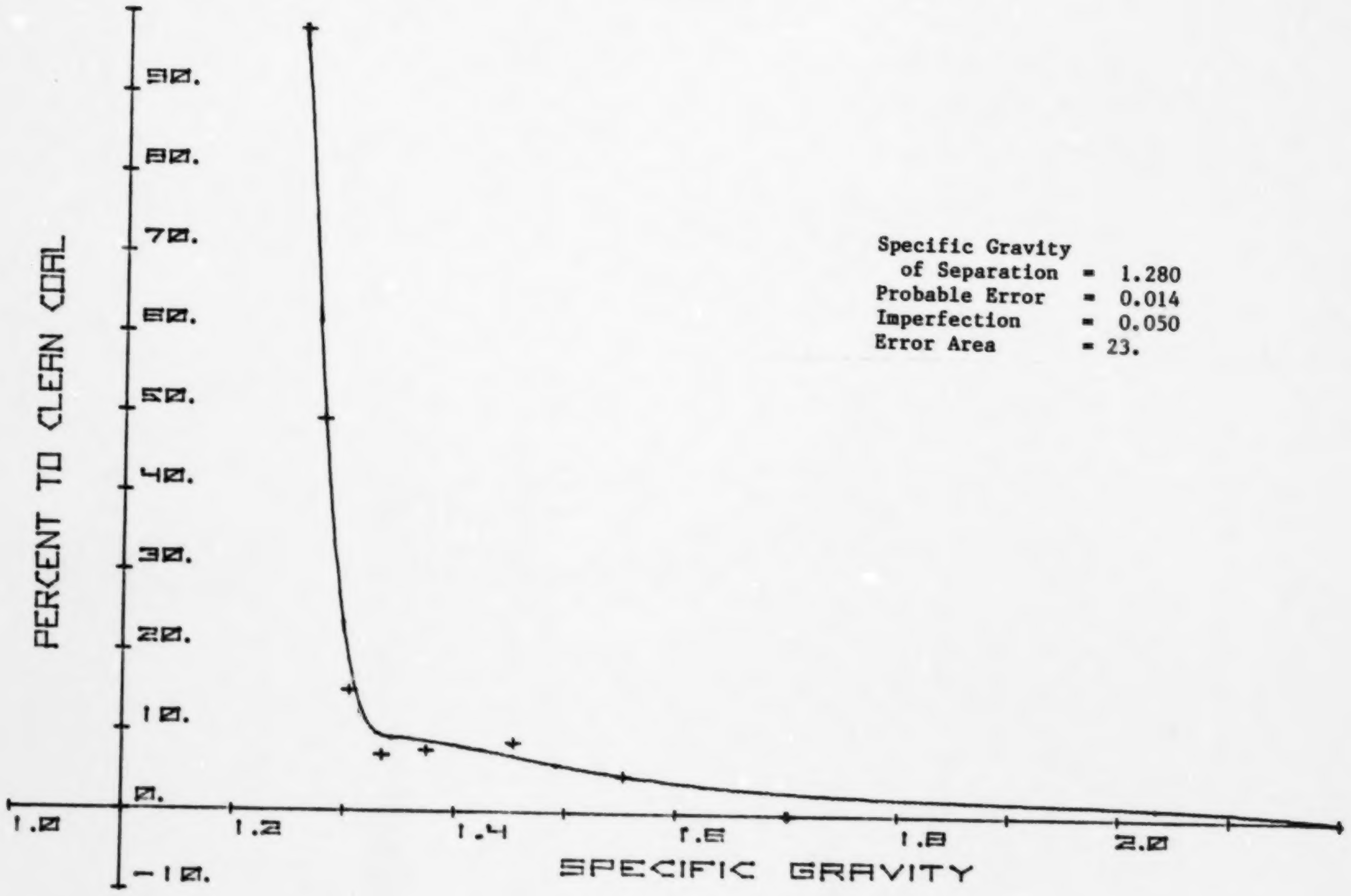
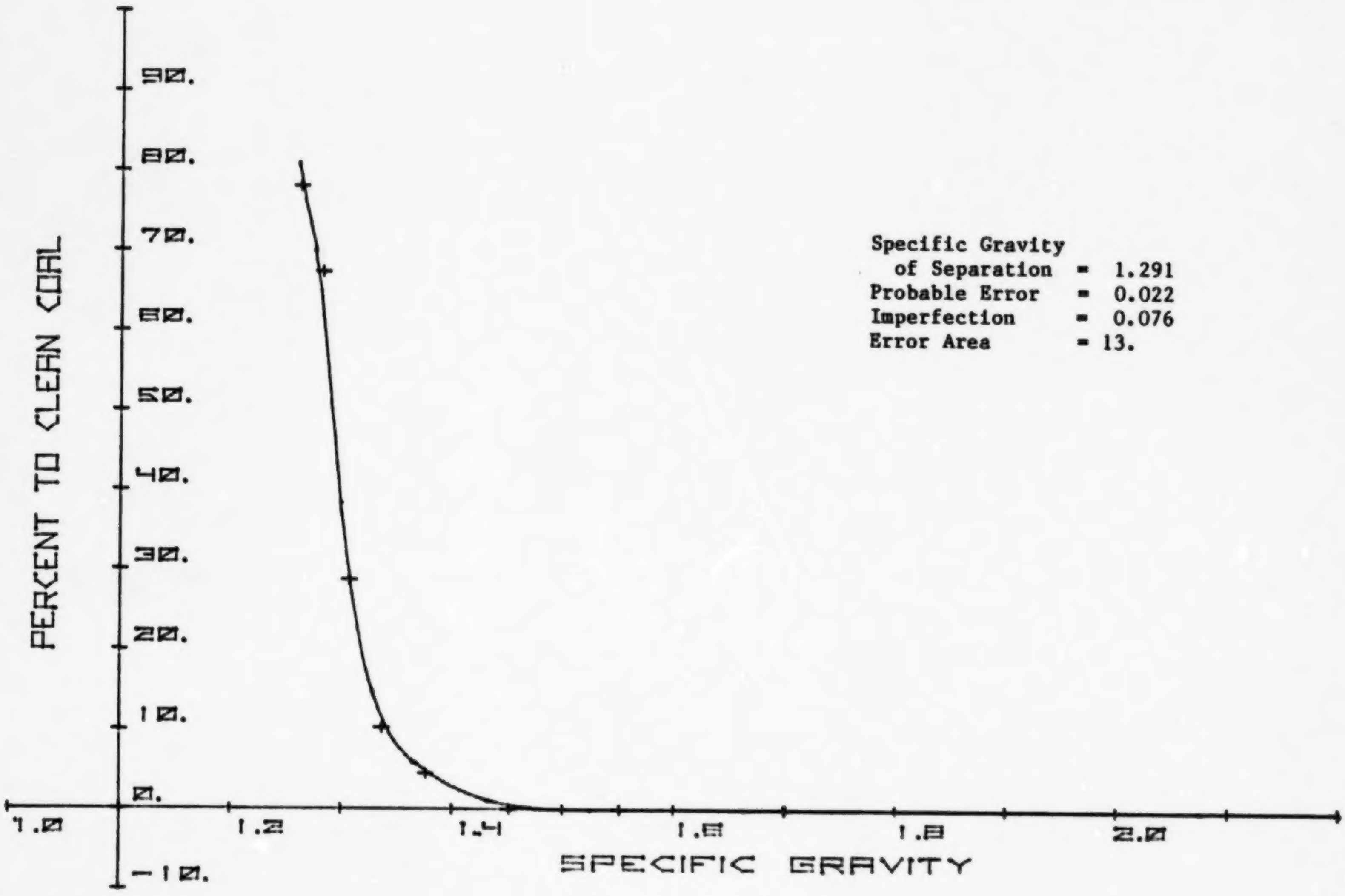


Figure A-108. Distribution Curve, Test No. 97 (9 x 100)

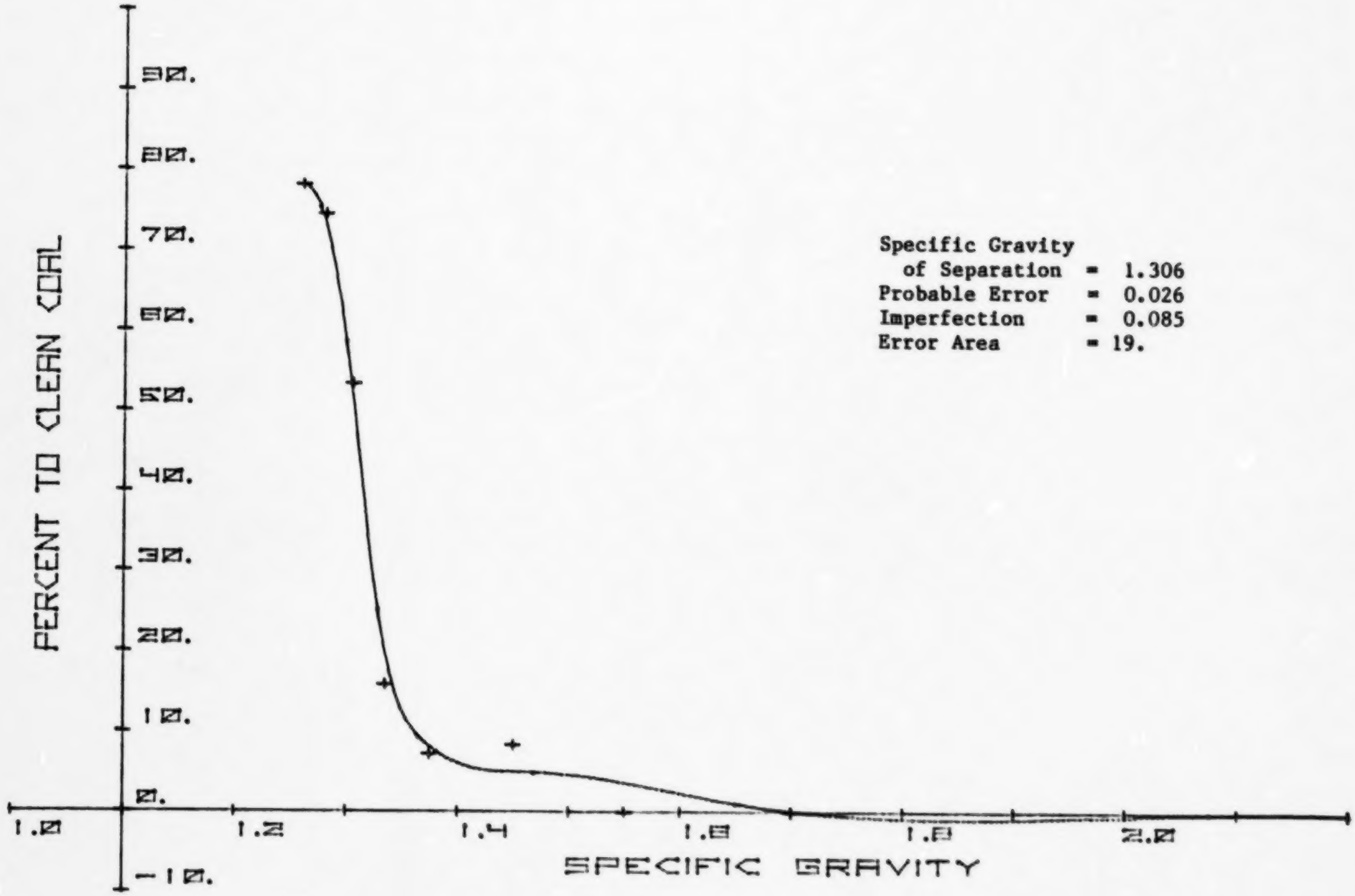
TABLE A-28. TEST 98 DATA SUMMARY

	9x14	14x28	28x100	9x100
SCREEN ANALYSIS, PERCENT:				
FEED	29.4	29.0	36.0	94.4
CLEAN COAL	22.3	29.3	41.8	93.4
REFUSE	34.6	28.7	31.7	95.1
ASH, PERCENT:				
FEED	23.0	20.8	21.9	21.9
CLEAN COAL	3.6	3.4	4.9	4.1
REFUSE	32.2	33.8	38.5	34.8
TOTAL SULFUR, PERCENT:				
FEED	2.90	2.95	3.70	3.22
CLEAN COAL	0.97	0.91	1.34	1.11
REFUSE	3.82	4.49	6.00	4.75
WEIGHT RECOVERY (YIELD)	32.2	42.9	49.3	42.0
THEORETICAL WEIGHT RECOVERY	38.4	54.6	66.6	57.3
WEIGHT RECOVERY EFFICIENCY	84.0	78.5	74.0	73.3
ASH ERROR	0.5	0.8	1.9	1.2
FLOAT IN REFUSE...% OF PRODUCT	15.0	24.6	26.3	22.8
SINK IN CLEAN COAL	14.2	10.3	9.8	13.9
TOTAL MISPLACED MATERIAL..% OF FEED	14.7	18.5	18.2	19.1
NEAR GRAVITY MATERIAL	64.2	70.1	66.8	67.4
SPECIFIC GRAVITY OF SEPARATION	1.291	1.306	1.313	1.306
PROBABLE ERROR	0.022	0.026	0.069	0.028
IMPERFECTION	0.076	0.085	0.220	0.092
ERROR AREA	13.	19.	65.	30.
DISTRIBUTION, PERCENT TO CLEANED COAL (SPECIFIC GRAVITY FRACTION):				
FLOAT 1.27	78.0	78.1	78.1	78.1
1.27-1.29	67.3	74.4	81.7	74.0
1.29-1.32	28.7	53.3	74.1	53.0
1.32-1.35	10.2	15.8	32.8	19.9
1.35-1.40	4.5	7.2	30.2	12.1
1.40-1.50	0.0	8.3	25.8	11.4
1.50-1.60	0.0	0.0	12.5	4.3
1.60-1.80	0.0	0.0	6.0	2.0
SINK 1.80	0.0	0.0	3.2	1.4



Specific Gravity
of Separation = 1.291
Probable Error = 0.022
Imperfection = 0.076
Error Area = 13.

Figure A-109. Distribution Curve, Test No. 98 (9 x 14)



Specific Gravity
  of Separation = 1.306
Probable Error = 0.026
Imperfection = 0.085
Error Area = 19.

Figure A-110. Distribution Curve, Test No. 98 (14 x 28)

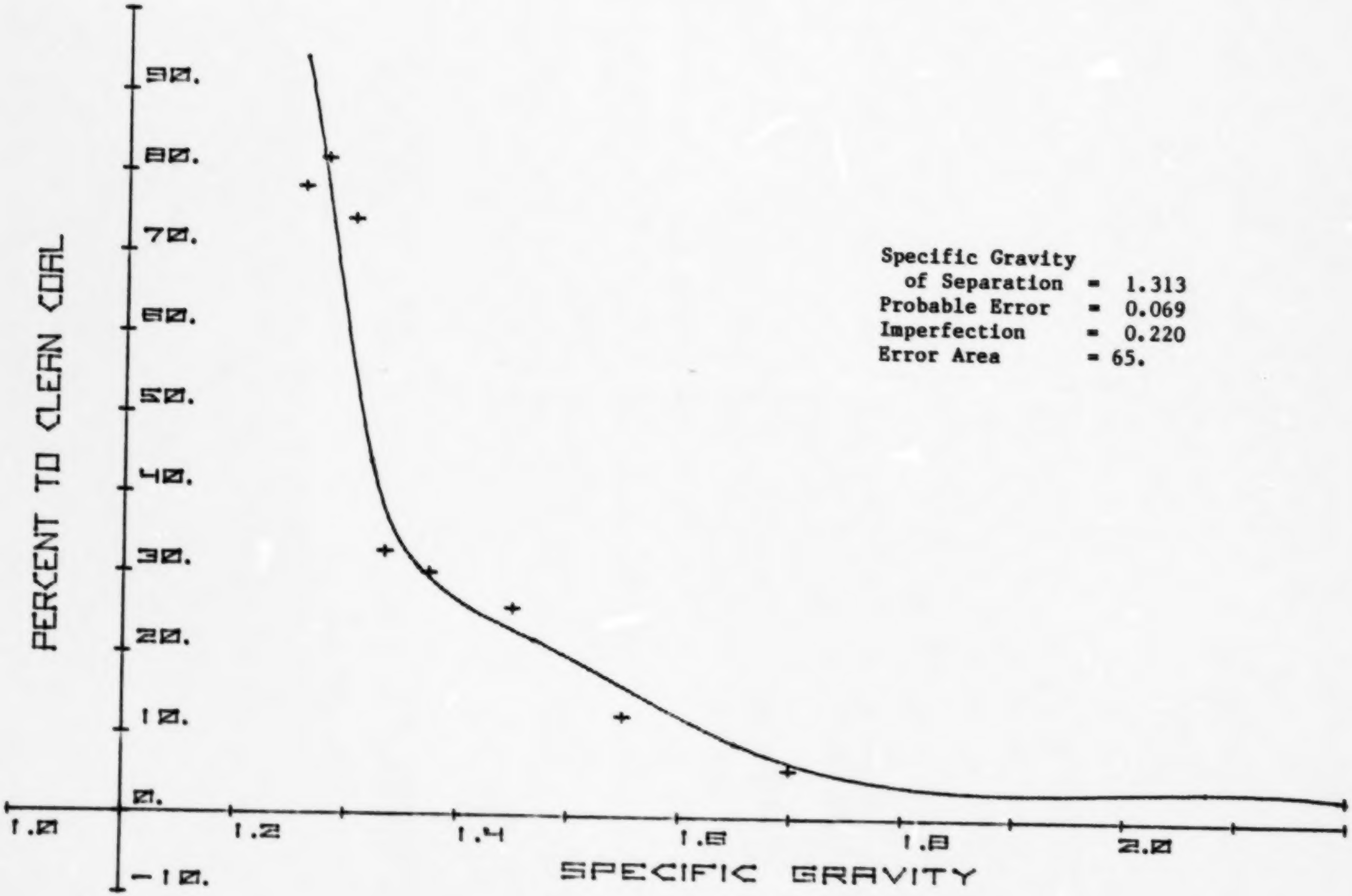
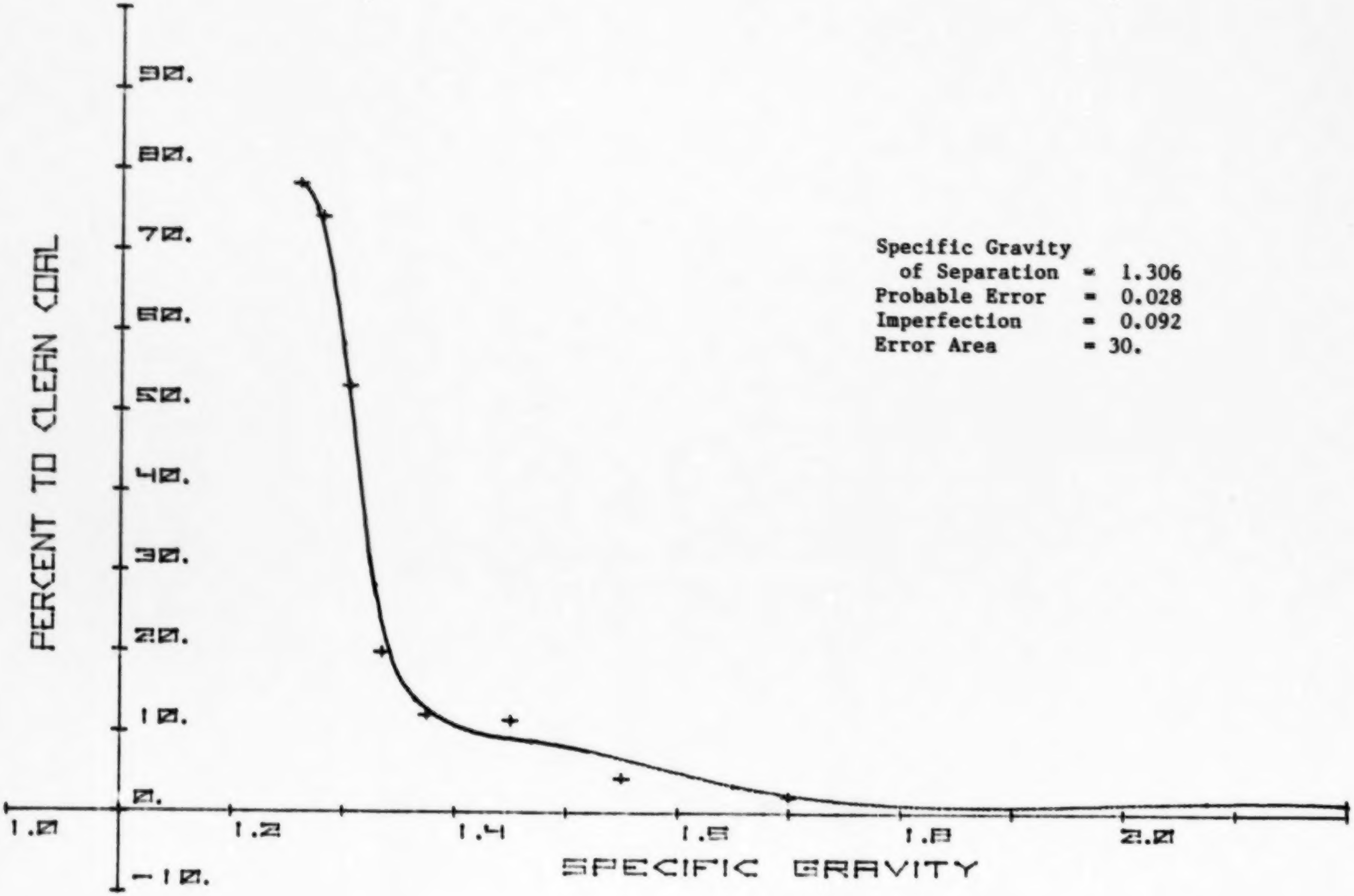


Figure A-111. Distribution Curve, Test No. 98 (28 x 100)

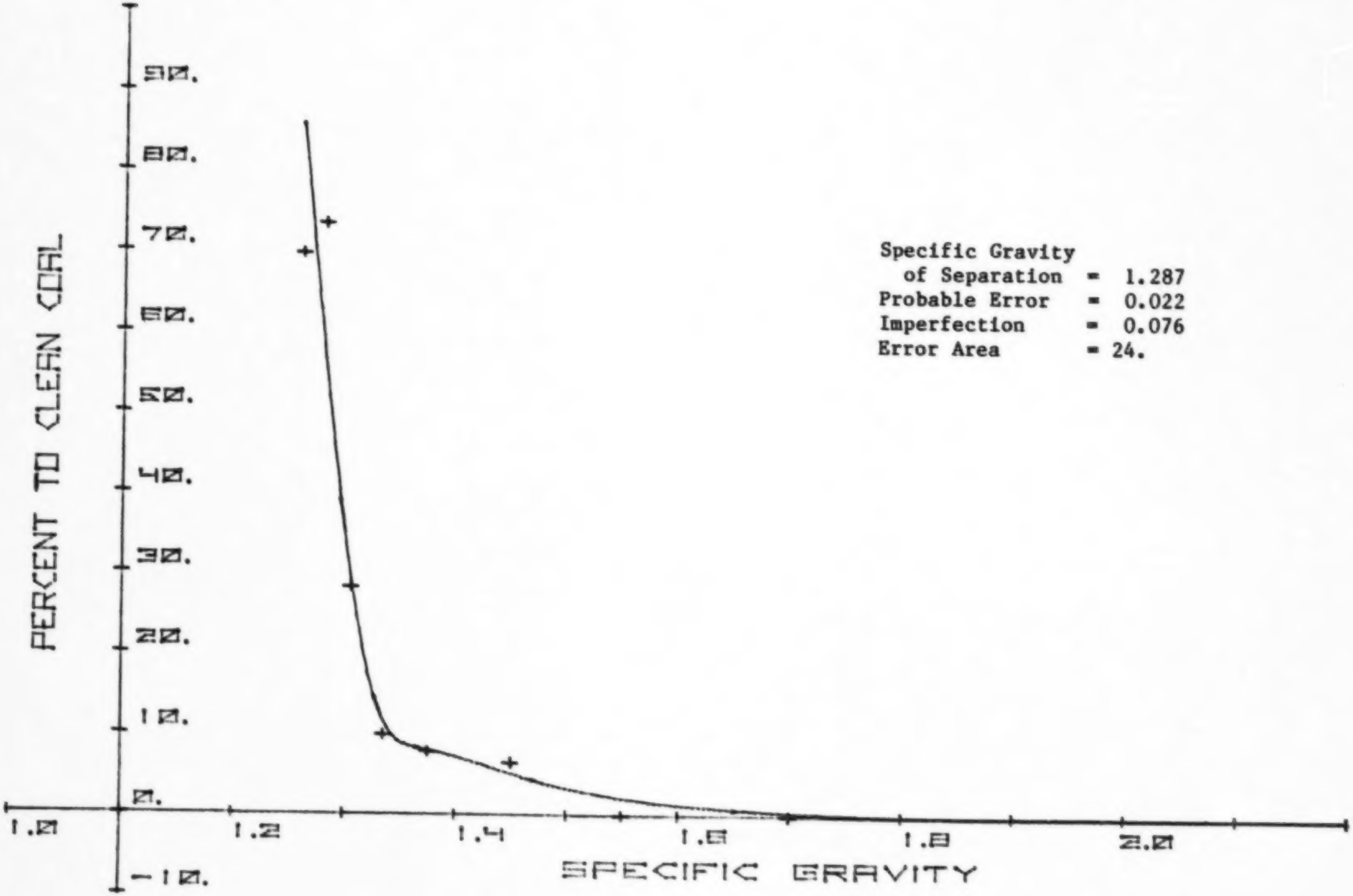


Specific Gravity
  of Separation = 1.306
Probable Error = 0.028
Imperfection = 0.092
Error Area = 30.

Figure A-112. Distribution Curve, Test No. 98 (9 x 100)

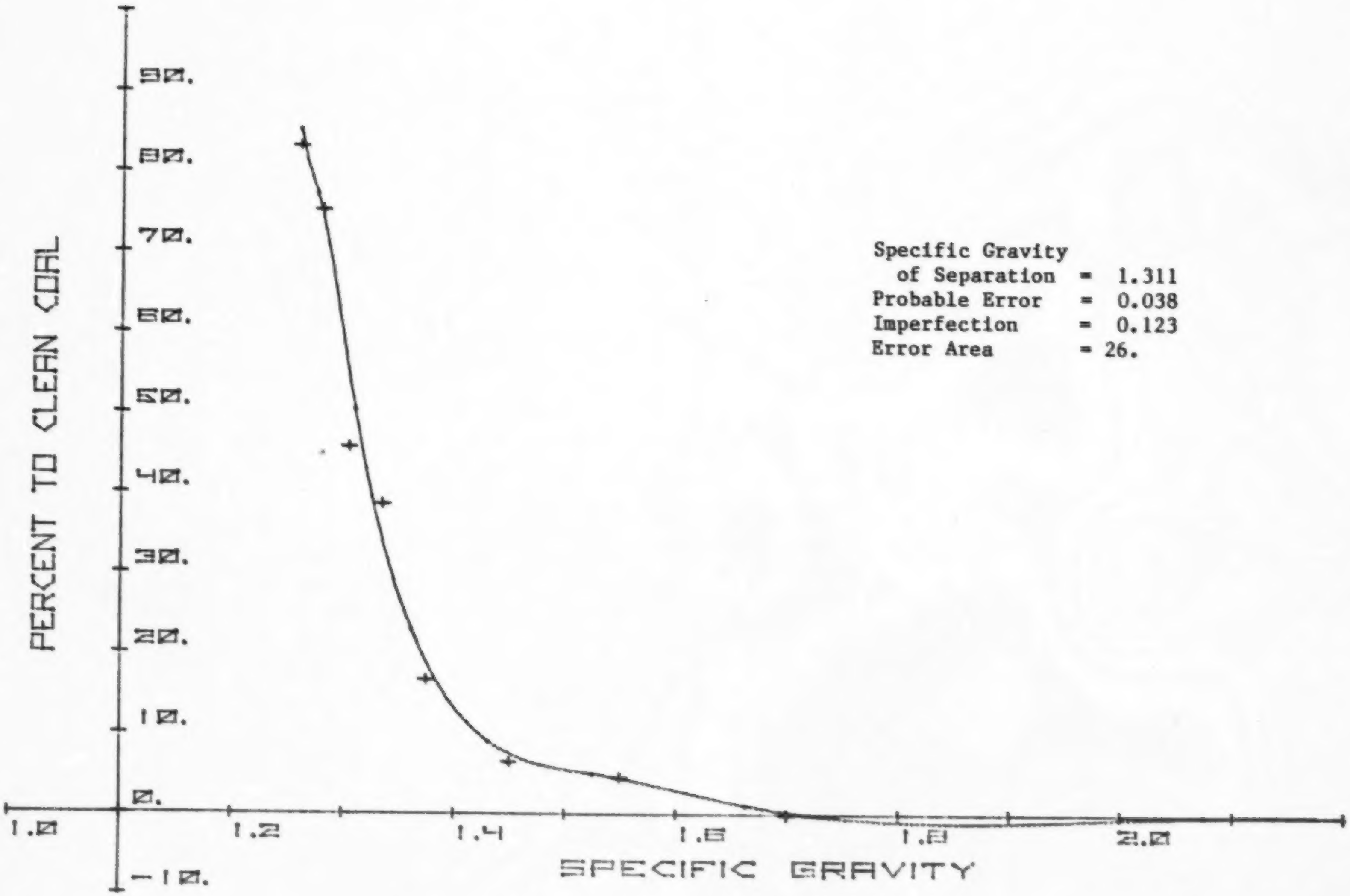
TABLE A-29. TEST 99 DATA SUMMARY

	9x14	14x28	28x100	9x100
SCREEN ANALYSIS, PERCENT:				
FEED	30.1	27.7	36.3	94.1
CLEAN COAL	18.9	26.9	46.4	92.2
REFUSE	40.8	28.4	26.7	95.9
ASH, PERCENT:				
FEED	23.1	19.0	17.6	19.8
CLEAN COAL	3.6	3.8	4.7	4.2
REFUSE	31.7	32.6	39.0	34.0
TOTAL SULFUR, PERCENT:				
FEED	2.86	2.88	3.41	3.08
CLEAN COAL	1.04	1.31	1.21	1.20
REFUSE	3.66	4.30	7.03	4.79
WEIGHT RECOVERY (YIELD)	30.5	47.3	62.2	47.7
THEORETICAL WEIGHT RECOVERY	38.8	60.3	75.7	62.3
WEIGHT RECOVERY EFFICIENCY	78.6	78.6	82.2	76.5
ASH ERROR	0.6	1.0	1.6	1.3
FLOAT IN REFUSE...% OF PRODUCT	16.0	21.6	35.7	23.9
SINK IN CLEAN COAL	10.5	13.0	4.0	15.4
TOTAL MISPLACED MATERIAL..% OF FEED	14.3	17.5	16.0	19.8
NEAR GRAVITY MATERIAL	64.4	71.9	22.9	71.1
SPECIFIC GRAVITY OF SEPARATION	1.287	1.311	1.405	1.319
PROBABLE ERROR	0.022	0.038	0.100	0.063
IMPERFECTION	0.076	0.123	0.247	0.197
ERROR AREA	24.	26.	71.	44.
DISTRIBUTION, PERCENT TO CLEANED COAL (SPECIFIC GRAVITY FRACTION):				
FLOAT 1.27	69.7	83.2	83.5	80.8
1.27-1.29	73.4	75.2	88.9	79.9
1.29-1.32	28.2	45.7	85.1	58.6
1.32-1.35	9.9	38.6	78.2	41.3
1.35-1.40	7.8	16.7	58.3	29.3
1.40-1.50	6.5	6.5	37.8	19.0
1.50-1.60	0.0	4.6	20.2	7.3
1.60-1.80	0.0	0.0	10.5	3.1
SINK 1.80	0.0	0.0	3.0	1.1



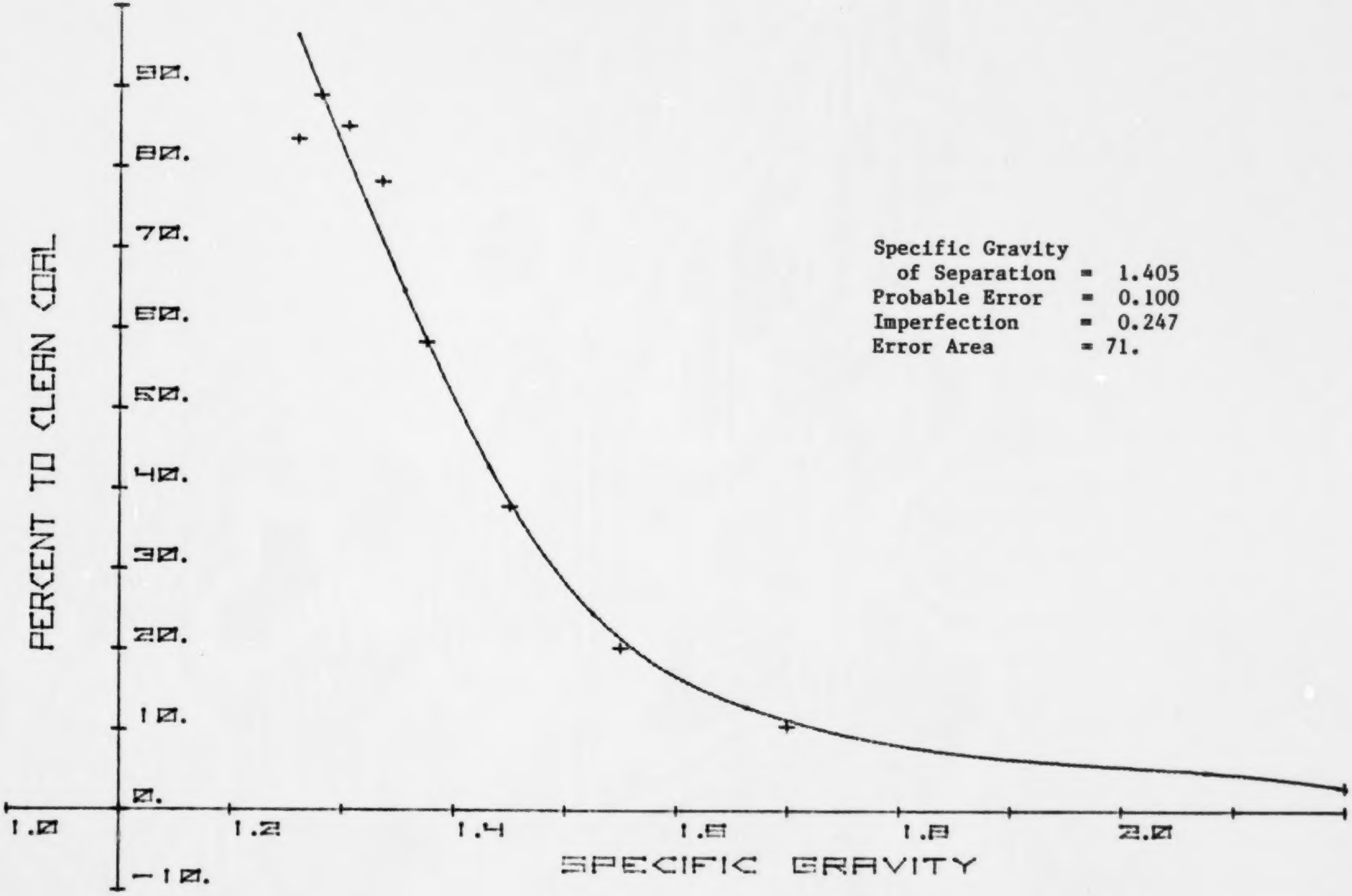
Specific Gravity
  of Separation = 1.287
Probable Error = 0.022
Imperfection = 0.076
Error Area = 24.

Figure A-113. Distribution Curve, Test No. 99 (9 x 14)



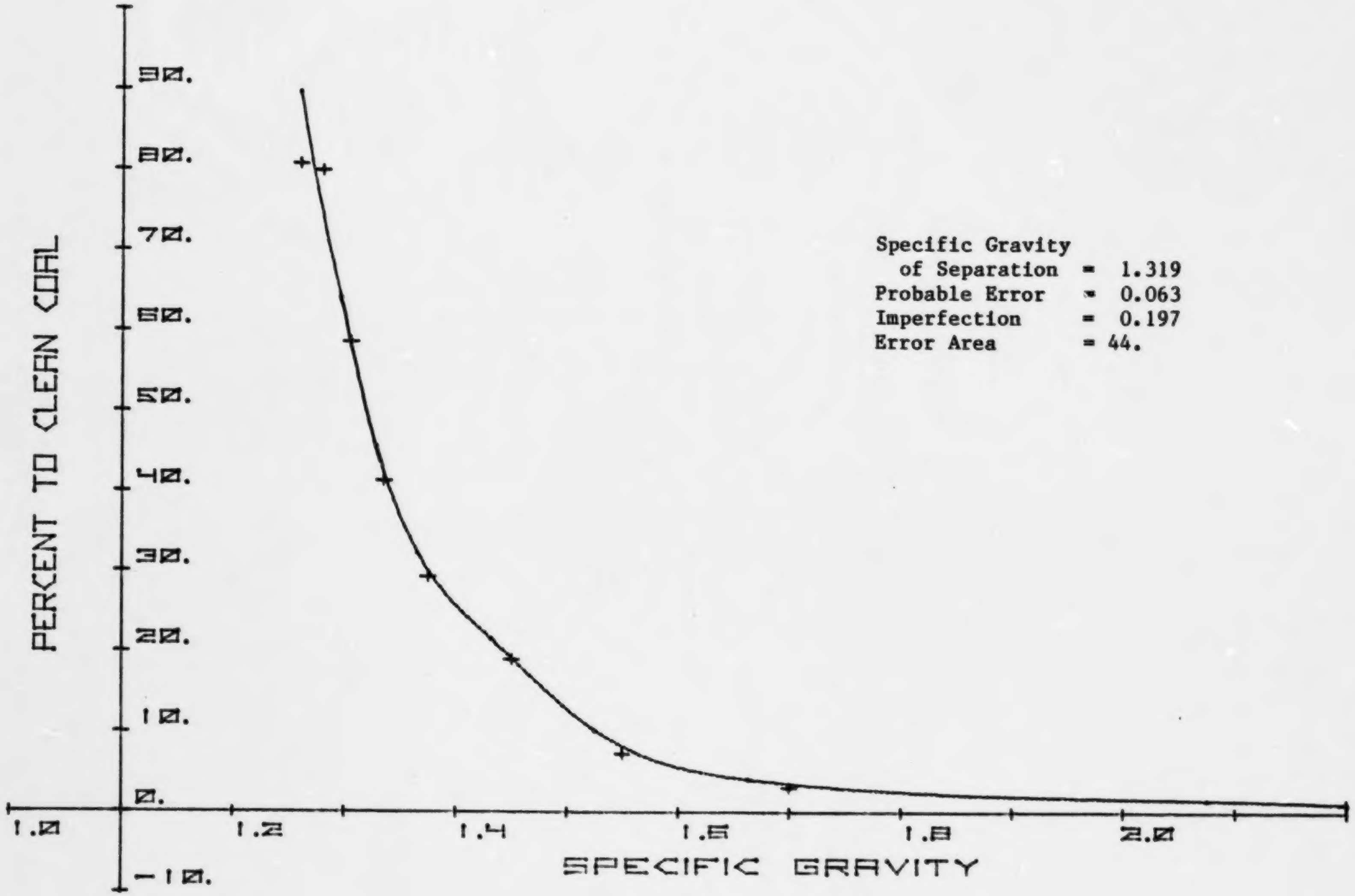
Specific Gravity
of Separation = 1.311
Probable Error = 0.038
Imperfection = 0.123
Error Area = 26.

Figure A-114. Distribution Curve, Test No. 99 (14 x 28)



Specific Gravity
of Separation = 1.405
Probable Error = 0.100
Imperfection = 0.247
Error Area = 71.

Figure A-115. Distribution Curve, Test No. 99 (28 x 100)

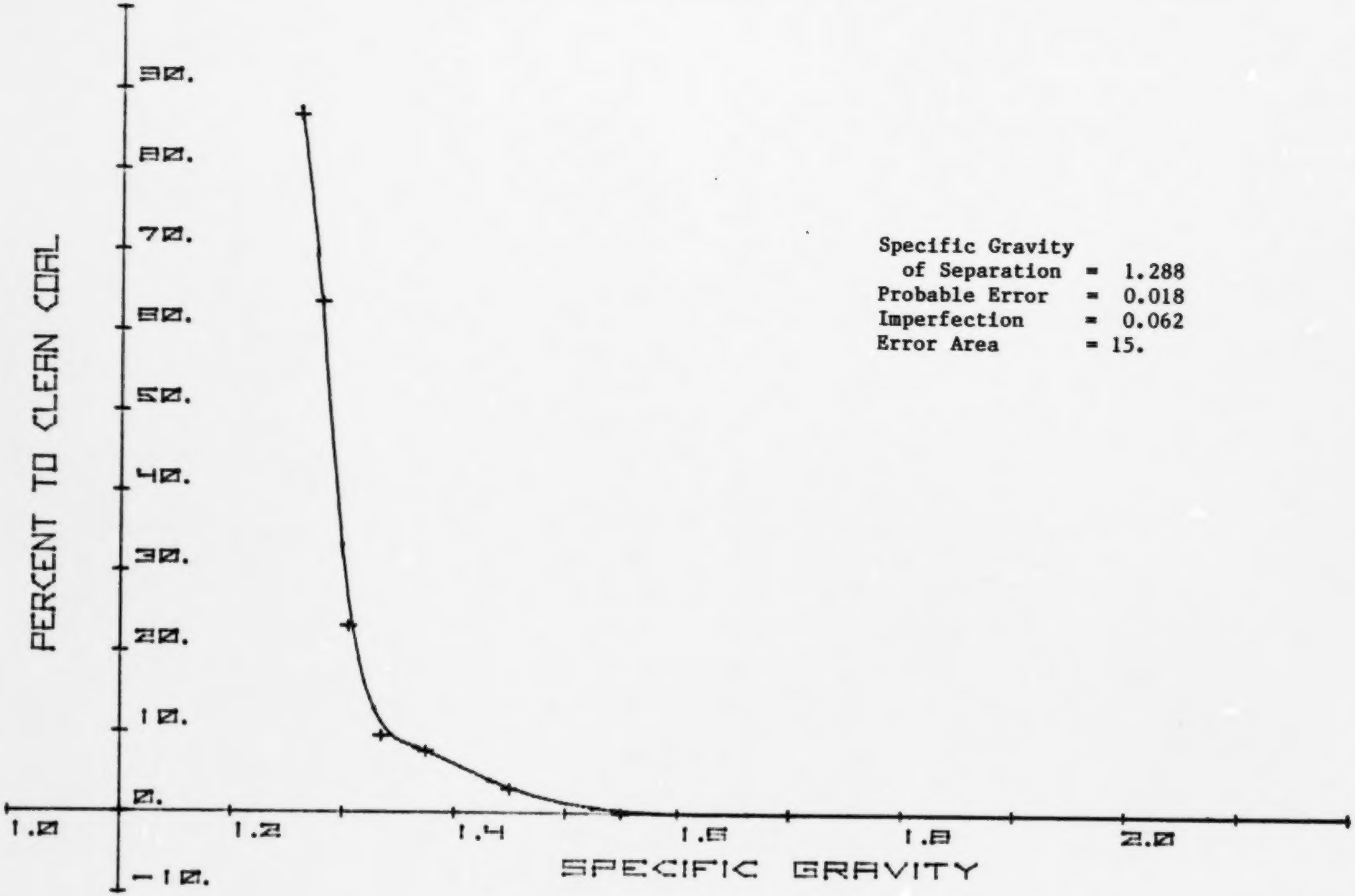


Specific Gravity
of Separation = 1.319
Probable Error = 0.063
Imperfection = 0.197
Error Area = 44.

Figure A-116. Distribution Curve, Test No. 99 (9 x 100)

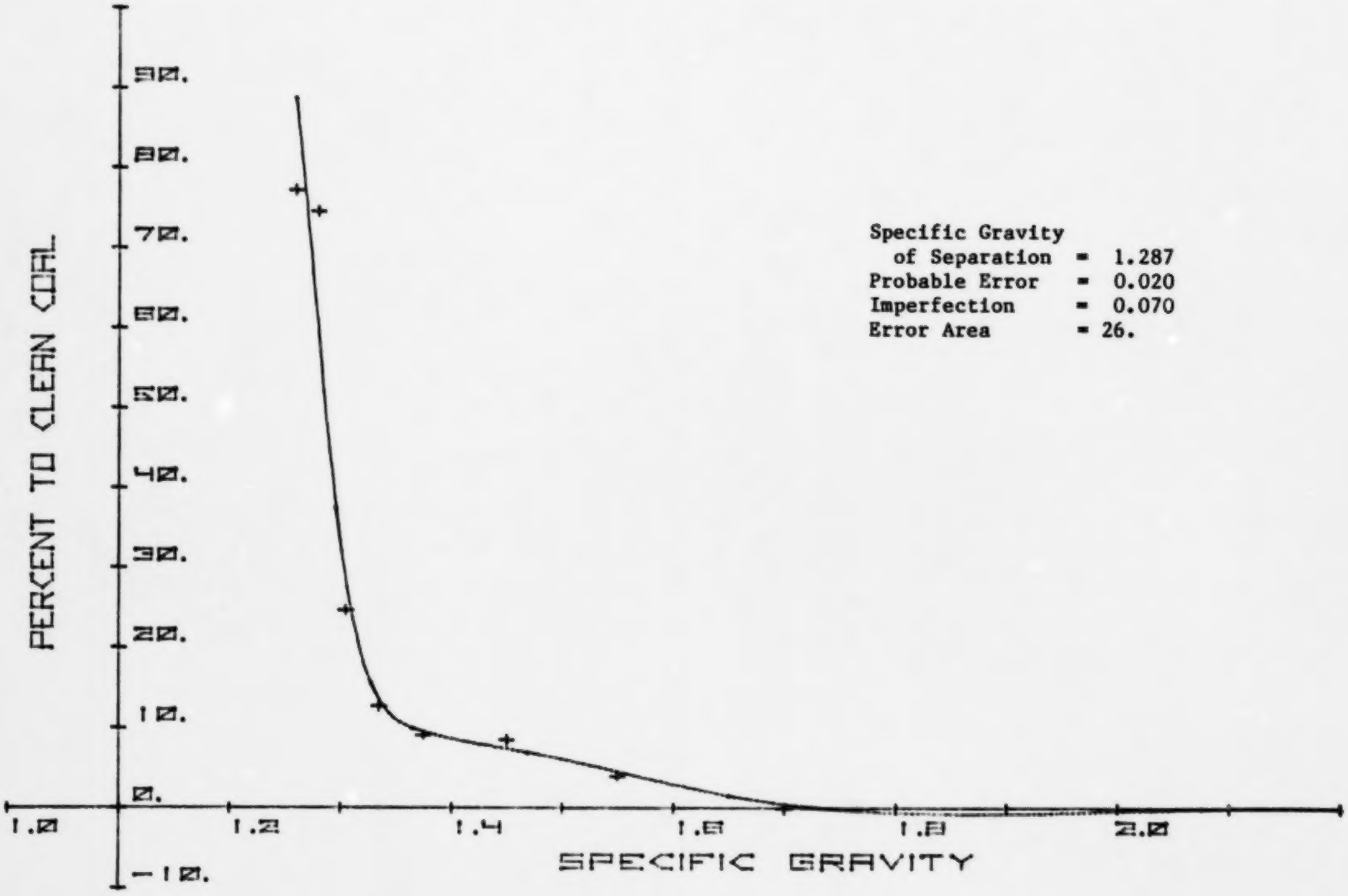
TABLE A-30. TEST 100 DATA SUMMARY

	9x14	14x28	28x100	9x100
SCREEN ANALYSIS, PERCENT:				
FEED	28.0	27.8	37.4	93.3
CLEAN COAL	17.6	25.9	47.8	91.4
REFUSE	35.8	29.3	29.6	94.7
ASH, PERCENT:				
FEED	21.9	19.5	18.5	19.8
CLEAN COAL	3.1	3.5	4.6	4.0
REFUSE	28.9	30.3	35.5	31.4
TOTAL SULFUR, PERCENT:				
FEED	2.66	2.44	3.22	2.82
CLEAN COAL	0.91	0.79	1.01	0.93
REFUSE	3.31	3.54	5.92	4.20
WEIGHT RECOVERY (YIELD)	27.1	40.1	55.0	42.2
THEORETICAL WEIGHT RECOVERY	41.2	54.2	71.3	61.2
WEIGHT RECOVERY EFFICIENCY	65.9	73.9	77.2	69.0
ASH ERROR	0.6	0.9	1.8	1.4
FLOAT IN REFUSE...% OF PRODUCT	12.3	21.6	27.2	20.0
SINK IN CLEAN COAL	26.5	5.3	10.3	11.5
TOTAL MISPLACED MATERIAL..% OF FEED	16.1	15.1	17.9	16.4
NEAR GRAVITY MATERIAL	59.7	69.3	71.7	67.2
SPECIFIC GRAVITY OF SEPARATION	1.288	1.287	1.329	1.296
PROBABLE ERROR	0.018	0.020	0.096	0.025
IMPERFECTION	0.062	0.070	0.292	0.083
ERROR AREA	15.	26.	68.	34.
DISTRIBUTION, PERCENT TO CLEANED COAL (SPECIFIC GRAVITY FRACTION):				
FLOAT 1.27	86.8	77.2	86.1	83.6
1.27-1.29	63.5	74.5	84.3	76.4
1.29-1.32	23.2	24.7	60.3	35.0
1.32-1.35	9.6	12.7	47.7	21.7
1.35-1.40	7.7	9.1	32.8	17.8
1.40-1.50	3.0	8.5	27.5	13.0
1.50-1.60	0.0	4.0	14.9	7.0
1.60-1.80	0.0	0.0	9.7	2.7
SINK 1.80	0.0	0.0	3.1	1.2



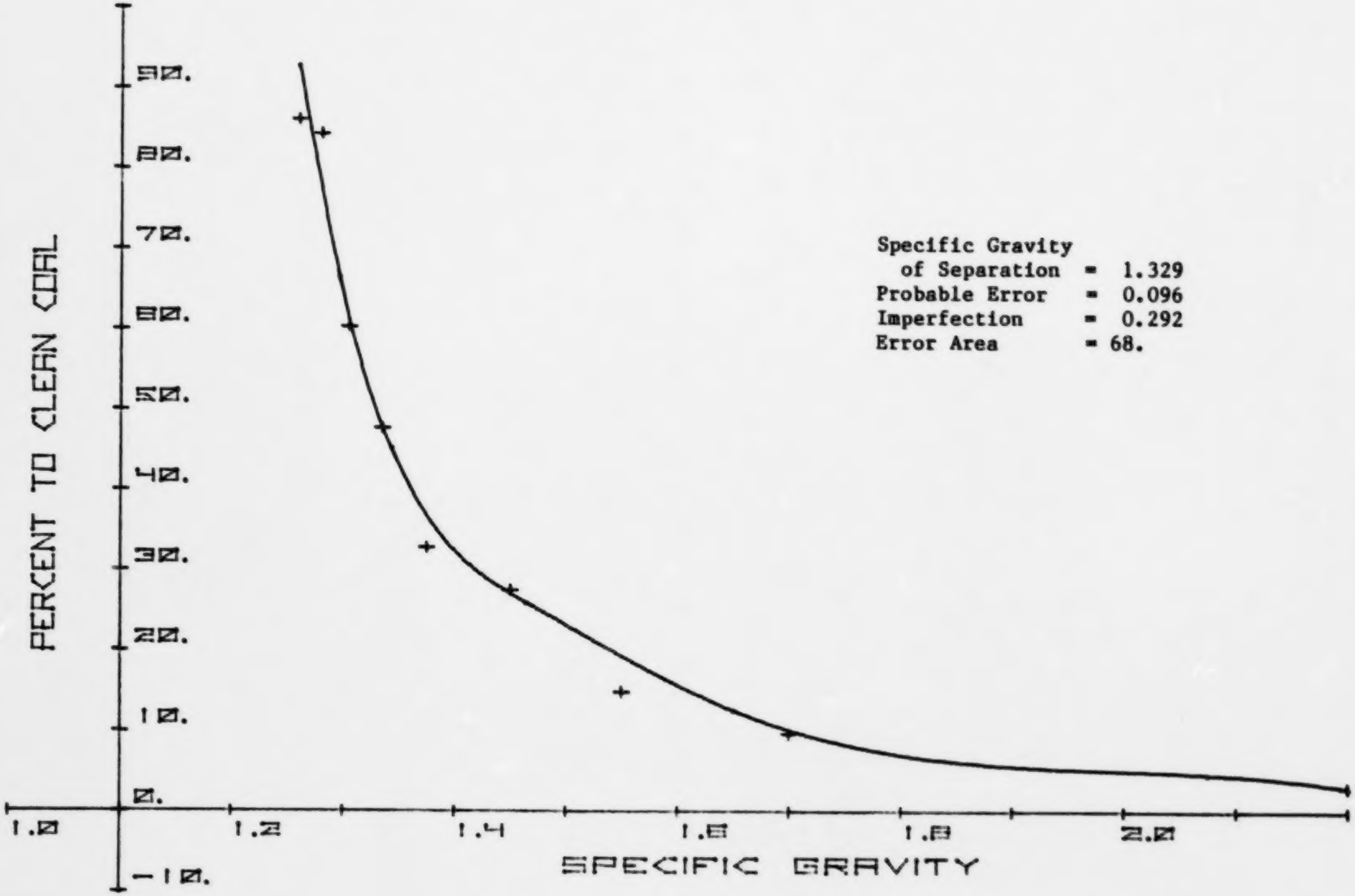
Specific Gravity
of Separation = 1.288
Probable Error = 0.018
Imperfection = 0.062
Error Area = 15.

Figure A-117. Distribution Curve, Test No. 100 (9 x 14)



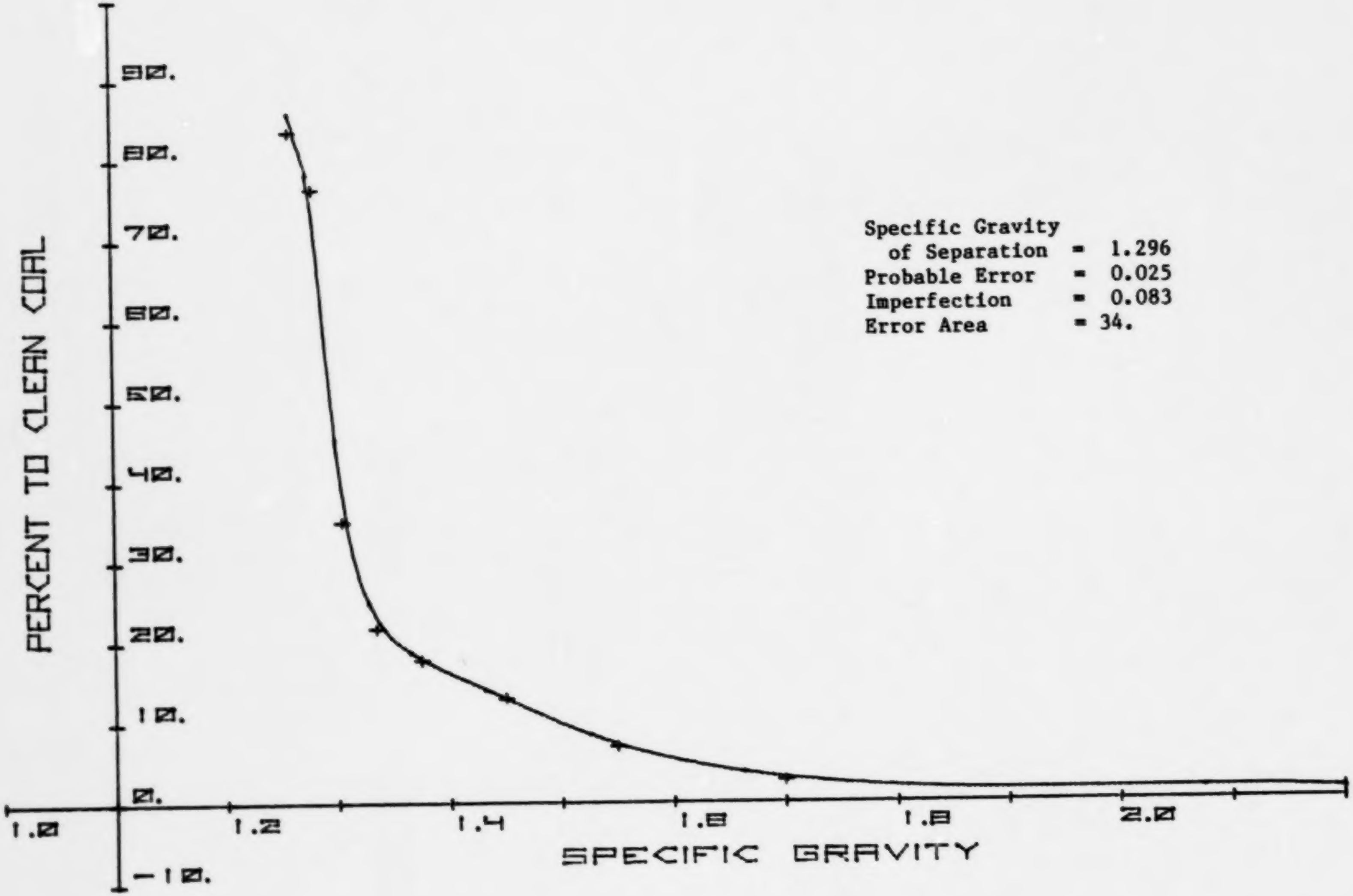
Specific Gravity
of Separation = 1.287
Probable Error = 0.020
Imperfection = 0.070
Error Area = 26.

Figure A-118. Distribution Curve, Test No. 100 (14 x 28)



Specific Gravity
of Separation = 1.329
Probable Error = 0.096
Imperfection = 0.292
Error Area = 68.

Figure A-119. Distribution Curve, Test No. 100 (28 x 100)

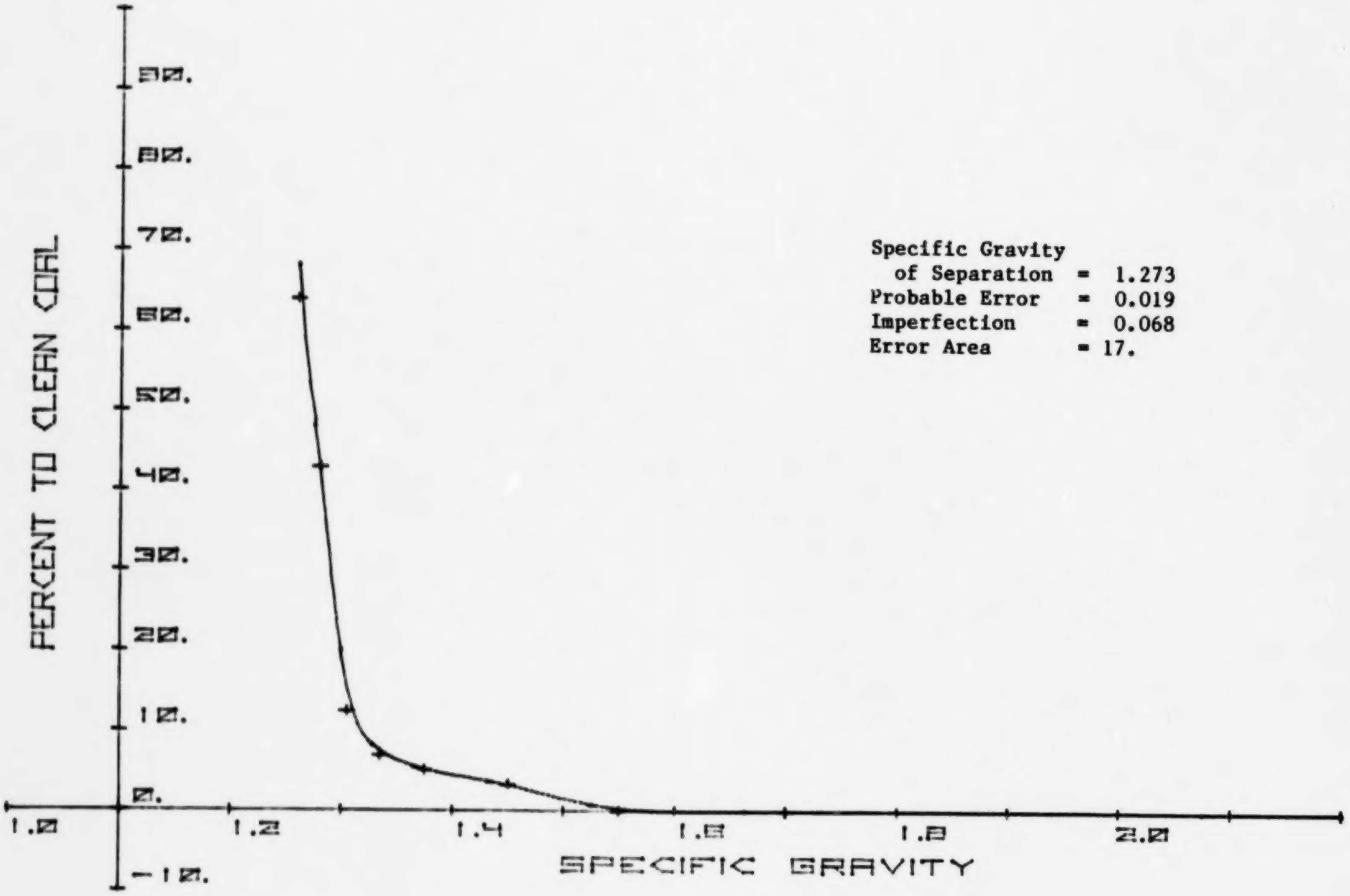


Specific Gravity
  of Separation = 1.296
Probable Error = 0.025
Imperfection = 0.083
Error Area = 34.

Figure A-120. Distribution Curve, Test No. 100 (9 x 100)

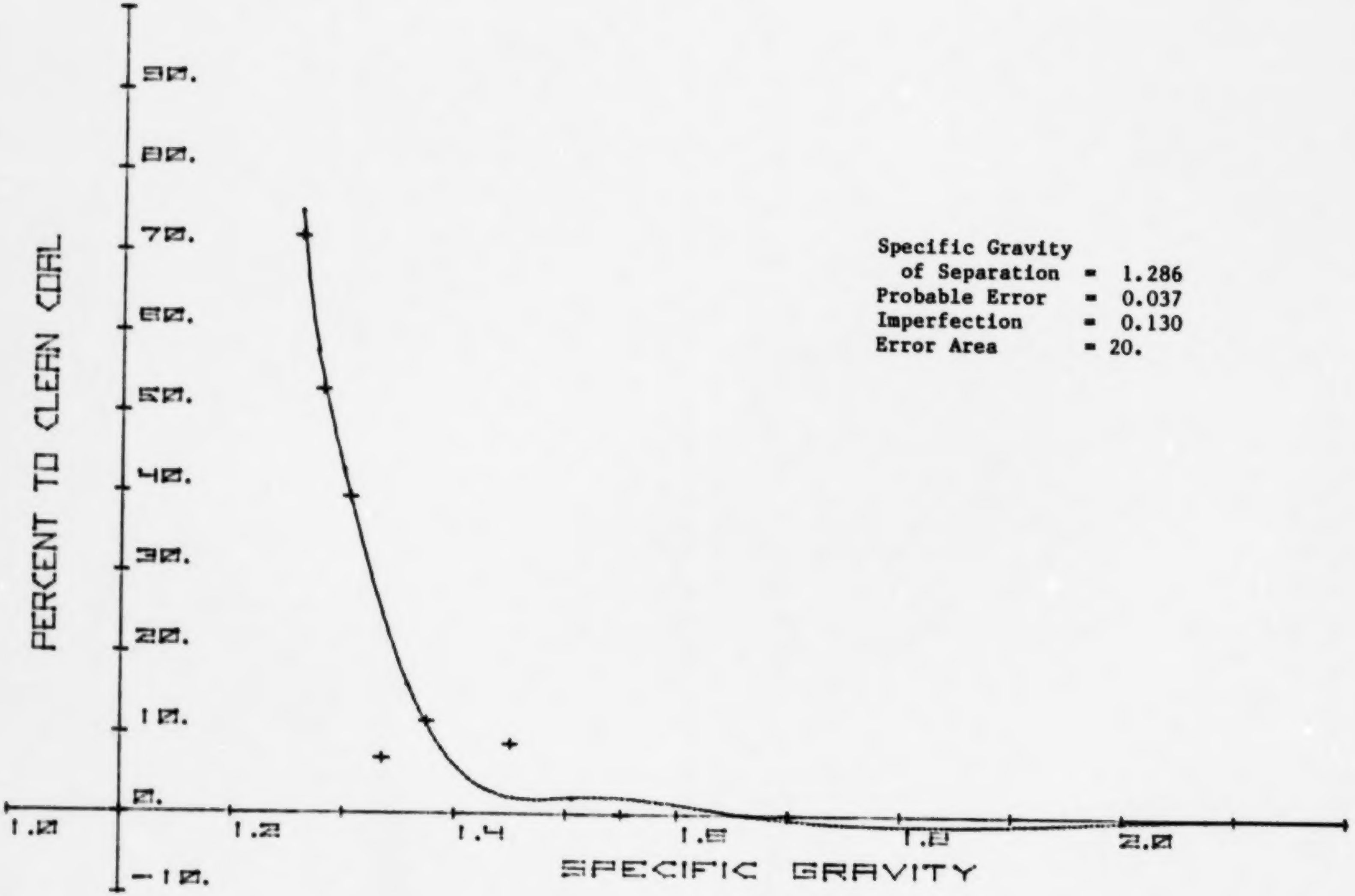
TABLE A-31. TEST 101 DATA SUMMARY

	9x14	14x28	28x100	9x100
SCREEN ANALYSIS, PERCENT:				
FEED	35.5	29.3	29.7	94.6
CLEAN COAL	20.4	27.5	42.9	90.8
REFUSE	43.8	30.3	22.5	96.6
ASH, PERCENT:				
FEED	25.5	18.9	19.1	21.4
CLEAN COAL	4.1	3.6	5.2	4.5
REFUSE	30.9	26.5	33.8	30.2
TOTAL SULFUR, PERCENT:				
FEED	2.40	2.12	2.41	2.32
CLEAN COAL	0.96	0.95	1.03	0.99
REFUSE	2.77	2.71	3.87	3.01
WEIGHT RECOVERY (YIELD)	20.4	33.3	51.2	34.1
THEORETICAL WEIGHT RECOVERY	40.3	51.6	71.0	56.8
WEIGHT RECOVERY EFFICIENCY	50.7	64.6	72.2	60.0
ASH ERROR	1.5	1.5	2.4	2.0
FLOAT IN REFUSE...% OF PRODUCT	12.2	19.8	24.1	16.3
SINK IN CLEAN COAL	21.6	16.0	14.6	21.2
TOTAL MISPLACED MATERIAL..% OF FEED	14.1	18.5	19.2	18.0
NEAR GRAVITY MATERIAL	53.3	66.4	70.1	62.0
SPECIFIC GRAVITY OF SEPARATION	1.273	1.286	1.328	1.289
PROBABLE ERROR	0.019	0.037	0.063	0.029
IMPERFECTION	0.068	0.130	0.191	0.099
ERROR AREA	17.	20.	61.	30.
DISTRIBUTION, PERCENT TO CLEANED COAL (SPECIFIC GRAVITY FRACTION):				
FLOAT 1.27	63.9	71.8	84.9	74.3
1.27-1.29	42.9	52.7	76.6	59.3
1.29-1.32	12.4	39.4	59.0	34.1
1.32-1.35	6.9	6.9	47.1	15.5
1.35-1.40	5.1	11.5	28.9	13.9
1.40-1.50	3.3	8.7	21.3	9.9
1.50-1.60	0.0	0.0	12.8	3.6
1.60-1.80	0.0	0.0	8.3	2.0
SINK 1.80	0.0	0.0	3.1	0.9



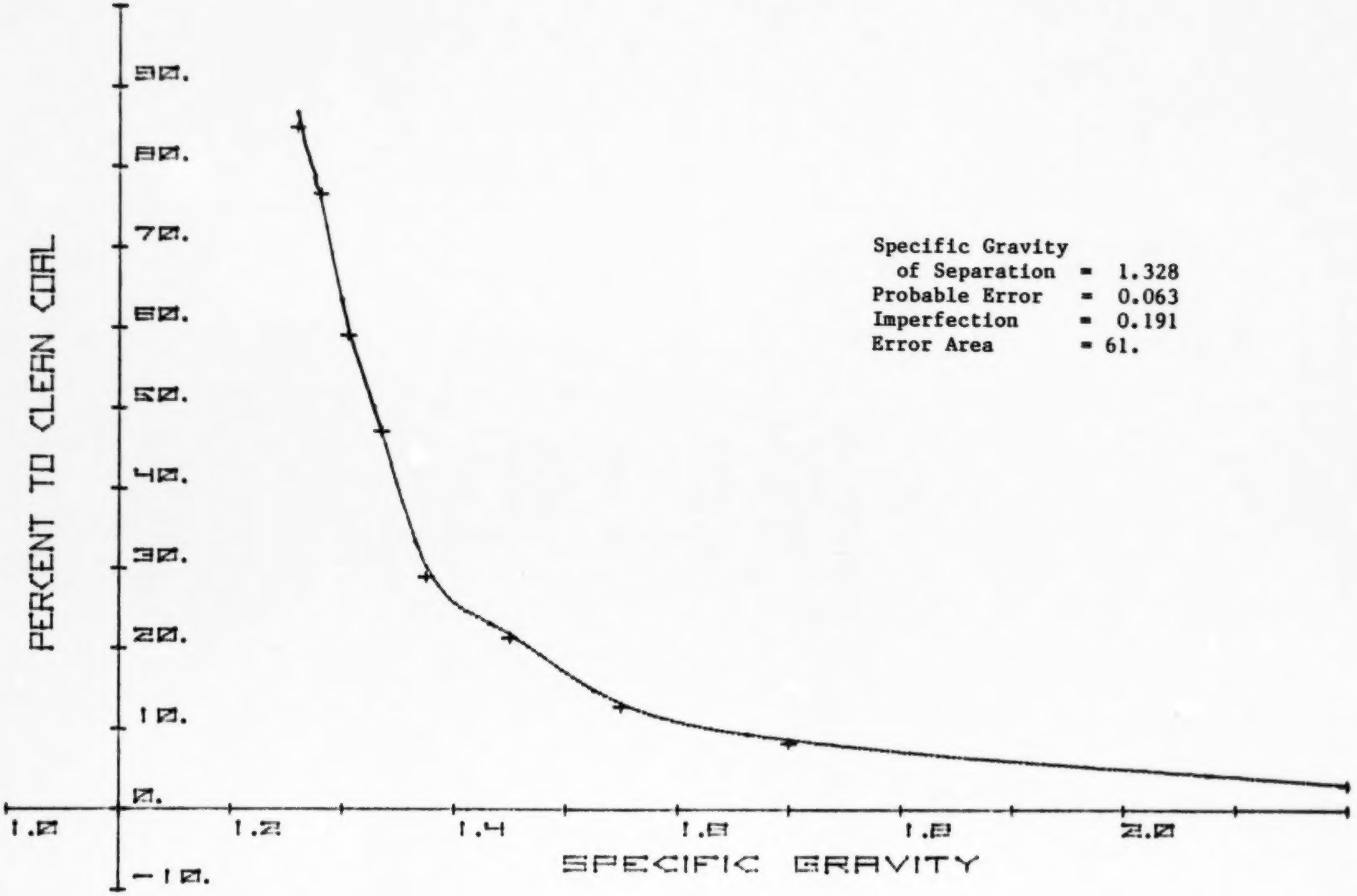
Specific Gravity
  of Separation = 1.273
Probable Error = 0.019
Imperfection = 0.068
Error Area = 17.

Figure A-121. Distribution Curve, Test No. 101 (9 x 14)



Specific Gravity
of Separation = 1.286
Probable Error = 0.037
Imperfection = 0.130
Error Area = 20.

Figure A-122. Distribution Curve, Test No. 101 (14 x 28)



Specific Gravity
of Separation = 1.328
Probable Error = 0.063
Imperfection = 0.191
Error Area = 61.

Figure A-123. Distribution Curve, Test No. 101 (28 x 100)

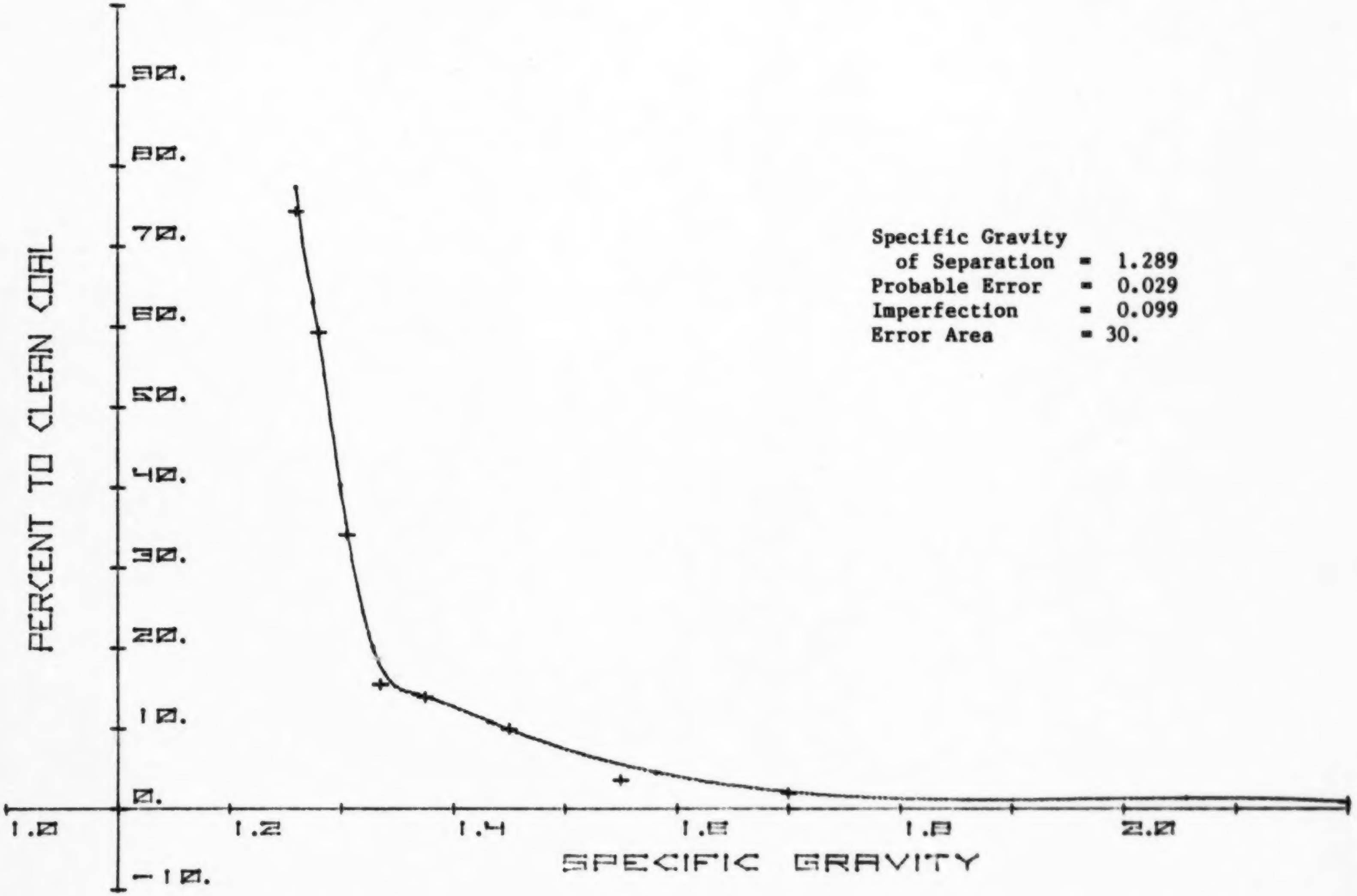
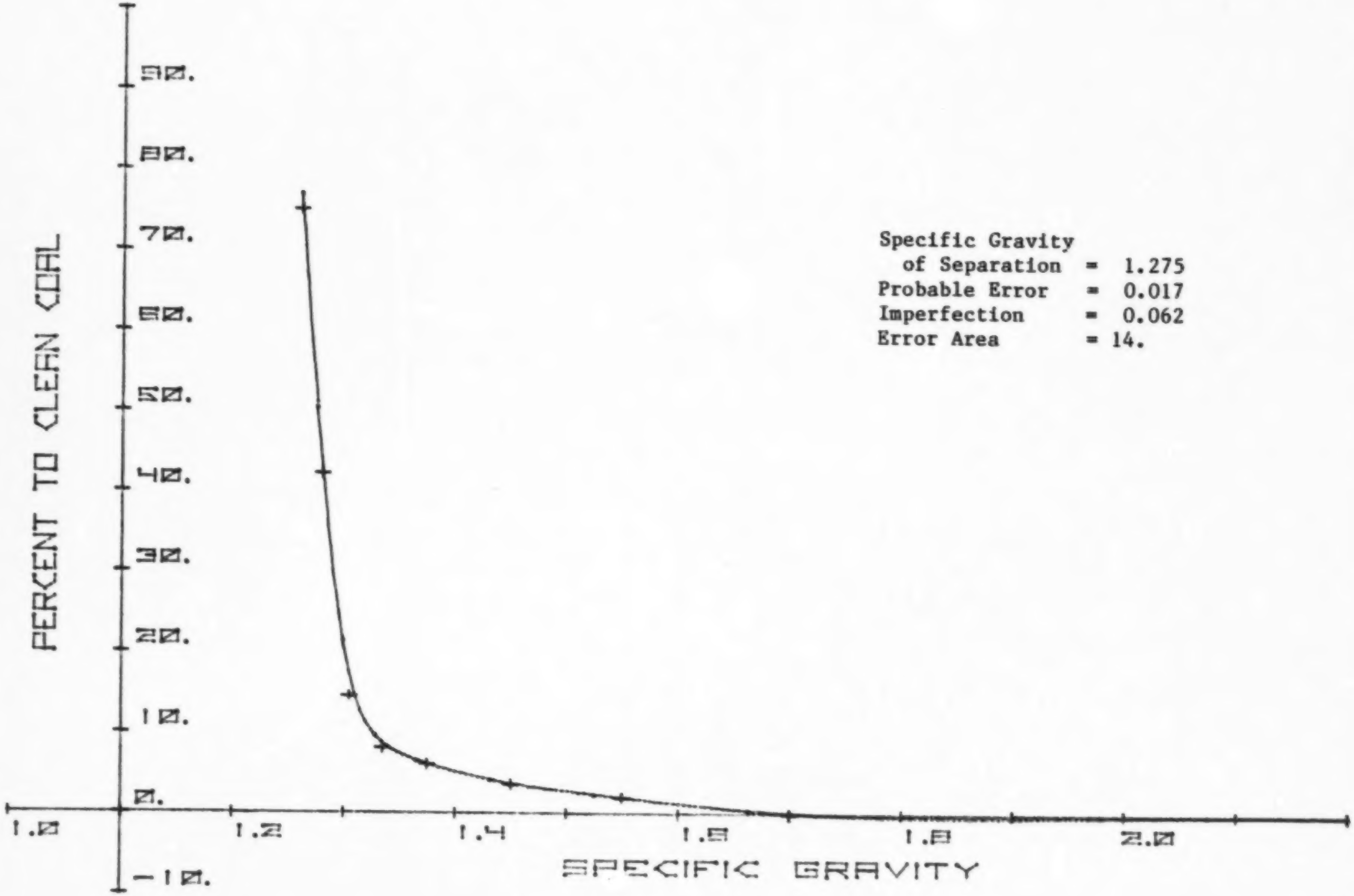


Figure A-124. Distribution Curve, Test No. 101 (9 x 100)

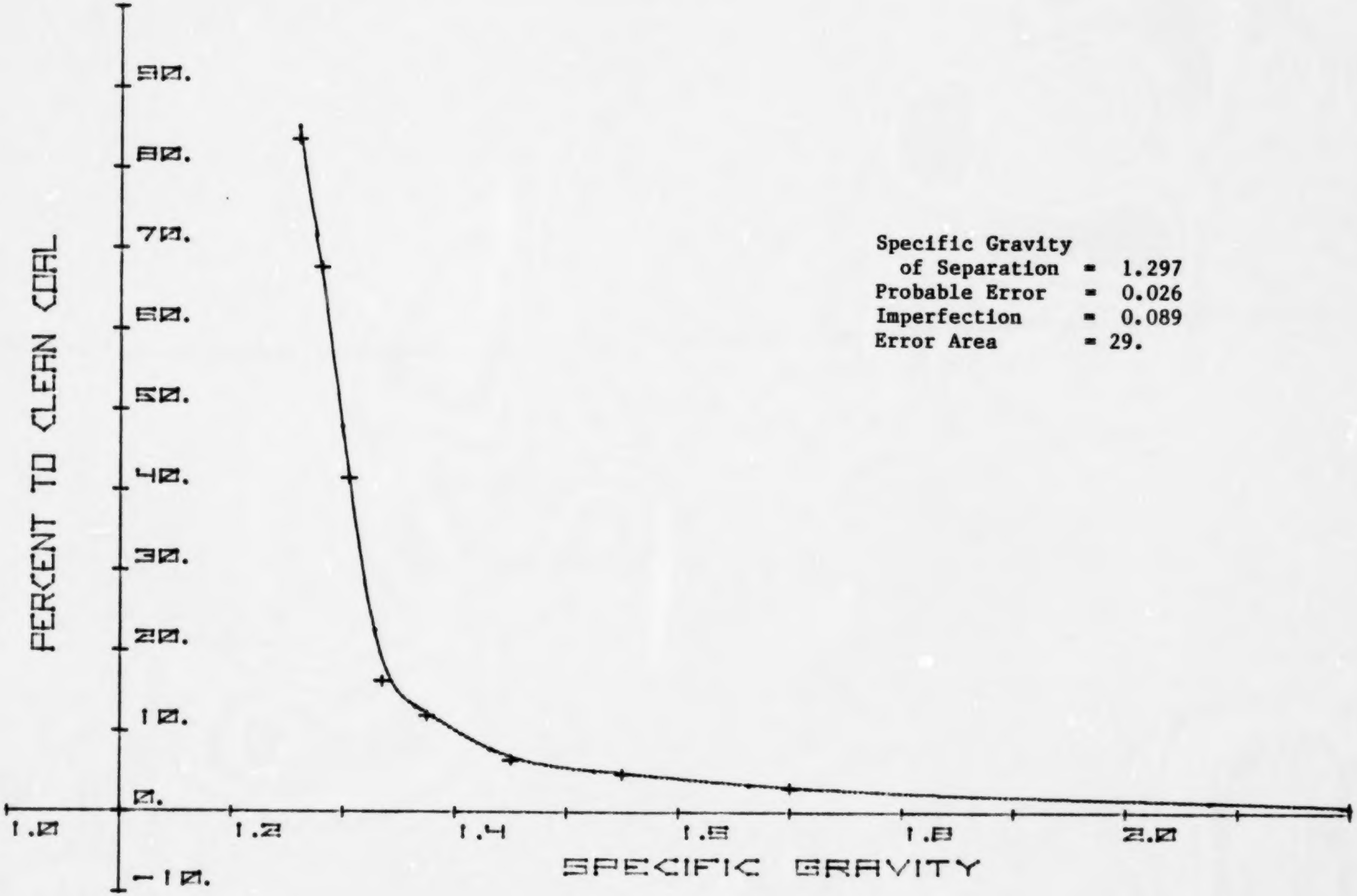
TABLE A-32. TEST 102 DATA SUMMARY

	9x14	14x28	28x100	9x100
SCREEN ANALYSIS, PERCENT:				
FEED	40.9	27.8	25.9	94.6
CLEAN COAL	26.4	28.1	35.5	90.1
REFUSE	50.0	27.6	19.8	97.4
ASH, PERCENT:				
FEED	22.9	19.7	19.3	21.0
CLEAN COAL	3.8	4.2	5.2	4.5
REFUSE	29.2	29.7	35.2	30.6
TOTAL SULFUR, PERCENT:				
FEED	2.22	2.19	2.42	2.27
CLEAN COAL	0.95	1.03	1.07	1.02
REFUSE	2.64	2.93	3.95	2.99
WEIGHT RECOVERY (YIELD)	25.0	39.1	53.1	36.8
THEORETICAL WEIGHT RECOVERY	41.7	57.5	69.9	56.8
WEIGHT RECOVERY EFFICIENCY	59.9	67.9	75.9	64.8
ASH ERROR	1.4	1.6	2.3	1.9
FLOAT IN REFUSE...% OF PRODUCT	10.0	13.9	25.0	13.0
SINK IN CLEAN COAL	22.3	16.3	11.9	23.3
TOTAL MISPLACED MATERIAL..% OF FEED	13.1	14.9	18.1	16.8
NEAR GRAVITY MATERIAL	56.8	64.6	69.9	61.8
SPECIFIC GRAVITY OF SEPARATION	1.275	1.297	1.351	1.289
PROBABLE ERROR	0.017	0.026	0.078	0.032
IMPERFECTION	0.062	0.089	0.223	0.112
ERROR AREA	14.	29.	62.	33.
DISTRIBUTION, PERCENT TO CLEANED COAL (SPECIFIC GRAVITY FRACTION):				
FLOAT 1.27	75.0	83.6	89.6	82.2
1.27-1.29	42.2	67.7	72.4	59.4
1.29-1.32	14.5	41.5	70.1	35.8
1.32-1.35	8.1	16.3	58.7	22.9
1.35-1.40	6.1	12.1	39.2	15.0
1.40-1.50	3.6	6.5	22.7	9.6
1.50-1.60	2.0	4.8	14.0	5.6
1.60-1.80	0.0	3.1	8.6	2.9
SINK 1.80	0.0	0.8	2.5	0.9



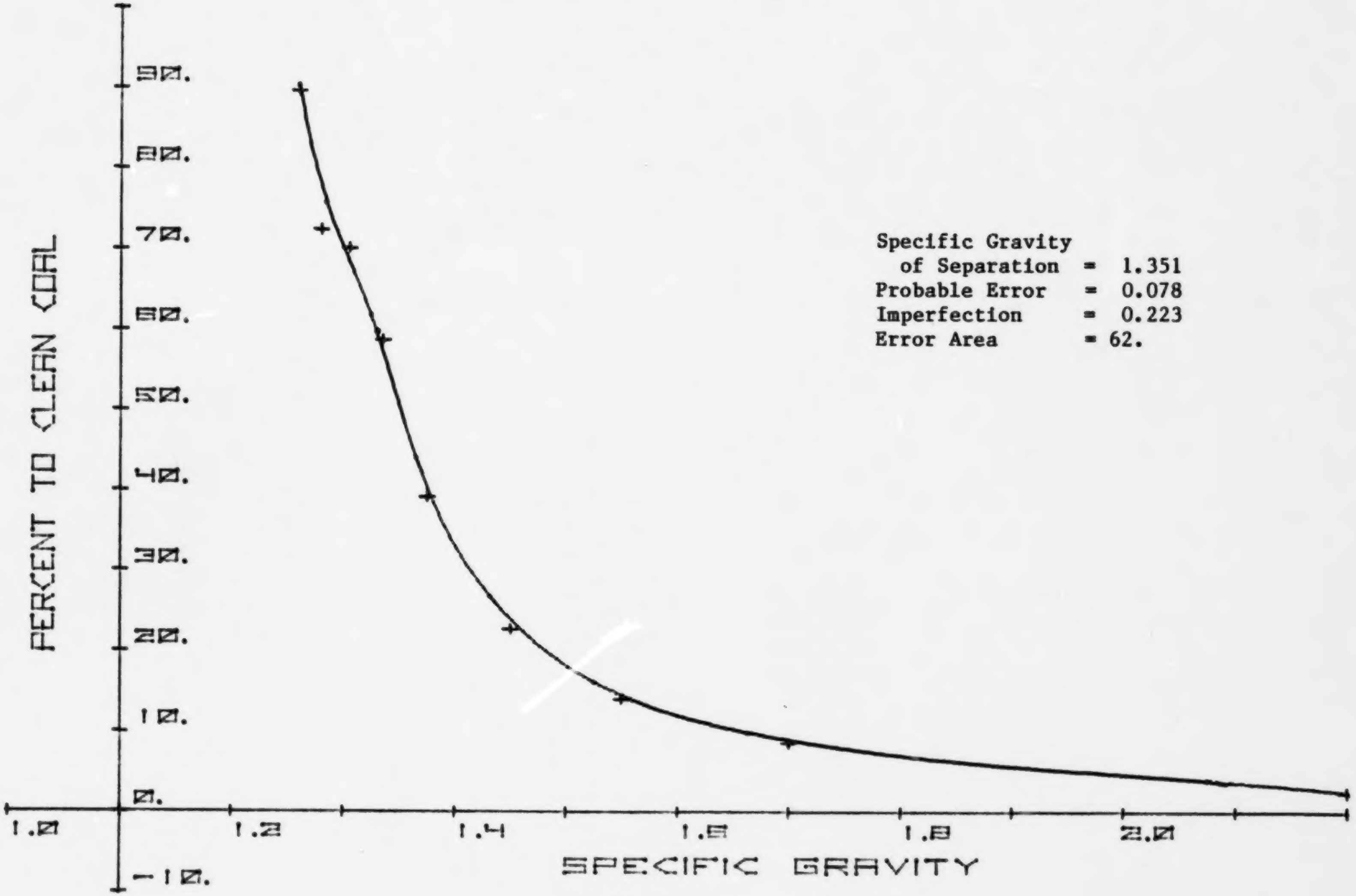
Specific Gravity
  of Separation = 1.275
Probable Error = 0.017
Imperfection = 0.062
Error Area = 14.

Figure A-125. Distribution Curve, Test No. 102 (9 x 14)



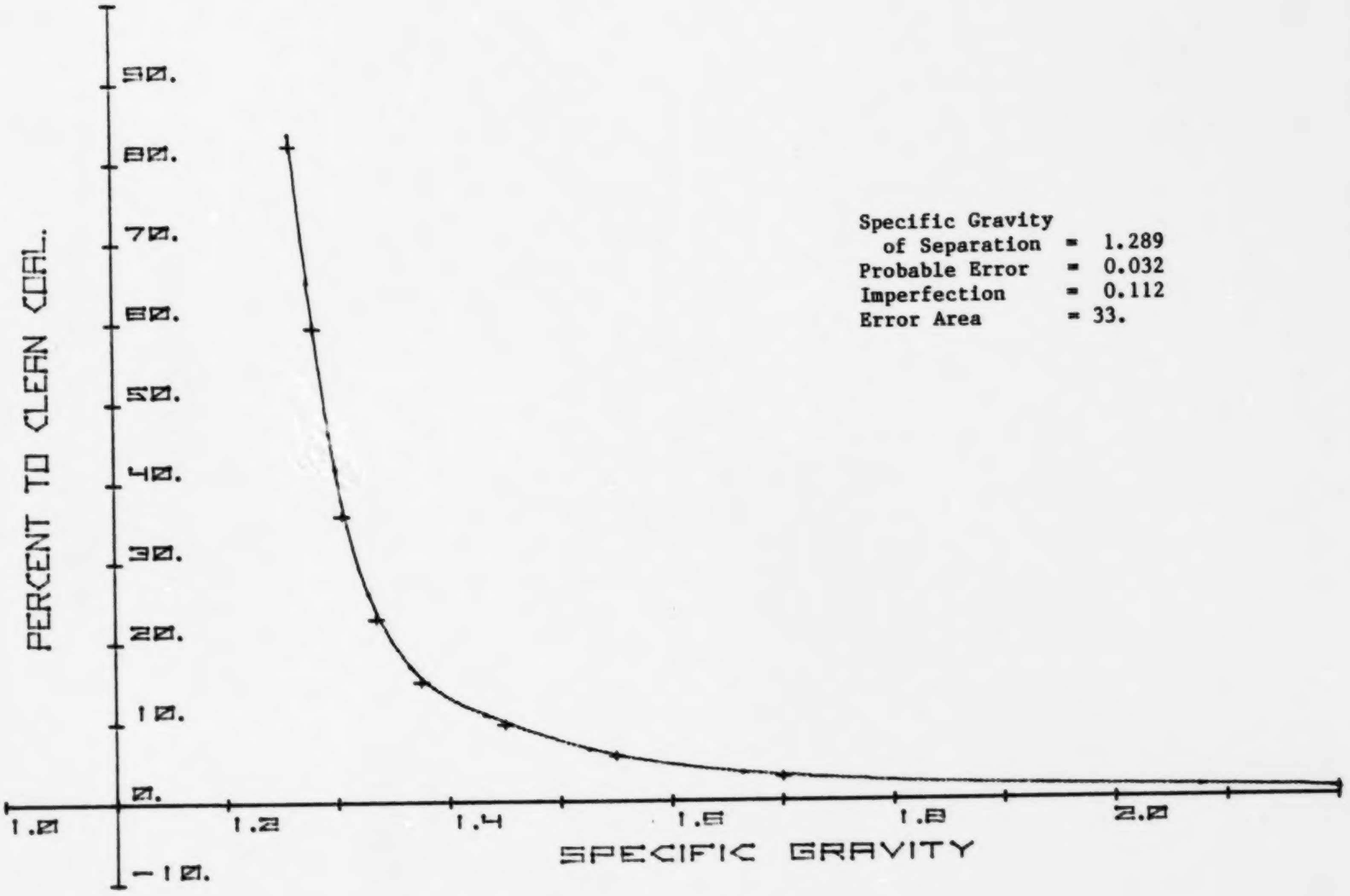
Specific Gravity
  of Separation = 1.297
Probable Error = 0.026
Imperfection = 0.089
Error Area = 29.

Figure A-126. Distribution Curve, Test No. 102 (14 x 28)



Specific Gravity
  of Separation = 1.351
Probable Error = 0.078
Imperfection = 0.223
Error Area = 62.

Figure A-127. Distribution Curve, Test No. 102 (28 x 100)



Specific Gravity
  of Separation = 1.289
Probable Error = 0.032
Imperfection = 0.112
Error Area = 33.

Figure A-128. Distribution Curve, Test No. 102 (9 x 100)

TABLE A-33. TEST 103 DATA SUMMARY

	9x14	14x28	28x100	9x100
SCREEN ANALYSIS, PERCENT:				
FEED..............................	40.6	27.6	27.8	96.1
CLEAN COAL........................	27.6	28.7	37.6	93.9
REFUSE............................	50.0	26.8	20.8	97.6
ASH, PERCENT:				
FEED..............................	25.5	16.6	20.6	21.5
CLEAN COAL........................	4.5	4.9	6.4	5.4
REFUSE............................	33.9	25.7	39.2	32.8
TOTAL SULFUR, PERCENT:				
FEED..............................	1.39	2.26	2.61	1.99
CLEAN COAL........................	1.04	0.97	1.26	1.10
REFUSE............................	1.54	3.26	4.38	2.62
WEIGHT RECOVERY (YIELD)............	28.6	43.7	56.7	41.1
THEORETICAL WEIGHT RECOVERY........	42.3	60.0	73.3	59.3
WEIGHT RECOVERY EFFICIENCY.........	67.6	72.8	77.3	69.2
ASH ERROR..........................	1.4	1.6	2.7	2.1
FLOAT IN REFUSE...% OF PRODUCT.....	10.9	21.2	25.5	15.7
SINK IN CLEAN COAL.................	16.8	13.0	10.9	20.6
TOTAL MISPLACED MATERIAL..% OF FEED	12.6	17.6	17.2	17.7
NEAR GRAVITY MATERIAL..............	55.6	67.6	36.8	62.7
SPECIFIC GRAVITY OF SEPARATION.....	1.289	1.310	1.381	1.307
PROBABLE ERROR.....................	0.023	0.041	0.100	0.041
IMPERFECTION.......................	0.078	0.131	0.263	0.134
ERROR AREA.........................	17.	27.	74.	37.
DISTRIBUTION, PERCENT TO CLEANED COAL (SPECIFIC GRAVITY FRACTION):				
FLOAT 1.27	83.0	84.3	93.1	87.0
1.27-1.29	62.4	54.4	71.8	62.7
1.29-1.32	31.0	70.9	78.9	58.2
1.32-1.35	10.9	32.2	67.5	30.6
1.35-1.40	8.3	17.1	45.4	20.1
1.40-1.50	4.8	11.5	33.1	14.0
1.50-1.60	0.0	0.0	16.7	4.4
1.60-1.80	0.0	0.0	10.2	2.7
SINK 1.80	0.0	0.0	4.1	1.1

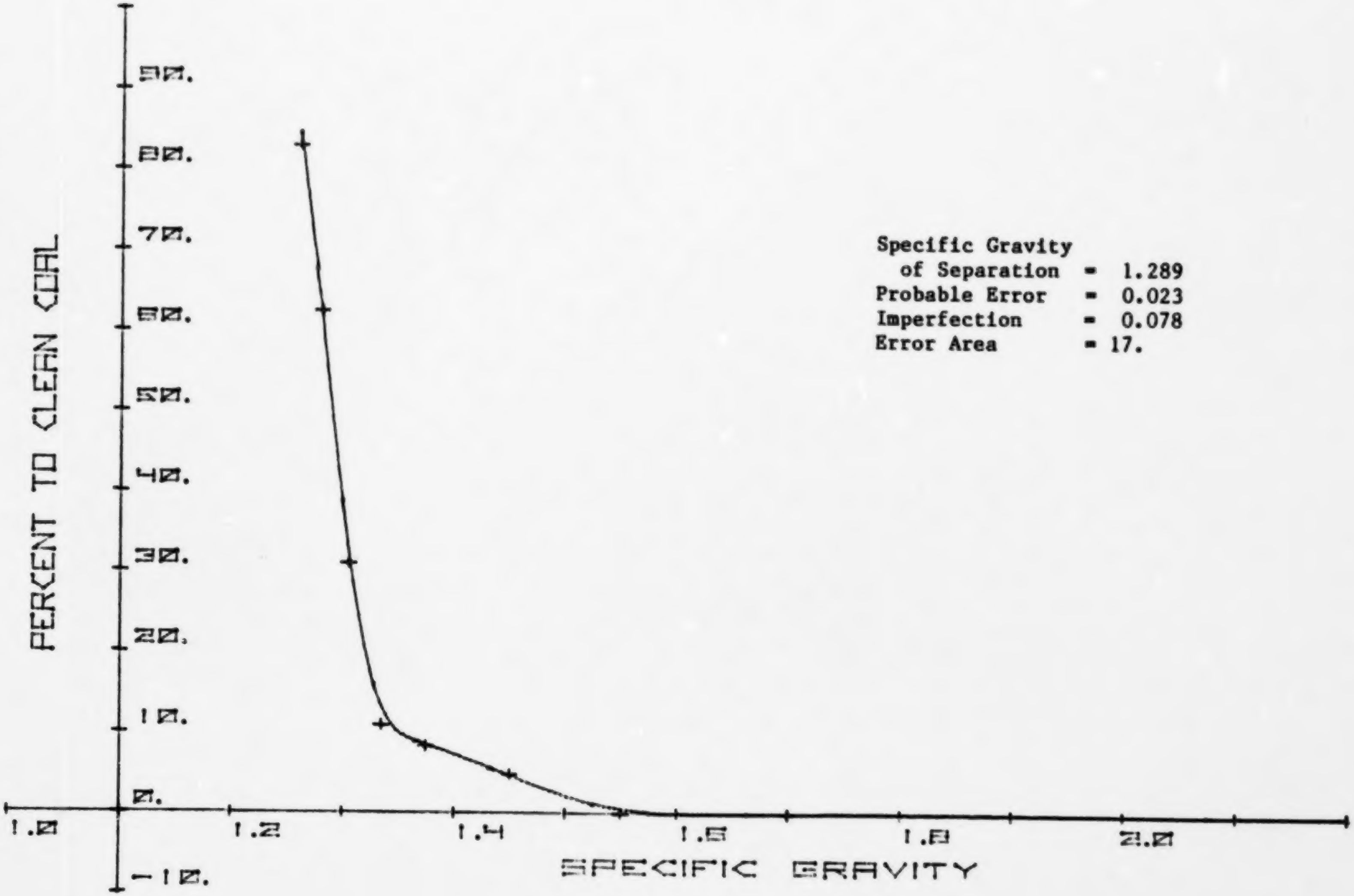
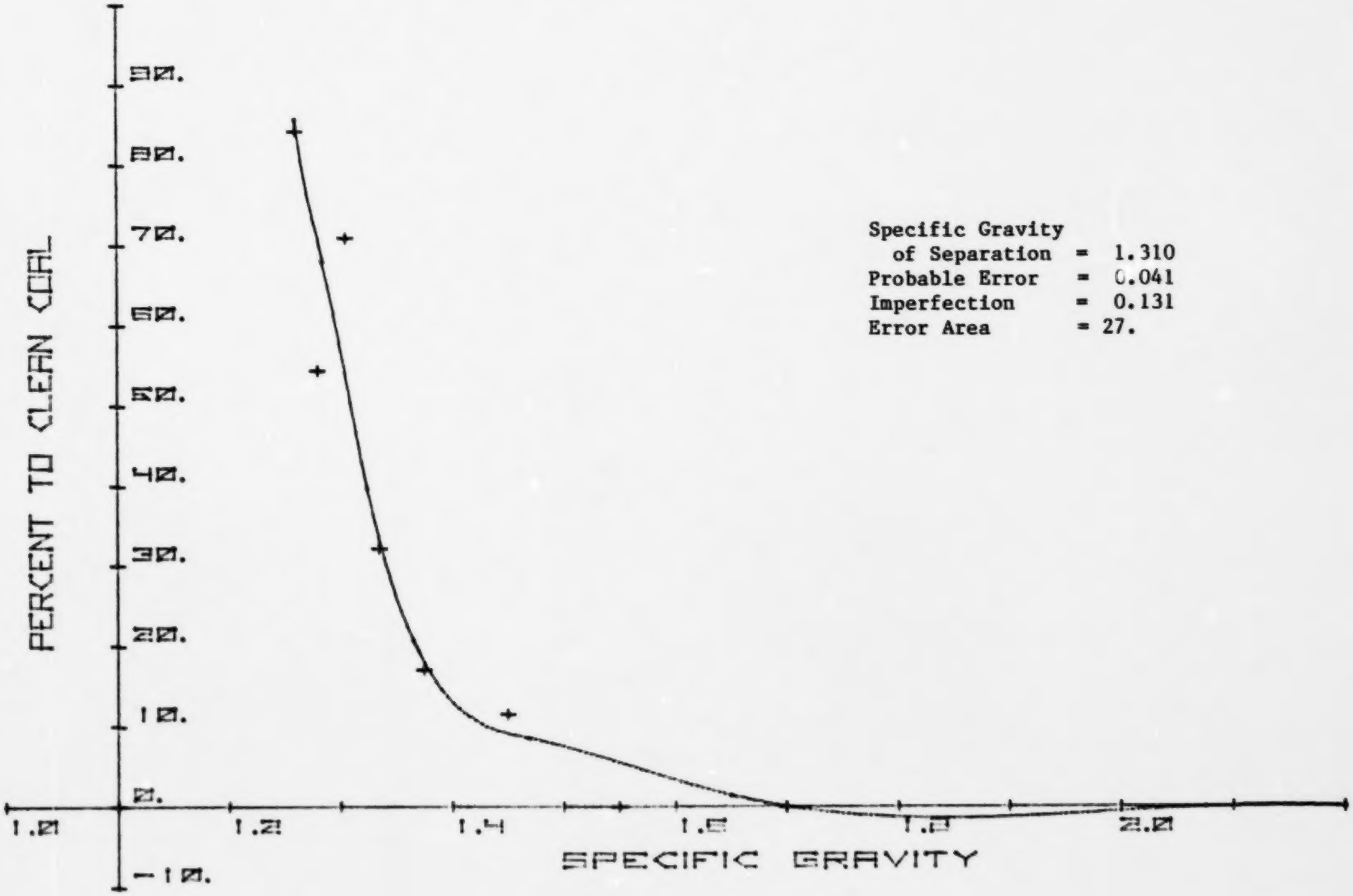


Figure A-129. Distribution Curve, Test No. 103 (9 x 14)



Specific Gravity
of Separation = 1.310
Probable Error = 0.041
Imperfection = 0.131
Error Area = 27.

Figure A-130. Distribution Curve, Test No. 103 (14 x 28)

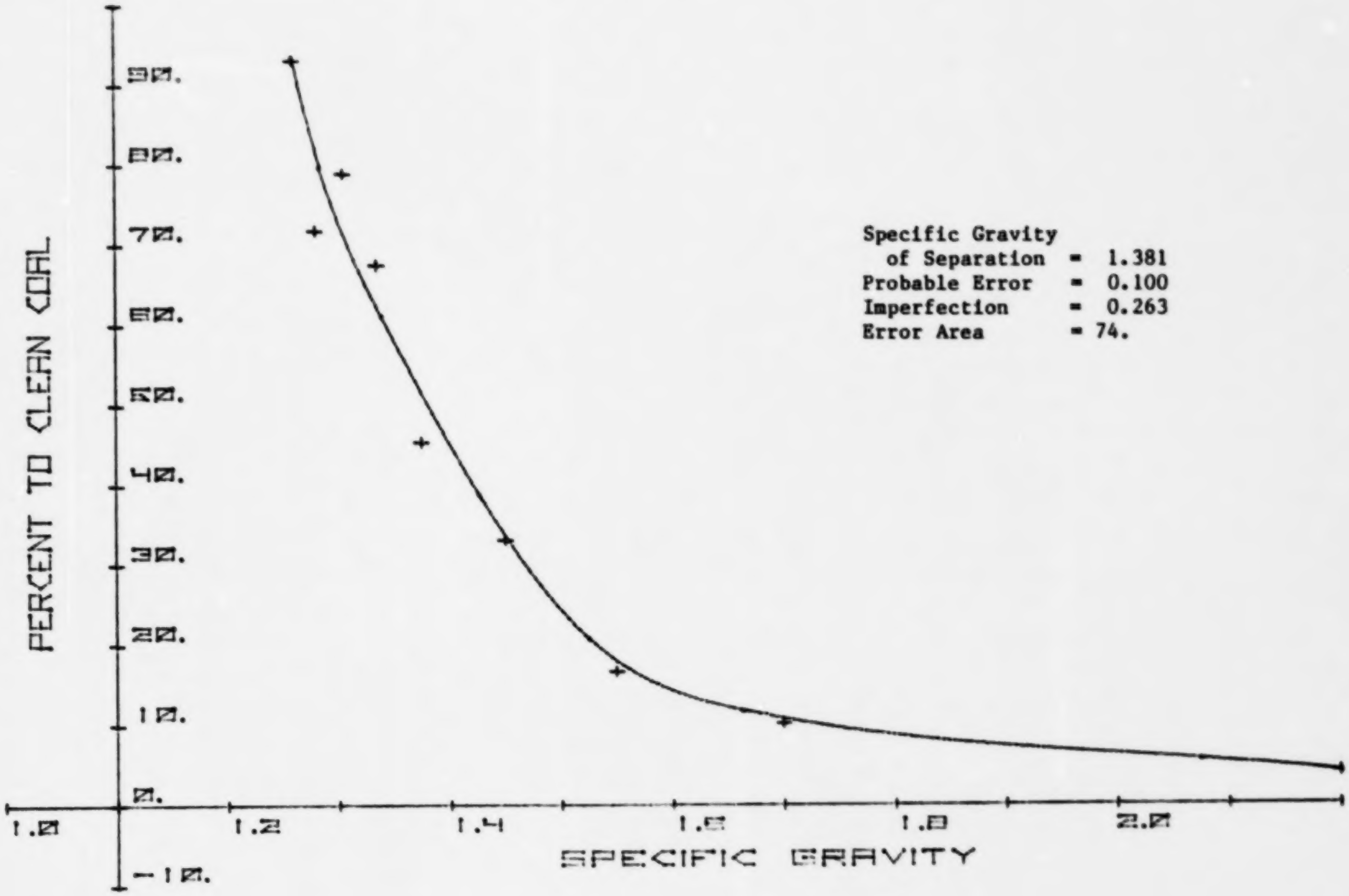


Figure A-131. Distribution Curve, Test No. 103 (28 x 100)

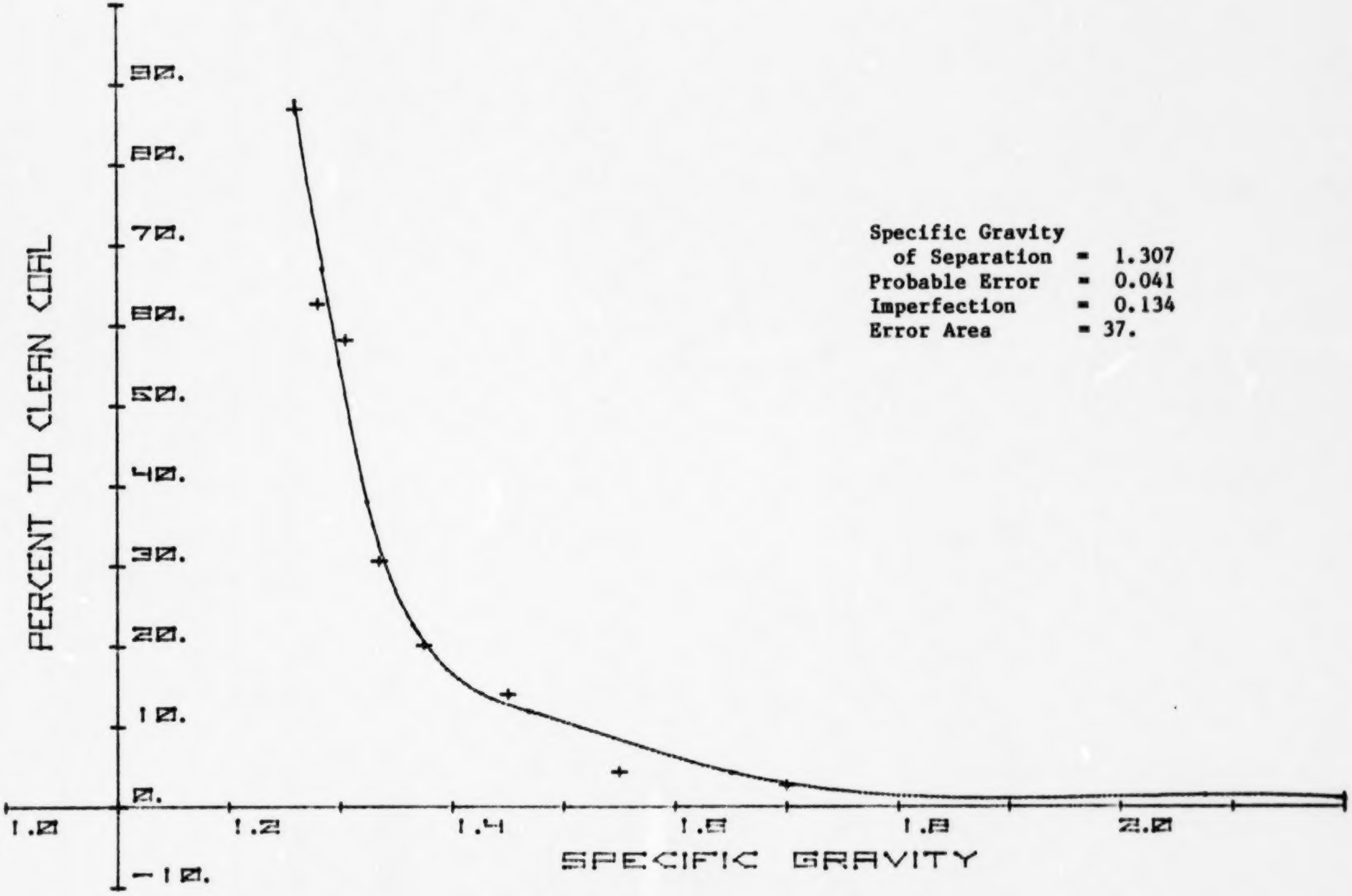
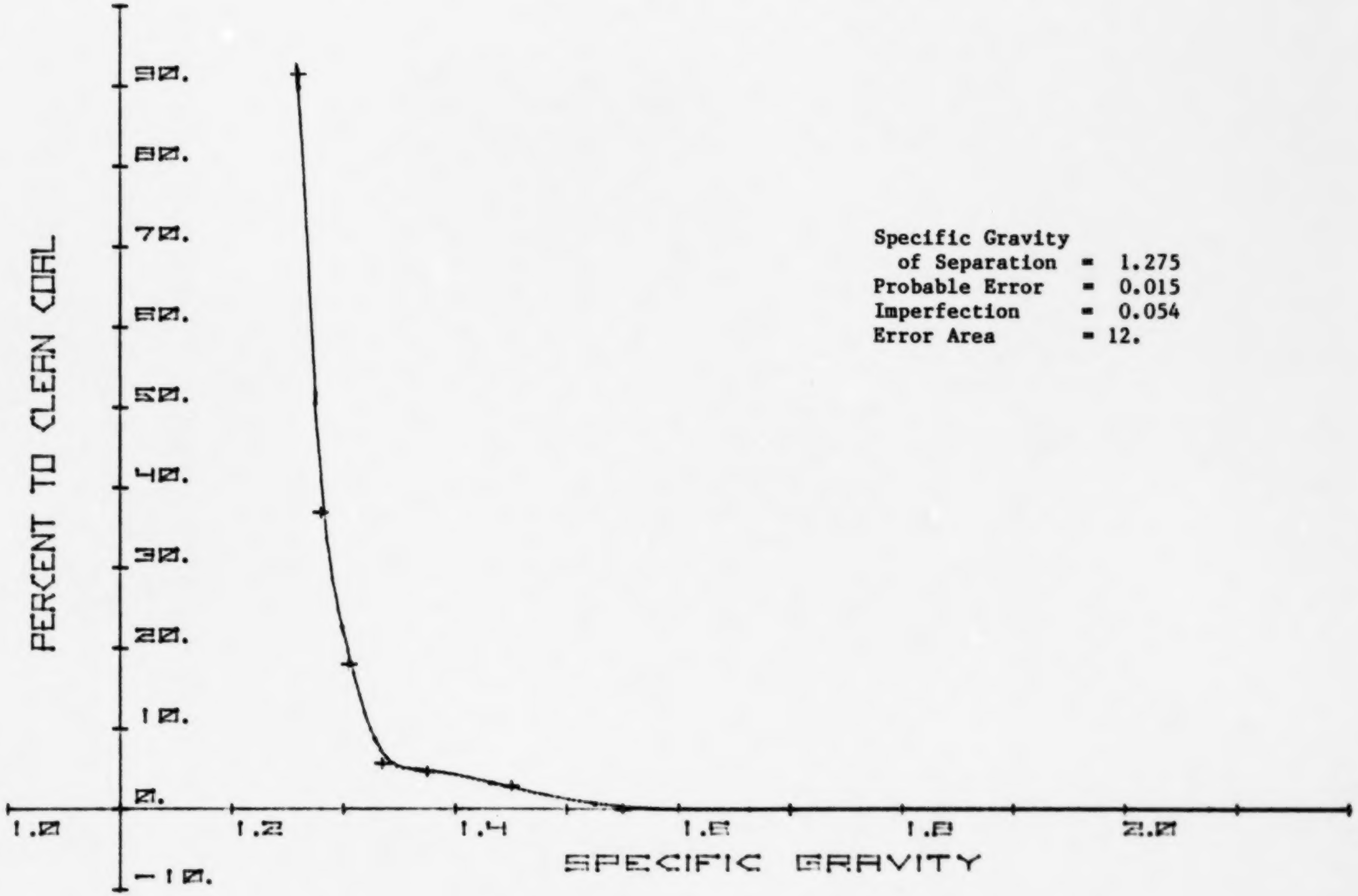


Figure A-132. Distribution Curve, Test No. 103 (9 x 100)

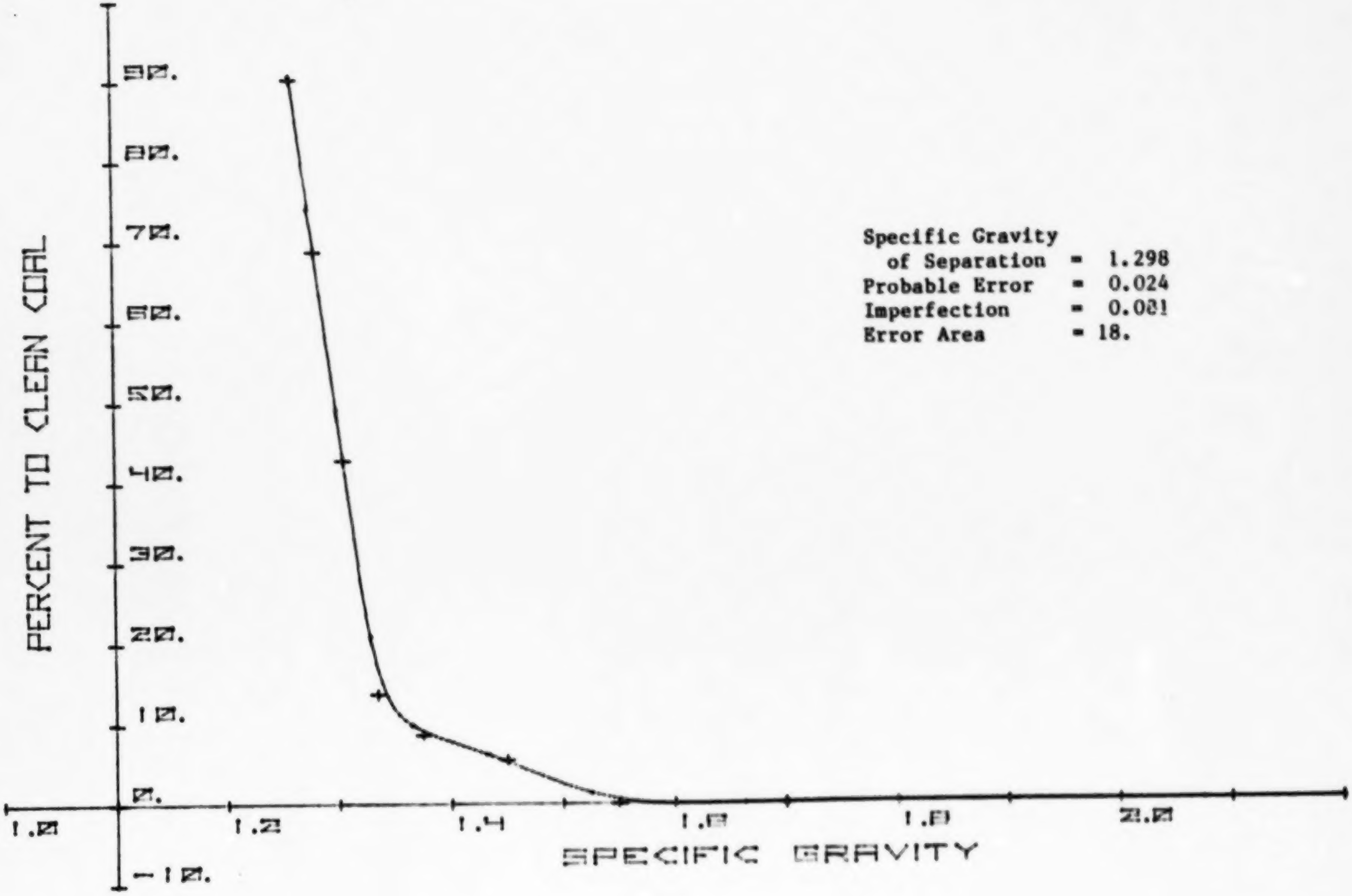
TABLE A-34. TEST 104 DATA SUMMARY

	9x14	14x28	28x100	9x100
SCREEN ANALYSIS, PERCENT:				
FEED	40.1	27.4	27.8	95.2
CLEAN COAL	27.9	26.0	38.5	92.3
REFUSE	48.7	28.4	20.3	97.3
ASH, PERCENT:				
FEED	24.5	21.3	18.7	21.9
CLEAN COAL	3.4	3.5	5.2	4.2
REFUSE	33.1	32.7	36.7	33.7
TOTAL SULFUR, PERCENT:				
FEED	2.47	2.35	2.40	2.42
CLEAN COAL	0.97	0.99	1.07	1.02
REFUSE	3.08	3.23	4.18	3.35
WEIGHT RECOVERY (YIELD)	28.7	39.1	57.1	40.0
THEORETICAL WEIGHT RECOVERY	37.2	50.3	71.5	54.4
WEIGHT RECOVERY EFFICIENCY	77.3	77.8	79.9	73.5
ASH ERROR	0.8	1.0	2.0	1.5
FLOAT IN REFUSE...% OF PRODUCT	3.9	11.3	23.6	9.7
SINK IN CLEAN COAL	16.4	12.8	10.0	18.5
TOTAL MISPLACED MATERIAL..% OF FEED	7.5	11.9	15.9	13.2
NEAR GRAVITY MATERIAL	53.5	63.7	72.5	61.8
SPECIFIC GRAVITY OF SEPARATION	1.275	1.298	1.351	1.294
PROBABLE ERROR	0.015	0.024	0.078	0.029
IMPERFECTION	0.054	0.081	0.221	0.099
ERROR AREA	12.	18.	62.	29.
DISTRIBUTION, PERCENT TO CLEANED COAL (SPECIFIC GRAVITY FRACTION):				
FLOAT 1.27	91.5	90.3	91.2	91.1
1.27-1.29	36.9	68.8	79.9	62.8
1.29-1.32	18.0	42.8	73.7	40.0
1.32-1.35	5.7	13.7	59.3	21.2
1.35-1.40	4.7	8.6	38.2	14.2
1.40-1.50	2.9	5.5	25.0	9.4
1.50-1.60	0.0	0.0	12.3	3.2
1.60-1.80	0.0	0.0	8.5	1.9
SINK 1.80	0.0	0.0	2.9	0.7



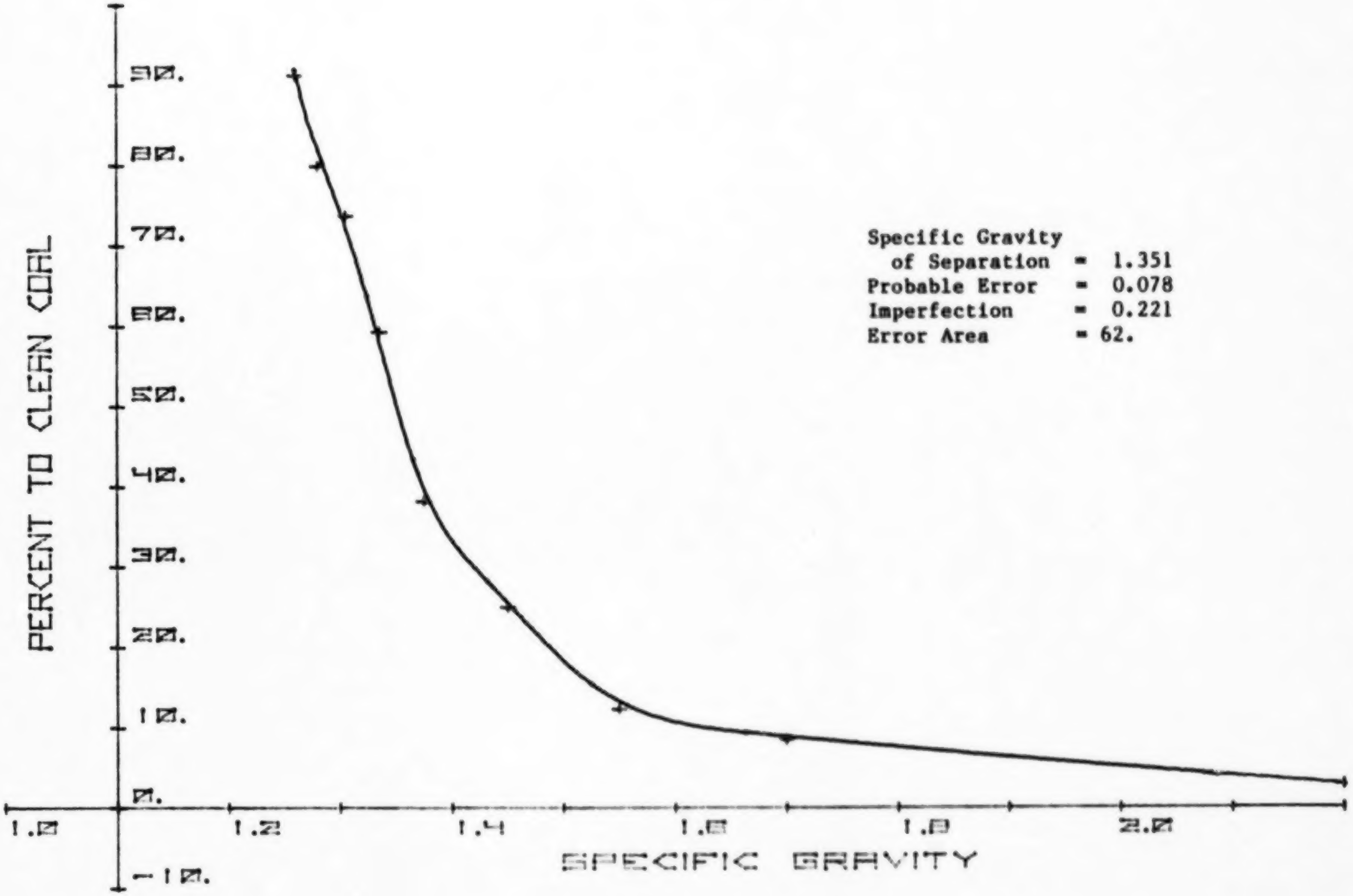
Specific Gravity
of Separation = 1.275
Probable Error = 0.015
Imperfection = 0.054
Error Area = 12.

Figure A-133. Distribution Curve, Test No. 104 (9 x 14)



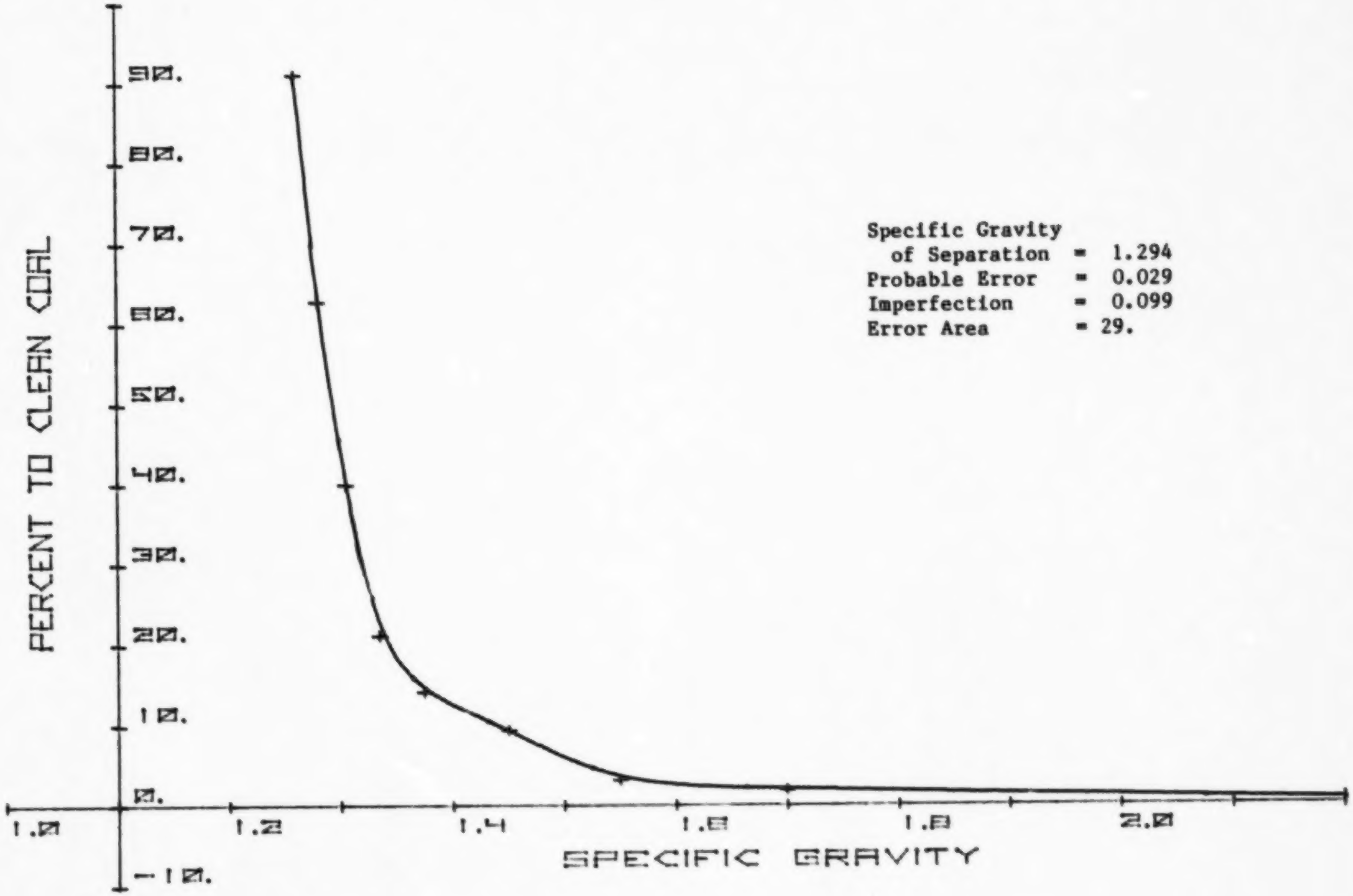
Specific Gravity
  of Separation = 1.298
Probable Error = 0.024
Imperfection = 0.081
Error Area = 18.

Figure A-134. Distribution Curve, Test No. 104 (14 x 28)



Specific Gravity
of Separation = 1.351
Probable Error = 0.078
Imperfection = 0.221
Error Area = 62.

Figure A-135. Distribution Curve, Test No. 104 (28 x 100)



Specific Gravity
  of Separation = 1.294
Probable Error = 0.029
Imperfection = 0.099
Error Area = 29.

Figure A-136. Distribution Curve, Test No. 104 (9 x 100)

TABLE A-35. TEST 105 DATA SUMMARY

	9x14	14x28	28x100	9x100
SCREEN ANALYSIS, PERCENT:				
FEED	34.4	27.6	33.5	95.6
CLEAN COAL	21.5	26.8	45.6	94.0
REFUSE	44.5	28.3	24.1	96.9
ASH, PERCENT:				
FEED	25.6	20.3	18.7	21.6
CLEAN COAL	3.7	3.9	5.4	4.6
REFUSE	33.8	32.5	38.4	34.6
TOTAL SULFUR, PERCENT:				
FEED	2.37	2.21	2.33	2.31
CLEAN COAL	0.97	0.98	0.97	0.97
REFUSE	2.90	3.13	4.34	3.32
WEIGHT RECOVERY (YIELD)	27.4	42.6	59.6	43.1
THEORETICAL WEIGHT RECOVERY	37.5	54.4	72.4	57.6
WEIGHT RECOVERY EFFICIENCY	72.9	78.3	82.4	74.9
ASH ERROR	1.0	1.1	1.9	1.6
FLOAT IN REFUSE...% OF PRODUCT	7.8	14.6	26.4	14.2
SINK IN CLEAN COAL	15.0	13.6	8.3	16.5
TOTAL MISPLACED MATERIAL..% OF FEED	9.8	14.2	15.6	15.2
NEAR GRAVITY MATERIAL	54.5	66.7	33.8	64.4
SPECIFIC GRAVITY OF SEPARATION	1.290	1.306	1.386	1.309
PROBABLE ERROR	0.020	0.028	0.078	0.036
IMPERFECTION	0.070	0.093	0.202	0.115
ERROR AREA	14.	21.	61.	34.
DISTRIBUTION, PERCENT TO CLEANED COAL (SPECIFIC GRAVITY FRACTION):				
FLOAT 1.27	87.0	88.0	87.0	87.3
1.27-1.29	63.9	75.1	96.6	80.2
1.29-1.32	30.1	50.9	83.4	55.8
1.32-1.35	7.5	23.7	61.9	26.1
1.35-1.40	5.2	11.1	54.3	22.0
1.40-1.50	3.2	6.0	27.9	11.9
1.50-1.60	0.0	0.0	13.1	4.0
1.60-1.80	0.0	0.0	7.9	2.1
SINK 1.80	0.0	0.0	2.8	0.9

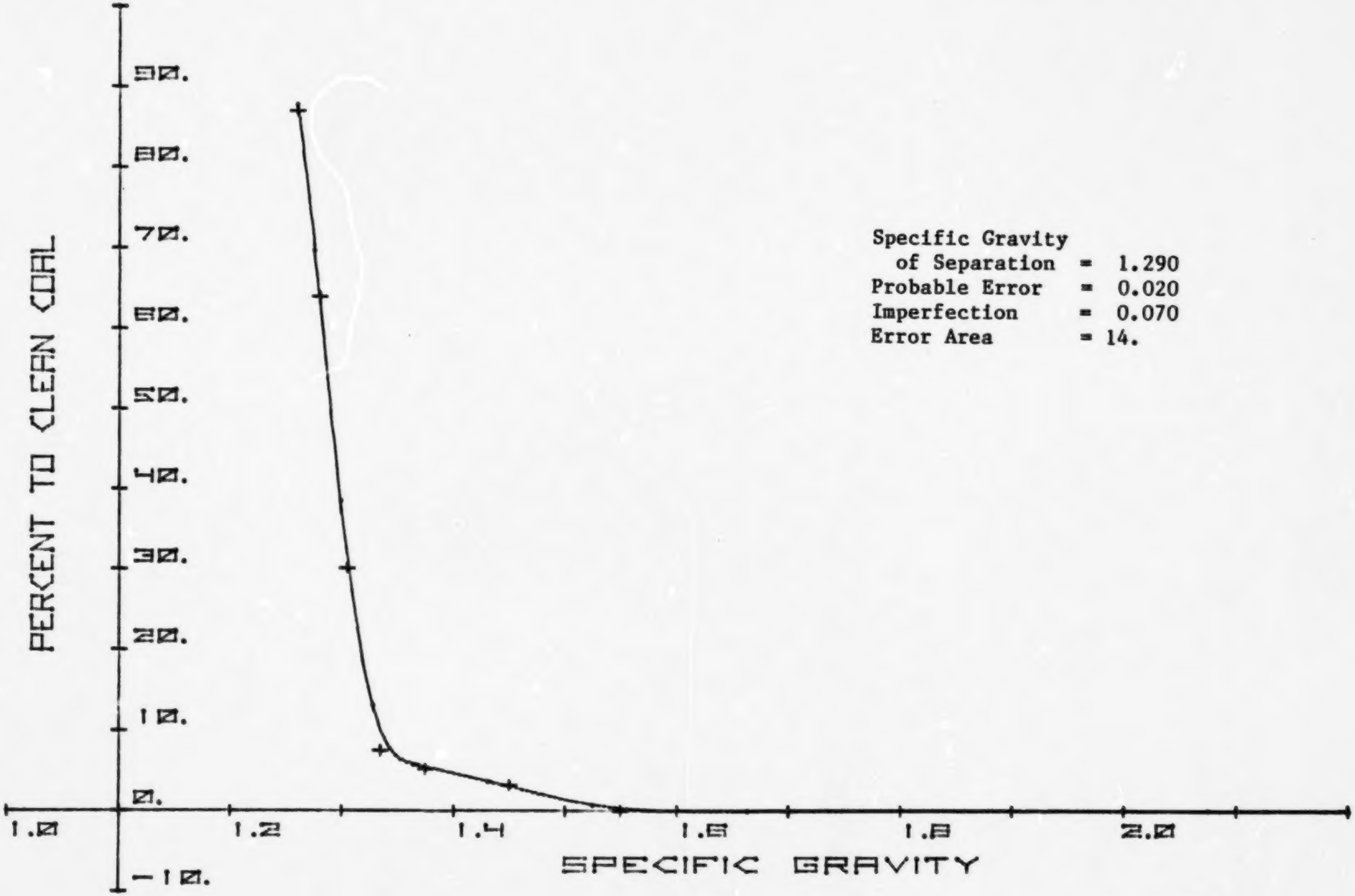
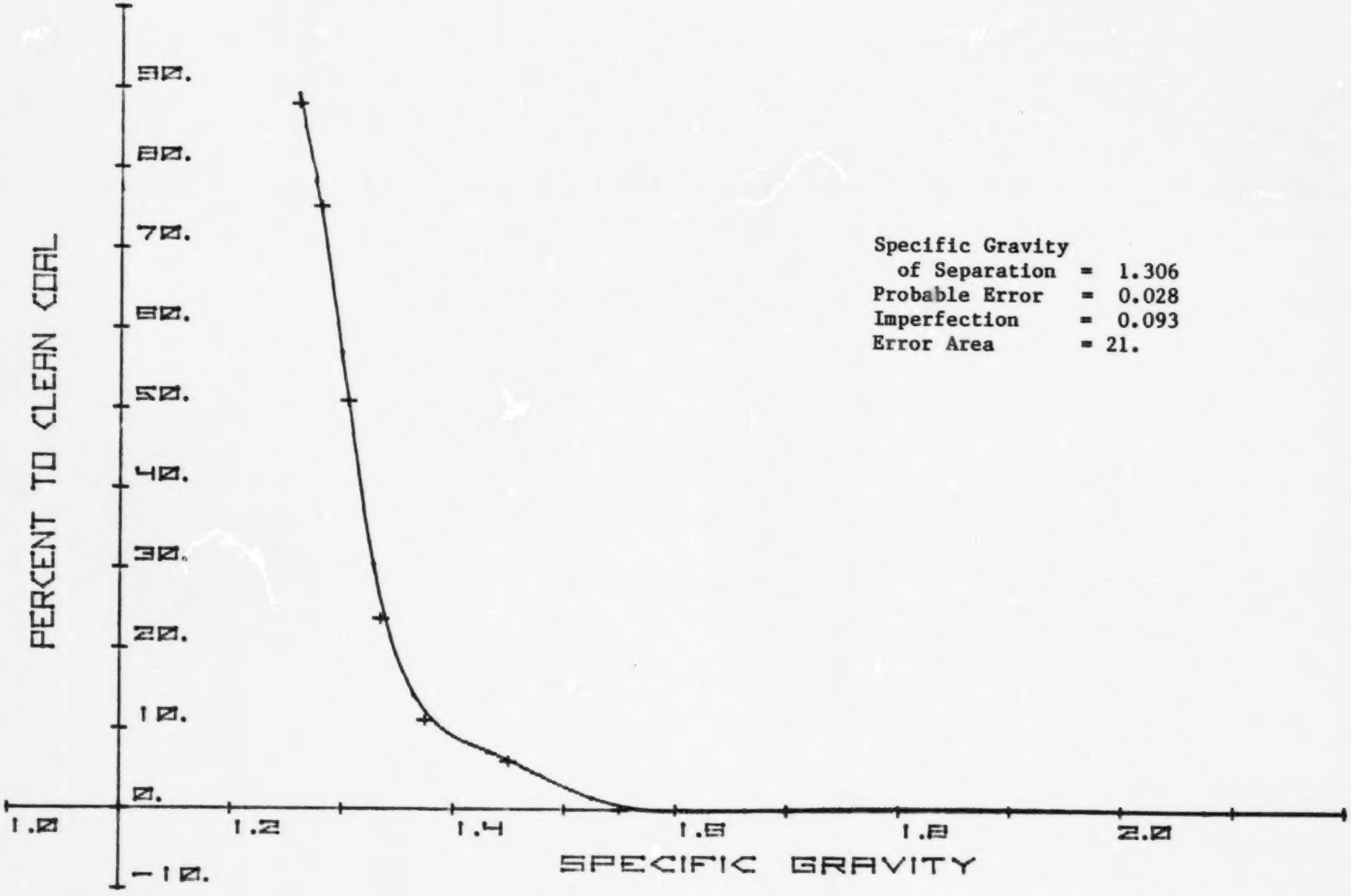
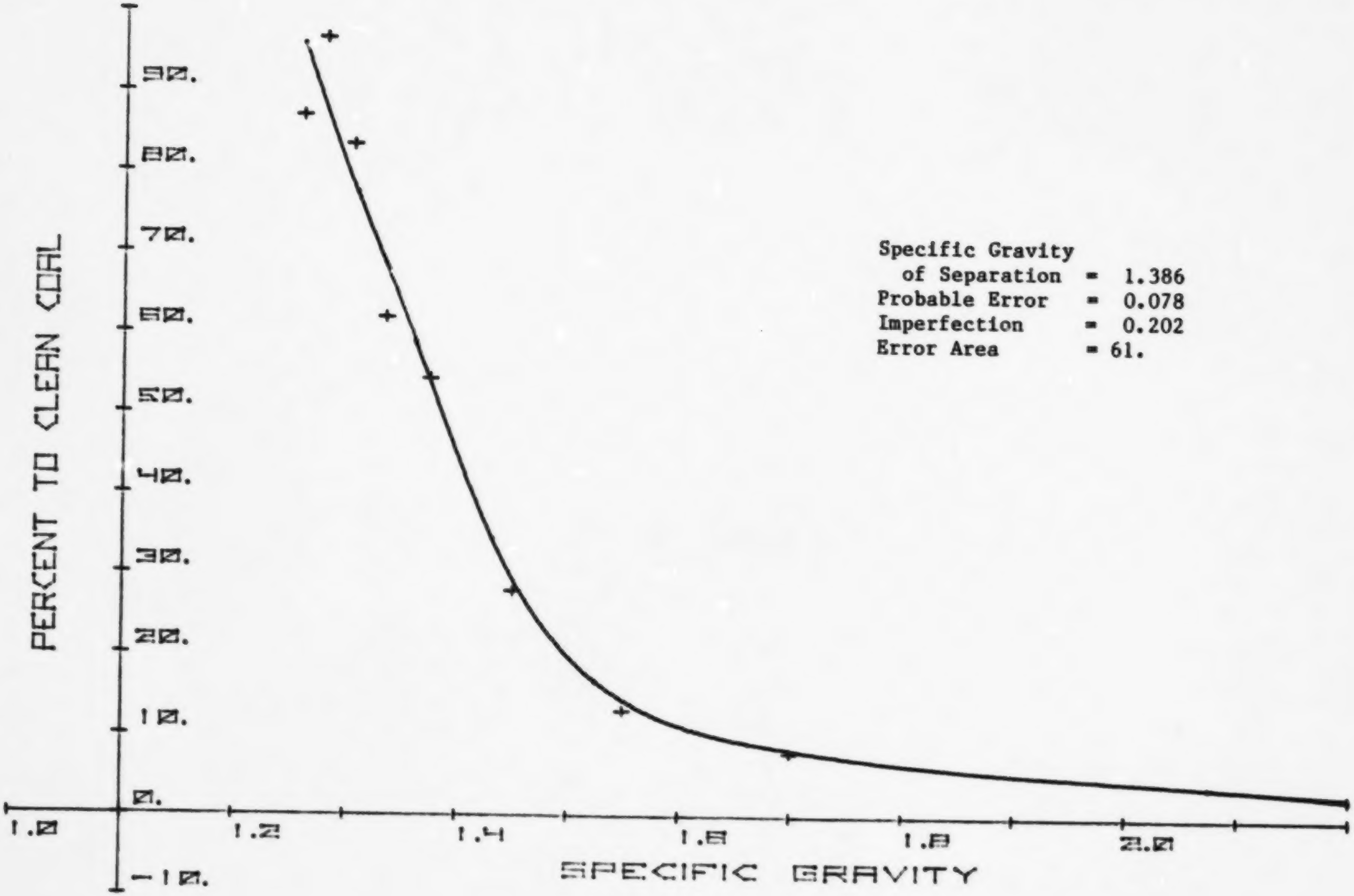


Figure A-137. Distribution Curve, Test No. 105 (9 x 14)



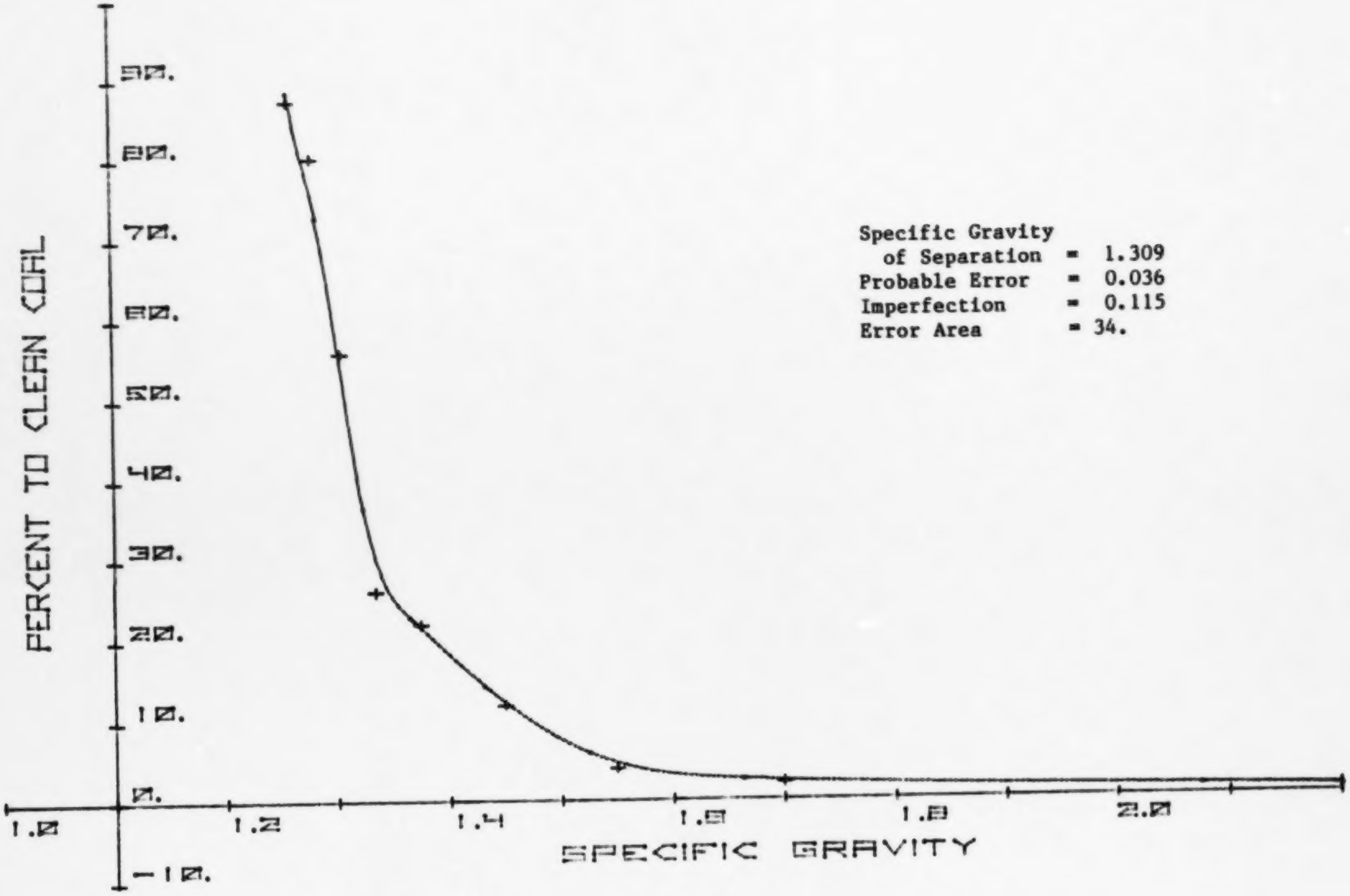
Specific Gravity
of Separation = 1.306
Probable Error = 0.028
Imperfection = 0.093
Error Area = 21.

Figure A-138. Distribution Curve, Test No. 105 (14 x 28)



Specific Gravity
  of Separation = 1.386
Probable Error = 0.078
Imperfection = 0.202
Error Area = 61.

Figure A-139. Distribution Curve, Test No. 105 (28 x 100)

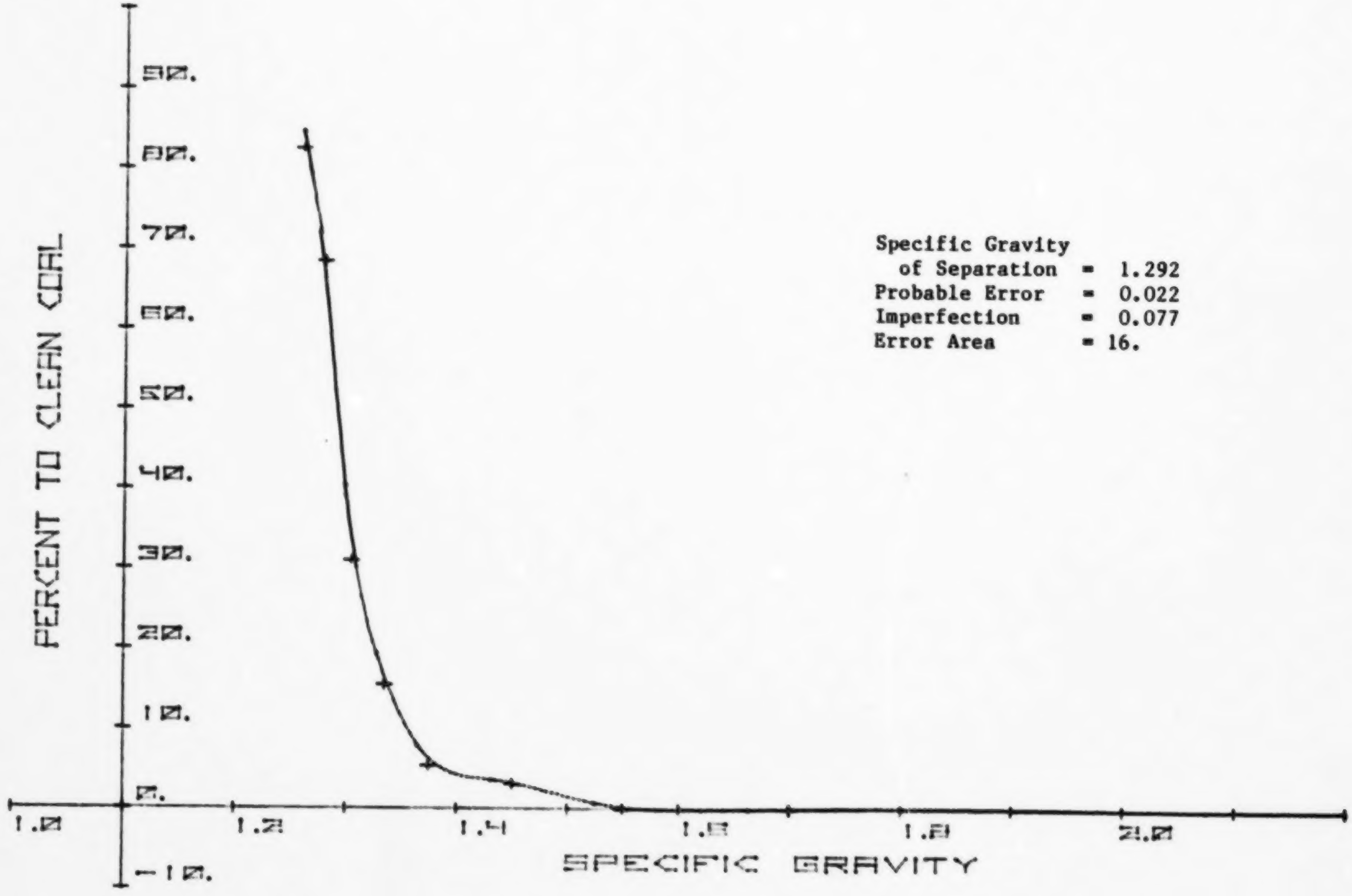


Specific Gravity
  of Separation = 1.309
Probable Error = 0.036
Imperfection = 0.115
Error Area = 34.

Figure A-140. Distribution Curve, Test No. 105 (9 x 100)

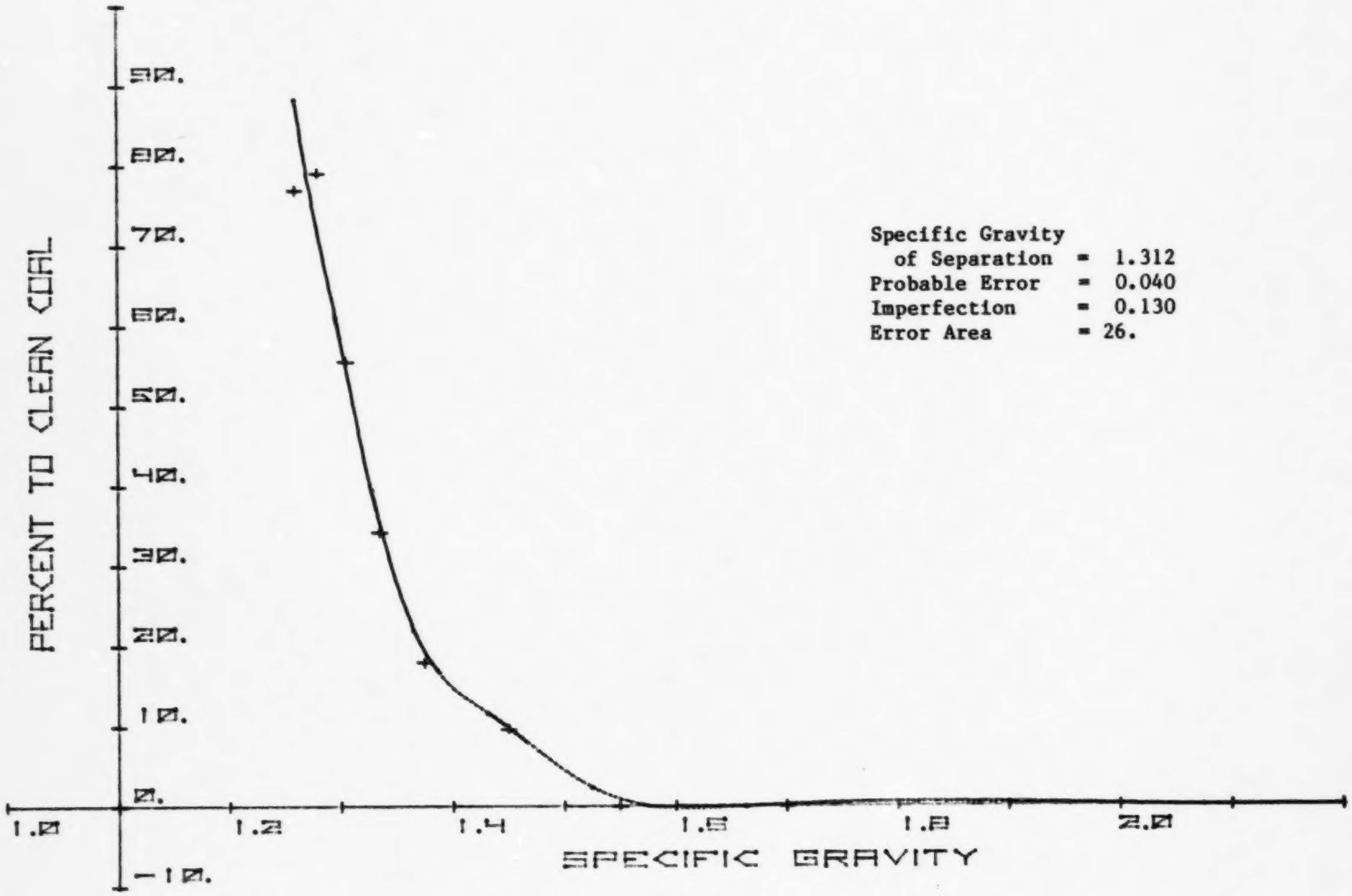
TABLE A-36. TEST 106 DATA SUMMARY

	9x14	14x28	28x100	9x100
SCREEN ANALYSIS, PERCENT:				
FEED	42.2	27.6	24.0	93.9
CLEAN COAL	28.7	28.7	34.0	91.4
REFUSE	51.9	26.9	16.9	95.8
ASH, PERCENT:				
FEED	23.7	19.5	19.0	21.3
CLEAN COAL	4.2	4.3	6.0	4.9
REFUSE	31.4	31.1	37.6	32.4
TOTAL SULFUR, PERCENT:				
FEED	2.31	2.13	2.34	2.26
CLEAN COAL	1.04	1.02	1.10	1.06
REFUSE	2.80	2.97	4.10	3.08
WEIGHT RECOVERY (YIELD)	28.2	43.1	58.9	40.4
THEORETICAL WEIGHT RECOVERY	43.5	59.8	74.4	59.3
WEIGHT RECOVERY EFFICIENCY	64.8	72.0	79.1	68.2
ASH ERROR	1.3	1.6	2.4	1.9
FLOAT IN REFUSE...% OF PRODUCT	9.7	22.8	34.7	18.6
SINK IN CLEAN COAL	19.2	13.7	6.4	19.2
TOTAL MISPLACED MATERIAL..% OF FEED	12.4	18.9	18.0	18.8
NEAR GRAVITY MATERIAL	57.8	69.6	26.3	65.5
SPECIFIC GRAVITY OF SEPARATION	1.292	1.312	1.401	1.308
PROBABLE ERROR	0.022	0.040	0.090	0.043
IMPERFECTION	0.077	0.130	0.224	0.138
ERROR AREA	16.	26.	68.	34.
DISTRIBUTION, PERCENT TO CLEANED COAL (SPECIFIC GRAVITY FRACTION):				
FLOAT 1.27	82.5	76.9	78.3	79.1
1.27-1.29	68.4	79.0	96.2	78.3
1.29-1.32	31.0	55.5	87.8	51.3
1.32-1.35	15.5	34.2	73.3	33.2
1.35-1.40	5.4	18.0	58.3	20.6
1.40-1.50	3.1	9.6	35.7	12.5
1.50-1.60	0.0	0.0	17.9	3.8
1.60-1.80	0.0	0.0	9.6	1.9
SINK 1.80	0.0	0.0	3.1	0.7



Specific Gravity
  of Separation = 1.292
Probable Error = 0.022
Imperfection = 0.077
Error Area = 16.

Figure A-141. Distribution Curve, Test No. 106 (9 x 14)



Specific Gravity
of Separation = 1.312
Probable Error = 0.040
Imperfection = 0.130
Error Area = 26.

Figure A-142. Distribution Curve, Test No. 106 (14 x 28)

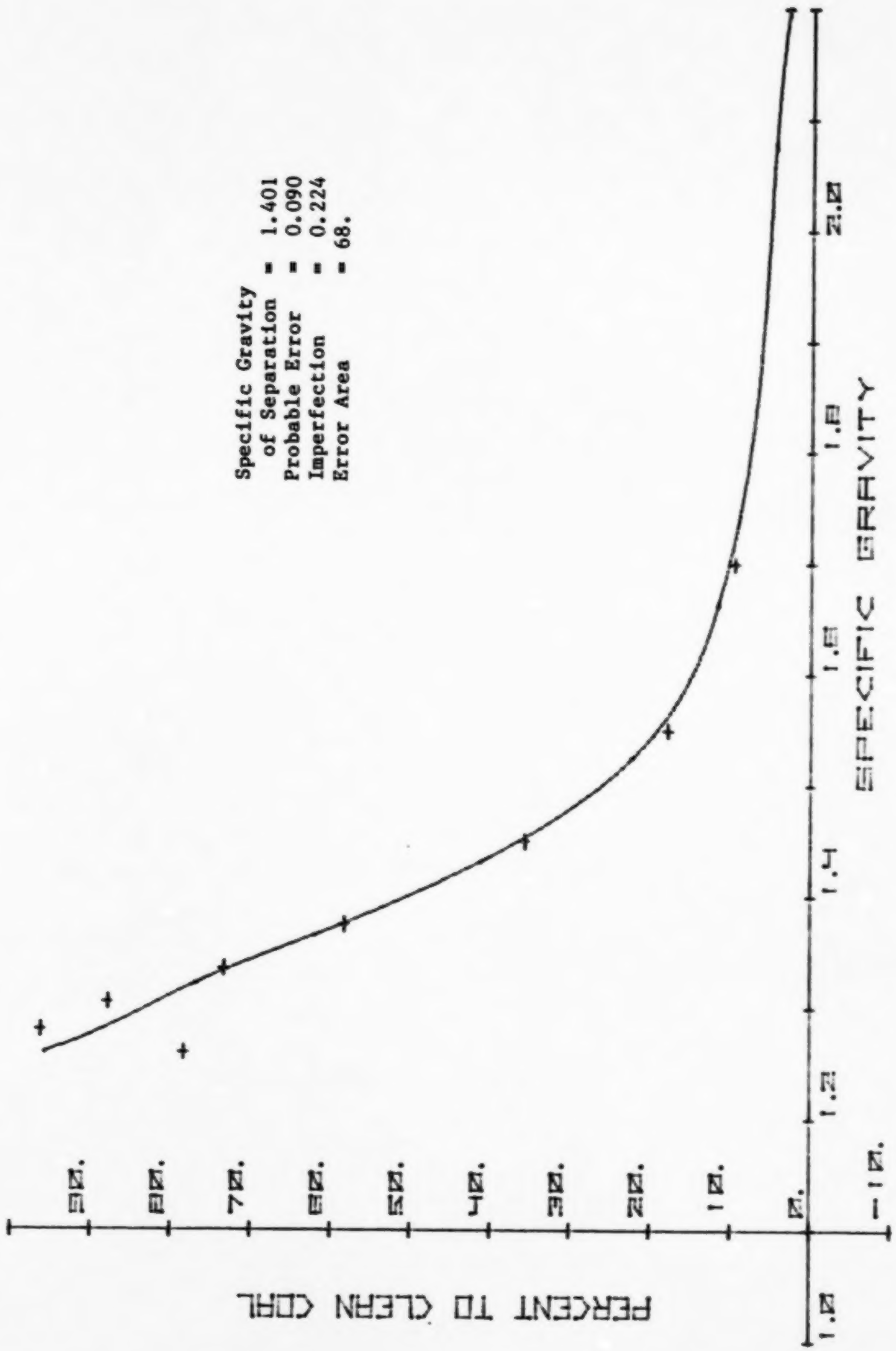
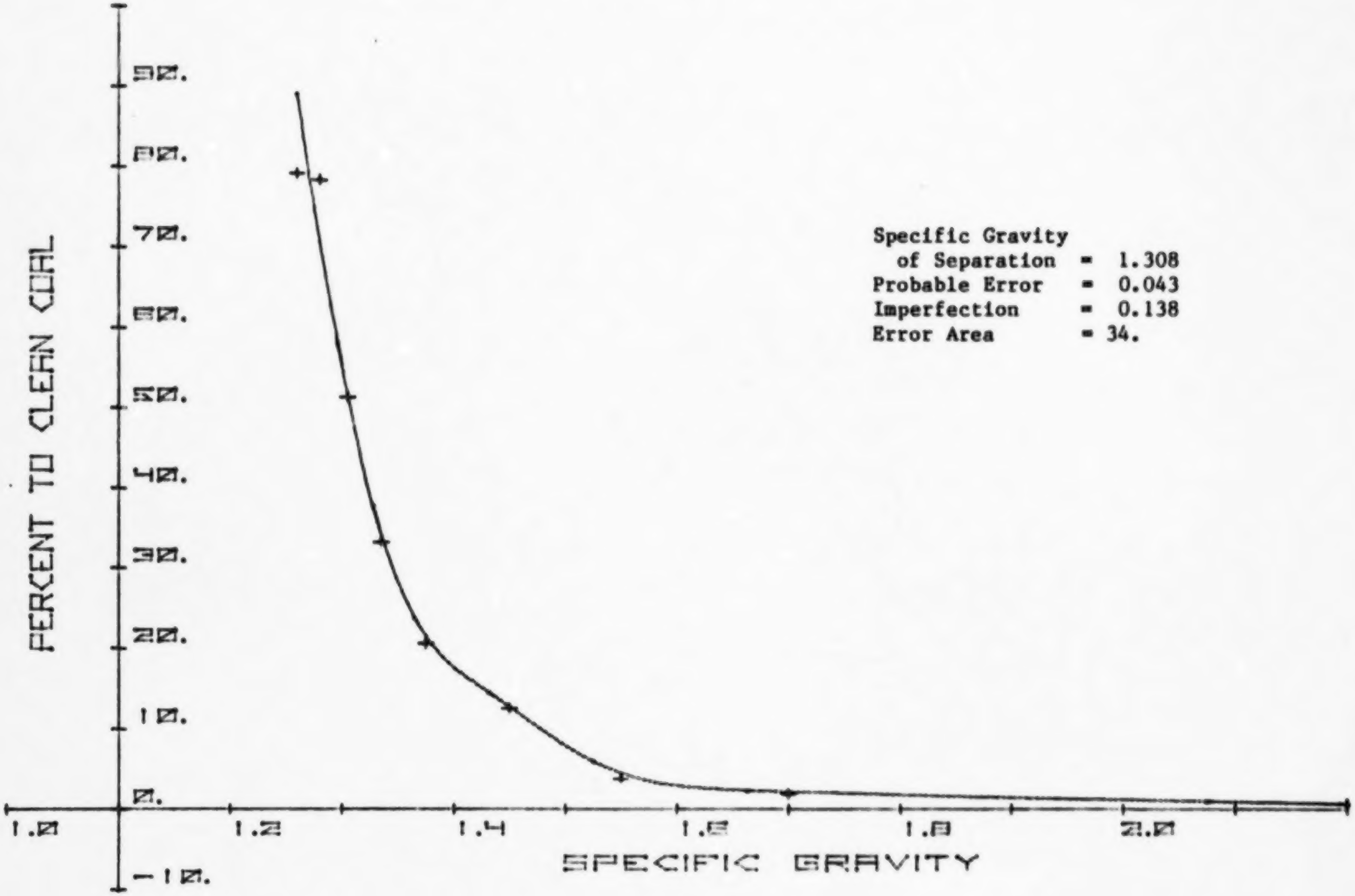


Figure A-143. Distribution Curve, Test No. 106 (28 x 100)



Specific Gravity
  of Separation = 1.308
Probable Error = 0.043
Imperfection = 0.138
Error Area = 34.

Figure A-144. Distribution Curve, Test No. 106 (9 x 100)

Appendix B. Estimated Btu and Normalized Sulfur Content Values

TABLE B-1. ESTIMATED BTU VALUES BY PARTICLE SIZE RANGE

	9 X 14		14 X 28		28 X 100	
	BTU / Lb.		BTU / Lb.		BTU / Lb.	
Test No.	Feed Coal	Clean Coal	Feed Coal	Clean Coal	Feed Coal	Clean Coal
71	9725.	15127.	10987.	15110.	11076.	15181.
72	10880.	15003.	11662.	14986.	9992.	14559.
73	9815.	15199.	11325.	15181.	11130.	14950.
74	11094.	15021.	11361.	15021.	10543.	14879.
75	9815.	15110.	11040.	15110.	10845.	14826.
76	10525.	15128.	10845.	15110.	10845.	14915.
77	9868.	14986.	11094.	15039.	10827.	14719.
78	10845.	15181.	12018.	15163.	11378.	14897.
79	10134.	15092.	11485.	15128.	10970.	14755.
80	9957.	15056.	10792.	14541.	11432.	14577.
81	10756.	15021.	11449.	15021.	11023.	14630.
82	10970.	15163.	11645.	15181.	11325.	15003.
83	11023.	15181.	11449.	15199.	11432.	15057.
84	11378.	15234.	11378.	15128.	11538.	15075.
85	10934.	15199.	11805.	15199.	11556.	15039.
86	10490.	15003.	11361.	14968.	11271.	14845.
87	10792.	15234.	11804.	15217.	11805.	15075.
88	11449.	15039.	11716.	14968.	11449.	14701.
89	11467.	15057.	11983.	15021.	12178.	14790.
90	11485.	15092.	12018.	15057.	12480.	14897.
91	12106.	15110.	12373.	15181.	12462.	14986.
92	11290.	15075.	12125.	15039.	12089.	14808.
93	11574.	15163.	11982.	15163.	12284.	14950.
94	11716.	15057.	11983.	15163.	12036.	14755.
95	11165.	15056.	12000.	15075.	12213.	14719.
96	10880.	15181.	11911.	15234.	12053.	14849.
97	11556.	15217.	12142.	15199.	12355.	15198.
98	11645.	15092.	12036.	15128.	11840.	14861.
99	11627.	15092.	12355.	15057.	12604.	14897.
100	11840.	15181.	12267.	15110.	12444.	14915.
101	11200.	15003.	12373.	15092.	12338.	14808.
102	11663.	15057.	12231.	14986.	12302.	14808.
103	11201.	14932.	12782.	14861.	12071.	14595.
104	11378.	15128.	11947.	15110.	12409.	14805.
105	11182.	15075.	12125.	15039.	12409.	14772.
106	11521.	14986.	12267.	14968.	12356.	14666.

TABLE B-2. NORMALIZED SULFUR VALUES BY PARTICLE SIZE RANGE

	9 X 14		14 X 28		28 X 100	
	Lb. Sulfur/MM BTU		Lb. Sulfur/MM BTU		Lb. Sulfur/MM BTU	
Test No.	Feed Coal	Clean Coal	Feed Coal	Clean Coal	Feed Coal	Clean Coal
71	4.30	0.58	3.37	0.44	5.07	0.72
72	3.07	0.70	2.90	0.64	6.18	0.84
73	3.59	0.69	2.98	0.72	3.33	0.61
74	2.80	0.73	3.38	0.68	5.12	0.79
75	3.32	0.64	3.09	0.30	4.59	0.67
76	3.33	0.61	3.30	0.59	5.03	0.77
77	3.75	0.63	3.22	0.64	4.66	0.74
78	3.42	0.68	2.56	0.59	4.16	0.74
79	3.70	0.60	3.01	0.59	5.21	0.76
80	3.90	0.58	3.70	0.68	4.53	0.84
81	3.16	0.67	3.09	0.57	4.62	0.90
82	3.00	0.48	2.94	0.46	4.08	0.67
83	2.88	0.59	3.24	0.66	4.20	0.58
84	3.17	0.73	3.28	0.77	4.40	0.60
85	2.79	0.53	2.63	0.53	4.09	0.59
86	3.18	0.79	2.96	0.61	3.96	0.61
87	2.94	0.56	2.37	0.51	3.30	0.75
88	2.39	0.63	2.92	0.66	3.99	0.73
89	2.07	0.58	2.10	0.57	2.56	0.66
90	2.00	0.68	2.18	0.66	2.16	0.63
91	2.17	0.75	2.25	0.61	2.56	0.64
92	2.72	0.67	2.27	0.51	3.10	0.67
93	2.45	0.67	2.71	0.67	2.98	0.68
94	2.30	0.76	2.57	0.77	3.06	0.81
95	2.55	0.52	2.51	0.54	2.79	0.70
96	3.22	0.91	2.73	0.95	3.54	0.88
97	2.35	0.63	2.48	0.62	2.96	0.50
98	2.49	0.64	2.45	0.60	3.12	0.90
99	2.46	0.69	2.33	0.87	2.70	0.81
100	2.25	0.60	1.99	0.52	2.59	0.68
101	2.14	0.64	1.71	0.63	1.95	0.70
102	1.90	0.63	1.79	0.69	1.97	0.72
103	1.24	0.70	1.77	0.65	2.16	0.86
104	2.17	0.64	1.97	0.66	1.93	0.72
105	2.12	0.64	1.82	0.65	1.88	0.66
106	2.01	0.69	1.74	0.68	1.89	0.75

APPENDIX C

CUBIC SPLINE METHODS

The fitting of curves to a set of data may be considered from several different points of view. For example, if the data are fairly precise (i.e., the error in the data is relatively small), one would try to fit a smooth curve to the points (much as a draftsman does with a French curve or a spline). Unfortunately, the draftsman's approach to curve fitting is highly subjective. Reading values from the resulting curve could be difficult and subject to human error. On the other hand, classical least-squares methods using high-degree polynomials tend to produce undulating curves and overly complicated functional expressions.

The cubic-spline method seems to be emerging as an effective method of fitting curves. Obviously, the word "spline" arises from the draftsman's device. A draftsman's spline is a flexible (typically plastic or rubber) strip which can be bent and held by weights so that it passes through each of the given data points while being smoothed from each point to the next. The following section briefly outlines the mathematical equivalent of this device, namely, the laws of thin beam flexure.

The basic conditions for defining a cubic spline are as follows:

(1) Given an ordered set of n data points:

$(x_i y_i)$ $i = 1,2,\ldots n$ where $x_1 < x_2 < \ldots < x_n$.

(2) A cubic spline, $s(x)$, is a function which consists of a set of $(n-1)$ cubic polynomials such that:

(a) In the i-th interval $[x_i, x_{i+1}]$, $s(x)$ is of the form:

$$s_i(x) = a_i(x-x_i)^3 + b_i(x-x_i)^2 + c_i(x-x_i) + d_i$$

(b) $s(x_i) = s_i(x_i) = y_i$ for $i = 1,2,\ldots,n-1$

(c) the two cubics immediately to the left and right of any x_i ($i = 2,3,\ldots,n-1$) have the same value at x_i, the same slope at x_i, the same curvature at x_i, and

(d) $s(x_n) = s_{n-1}(x_n) = y_n$.

In particular, by writing the mathematical equations for the three conditions stated in (c), and using (b) and (d) above, a set of 4n-6 linear equations in the 4n-4 unknowns (the a's, b's, c's, and d's) is obtained. In order to solve the resulting system of equations, two other conditions are needed. For the problem of fitting dense media cyclone distribution data, these two end conditions were chosen to be that the slope of the cubic spline at x_1 (and x_n) is a linear extrapolation of the slope from x_2 and x_3 (and x_{n-2} and x_{n-1}). Although other end conditions could have been selected, BCR felt that the extrapolation of the slope was appropriate for this curve-fitting problem.

The expanded system of 4n-4 linear equations in 4n-4 unknowns can be solved by standard matrix techniques and the resulting set of coefficients (a's, b's, c's, and d's) defines a unique cubic spline. Further details regarding the specific equations and methods for solving such systems of equations can be found in Gerald (1970) or Ahlberg, et al. (1967), for example.

The methodology described above defines a cubic spline as mathematically equivalent to a draftsman's spline, and, as such, passes through the n data points, (x_i, y_i) $i = 1,2,\ldots,n$. Unfortunately, the distribution data from the dense media cyclone are not highly precise. In fact, in several cases the distribution data values increase as the specific gravity increases--which cannot be physically correct, since the separation process is by its very nature monotone decreasing as specific gravity of separation increases.

Obviously, such inconsistencies can arise from incorrect operations, as well as from measurement and computation errors. In any case, the cubic-spline technique must be modified to produce a curve which is smoothing in nature rather than being interpolatory. The cubic-spline smoothing technique used with the PENELEC data is as follows:

(1) One additional (left) end point was added to the data set; namely, (1.25, 100). This point served only to get the curve started correctly. On occasion, this end point was adjusted downward to $(1.25, y_1)$ in order to provide a more suitable fit in the upper portion of the curve.

(2) The nine observed distribution percent values were assigned the following specific-gravity values: 1.26, 1.28, 1.305, 1.335, 1.375, 1.45, 1.55, 1.70, 2.20.

(3) The nine points were smoothed by the following linear procedure:

(a) Given (x_i, y_i) $i = 1,2,\ldots,n$

$$\text{Define } (w_i, z_i) = \left(\frac{x_i + x_{i+1}}{2}, \frac{y_i + y_{i+1}}{2}\right) \quad i = 1,2,\ldots,n-1$$

(b) $$\text{Define } (x'_k, y'_k) = (x_1, y_1), \left(\frac{w_i + x_{i+1}}{2}, \frac{z_i + y_{i+1}}{2}\right), (x_n, y_n)$$

$$k = 1,2,\ldots,n+1; \; i = 1,2,\ldots,n-1$$

The values (x'_k, y'_k) are used in lieu of the original (x_i, y_i) $i = 1,2,\ldots,n$ and are subsequently input to the cubic-spline methodology. The resulting cubic spline does not pass through the original data values but rather smooths out the fluctuation. For some data sets, selected adjacent (x_i, y_i) pairs were averaged prior to smoothing in order to remove the more erratic fluctuations from the data.

Evaluation of the spline function $s(x)$ is straightforward; to find the value of $s(x)$ for any particular x value, say x^*,

(1) find the value of i for which

$$x'_k \le x^* < x'_{k+1}$$

where (x'_k, y'_k) $k = 1,2,\ldots,n+1$ is the smoothed data set.

Then

(2) $s(x^*) = a_k (x^*-x'_k)^3 + b_k(x^*-x'_k)^2 + c_k(x^*-x'_k) + d_k$

can be directly calculated by the computer.

Hence, given any specific gravity and the n x 4 matrix of coefficients (a's, b's, c's, and d's) the corresponding value of percent to clean coal can be calculated by using equation (2) above.

To find the inverse value for any given distribution value, say y^* where $0 \le y^* \le 100$; simply find k such that

$$y'_k \ge y^* > y'_{k+1}$$

then use the interval halving (bisection) method to find the value of x, say x^*, such that

$$y'_k = s(x'_k) \ge y^* = s(x^*) > y'_{k+1} = s(x'_{k+1}).$$

Thus, given any distribution percentile ($0 \le y^* \le 100$), this process results in a corresponding specific gravity.

Parameters needed to compute values for the independent criteria in evaluating dense media cyclone performance, such as probable error ($w = 25$ and $w = 75$), specific gravity of separation ($w = 50$), etc., were found via the procedure outlined above. Integration of areas, such as when computing error area, were determined by direct algebraic integration of the explicit formulas for each subinterval.

APPENDIX D

MULTIPLE REGRESSION ANALYSIS

Regression analysis consists of a set of statistical procedures for approximating the relationship between two or more variables so as to predict one variable from the other(s). A regression model expresses the tendency of a dependent (response) variable to vary systematically with the independent (predictor) variable(s).

A basic regression model with only one independent variable can be stated as follows:

$$Y_i = \beta_o + \beta_1 X_i + \varepsilon_i$$

where:

Y_i is the value of the dependent variable in the ith trial,

X_i is the value of the independent variable in the ith trial,

β_o and β_1 are model parameters, and

ε_i is the random error in the ith trial.

Regression models containing more than one independent variable are called multiple regression models. A multiple regression model is, therefore, an extension of the above model and has the general form:

$$Y_i = \beta_o + \beta_1 X_{i1} + \beta_2 X_{i2} + \dots + \beta_k X_{ik} + \varepsilon_i$$

Multiple regression modeling uses standard least-squares techniques and sample values for the dependent and independent variables to calculate estimates of the regression parameters (β_o, β_1, etc.). The result is the "best fit" equation of the form:

$$Y = b_o + b_1 X_1 + b_2 X_2 + \dots + b_k X_k$$

where b_o, b_1, b_2, ..., b_k are the regression coefficients, i.e., parameter estimates.

In applying regression analysis, there is a practical need to limit the number of independent variables in the regression model. Consequently, a central task in building regression models is to determine a set of independent variables which is "best" in some sense. This choice is usually based upon the degree to which a selected variable contributes to reducing the remaining (unexplained) variation in the dependent variable after allowing for the contributions of other independent variables already in the regression model. Additional considerations include the importance of the variable as a causal agent in the process being analyzed; how accurately, quickly, or economically observations on the variable can be obtained as compared to other independent variables; and the extent to which the variable can be controlled during the process under investigation.

The following methods are fairly standard techniques found in many books on mathematical statistics.[1] The statistical procedure used to select the terms for a particular model determines the advantage of using an i + 1 term regression equation in lieu of an i term equation. A description of this procedure is provided below.

Suppose that n is the number of sample data points, Y is the dependent variable, and k independent variables, X_1, X_2, ..., X_k, affect Y. As usual, let the variation in Y be calculated by the sample variance

$$V(Y) = \frac{1}{n} \sum_{i=1}^{n} (Y_i - \overline{Y})^2 \text{ where } \overline{Y} = \frac{1}{n} \sum_{i=1}^{n} Y_i$$

---

[1] See bibliography

Note that $\overline{Y}$ is the sample mean of the dependent variable Y. Given that a regression equation, $\hat{Y} = b_o + b_1X_1$, has been determined for relating Y to the variable X_1, the ability of this equation (and hence, X_1) to explain the variation in Y is measured by the correlation coefficient for Y and X_1, denoted as r_{y1}, which satisfies the relationship

$$r_{y1}^2 \left(\sum_{i=1}^{n} (Y_i - \overline{Y})^2 \right) = \left(\sum_{i=1}^{n} (\overline{X}_{1i} - \overline{X}_1)(Y_1 - \overline{Y}) \right)^2 \Bigg/ \sum_{i=1}^{n} (X_{1i} - \overline{X}_1)^2$$

In a similar manner, the benefit in using a two-variable regression equation of the form $Y = c_o + c_1X_1 + c_2X_2$ instead of the single variable regression equation, $Y = b_o + b_1X_1$, can be measured by

$$\left(R_{y12}^2 - r_{y1}^2 \right) \sum_{i=1}^{n} (Y_i - \overline{Y})^2$$

where R_{y12} is the multiple correlation coefficient between Y and the independent variables X_1 and X_2.

The relative ability of the variable X_1 (and hence, the regression equation $\hat{Y} = b_o + b_1X_1$) to explain the variation in Y is computed as the F-ratio of the variation in Y accounted for by the equation to the remaining unexplained variation in Y; namely,

$$F(1, n-2) = \left(\frac{r_{y1}^2 \sum_{i=1}^{n} (Y_1 - \overline{Y})^2}{1} \right) \Bigg/ \left(\frac{(1 - r_{y1}^2) \sum_{i=1}^{n} (Y_i - \overline{Y})^2}{n - 2} \right) = \frac{(n-2)\, r_{y1}^2}{(1 - r_{y1}^2)}$$

The denominator has n-2 degrees of freedom since two parameters, β_o and β_1, are estimated in obtaining $\hat{Y}_i$.

The relative ability of X_2 to explain additional variation in Y is similarly given by the F-ratio of the additional variation explained to the remaining unexplained variation in Y; namely,

$$F(1, n-3) = \left(\frac{(R^2_{y12} - r^2_{y1}) \sum_{i=1}^{n} (Y_i - \overline{Y})^2}{1}\right) \Bigg/ \left(\frac{(1 - R^2_{y12}) \sum_{i=1}^{n} (Y_i - \overline{Y})^2}{n - 3}\right)$$

$$= \frac{(n-3)\ (R^2_{y12} - r^2_{y1})}{(1 - R^2_{y12})}$$

Since another independent variable was added to the model, the denominator has n-3 degrees of freedom. The significance of using three independent variables can be tested using the analogous F-ratio:

$$F(1, n-4) = \frac{(n-4)\ (R^2_{y123} - R^2_{y12})}{(1 - R^2_{y123})}$$

where R_{y123} is the multiple correlation coefficient between Y and the variables X_1, X_2 and X_3, and so on for four or more independent variables.

The F-distribution is a theoretical distribution for evaluating hypotheses about either (1) the equality of variances or (2) the equality of more than two means. The F-ratio is used here as a criterion for judging how well various regression functions fit an observed set of data. Specifically, it provides a method for describing the variation observed in a dependent variable in terms of other variables.

Mathematically, the F-distribution is defined as follows: if U_1 and U_2 are two independent random variables having chi-square distributions with v_1 and v_2 degrees of freedom respectively, then the distribution of the ratio

$$F = \frac{(U_1/v_1)}{(U_2/v_2)}$$

is called the F-distribution with v_1 and v_2 degrees of freedom.

In terms of building regression models, the F-ratio is utilized to evaluate the hypothesis that the inclusion of an additional independent variable, say X_i, in the model significantly reduces the unexplained variation in the dependent variable. Thus, for a given confidence level C, the statistical significance of using one independent variable, X_1, is determined by comparing the computed value of $F(1, n-2)$ to the theoretical F-value $F_c(1, n-2)$ obtained from any standard F-table. If $F(1, n-2)$ is greater than $F_c(1, n-2)$, then the independent variable is statistically significant and should be considered for inclusion in the model. Similarly, the appropriateness of using two independent variables instead of one is determined by comparing the computed $F(1, n-3)$ to $F_c(1, n-3)$. Again if $F(1, n-3)$ is greater than $F_c(1, n-3)$, then the inclusion of the second independent variable should also be considered. This process is continued until the computed value of the F-ratio, $F(1, n-i)$, is less than the tabulated critical value, $F_c(1, n-i)$.

For dense media cyclone data, the dependent variables (Y_i's) in the above discussion are the performance criteria values (ash error, percent sulfur in clean coal, error area, etc.). These values were obtained directly from the data base or computed from the distribution curves for each test. The independent variables (X_i's) are values for the cyclone operating conditions (flow rate, orifice size, media-to-coal ratio, and particle size range). Since media-to-coal ratios were nominal values (3:1, 5:1, 7:1), BCR used the percent coal in slurry for each test to quantify this variable. For particle size range, the mesh size corresponding to the medians of the three size ranges (9 x 14, 14 x 28, and 28 x 100) were converted to millimeters (1.590, .890, and .375). These values were used for all subsequent calculations.

Multiple regression models were constructed for each cyclone performance criterion using the following stepwise procedure:

(1) A confidence level of 95 percent was selected.

(2) For each independent variable (X), the one variable regression model, $Y = aX + b$, was computed.

(3) The independent variable, say X_1, having the "best" correlation (linear correlation coefficient) was selected for possible inclusion in the "final" model.

(4) The statistical significance of the resulting regression equation, say $Y = b_o + b_1X_1$, obtained for the "candidate" variable, X_1, was determined by comparing its computed F-ratio, $F(1, n-2)$ to the F-ratio critical value, $F_c(1, n-2)$ as described in the above subsection.

(5) If the "best" one variable model was significant, the candidate X_1 was included in the ensuing models.

(6) All models with X_1 and any other independent variable were then generated. Thus, all models of the form $Y = cX_1 + dX + f$ were computed.

(7) By comparing multiple correlation coefficients, the best model with two variables X_1, and say $X = X_2$, was identified. The significance

of this model, $Y = c_o + c_1X_1 + c_2X_2$, was determined by the comparison of the computed F-ratio and the F-ratio critical value.

(8) Again, if the best model with two independent variables was significant, all three variable models of the form $Y = gX_1 + hX_2 + mX + p$ were generated, and so forth.

(9) The above process was continued until a "candidate" best variable, say X_i, and a corresponding model of the form

$$Y = a_o + a_1X_1 + a_2X_2 + \ldots + a_iX_i$$

was obtained for which the computed F-ratio, F(1, n-i-1), was less than the tabular critical value $F_c(1, n-i-1)$. In this situation the "candidate" variable X_i was not added to the model and the preceding i-1 term model was selected as the final model.

(10) With the model finalized, its overall significance was determined by squaring the multiple correlation coefficient to obtain the coefficient of multiple determination. This value represents the fraction of the total variation in Y explained by the final model.

(11) The standard error of estimate (SEE) was computed for the final model using the following equation:

$$SEE = S_y\sqrt{1-R^2}$$

where S_y = standard deviation of observed Y values; i.e., the square root of the variance of the Y's

R^2 = coefficient of multiple determination

This value is a measure of the variability of the residual (predicted-observed) values obtained from using the final regression model.